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Note on Water Drag on Sea Ice for Different Surface Roughness

The approach in Myrhaug [1], where a simple analytical theory describing the motion in a turbulent planetary boundary layer near a rough seabed was presented, is extended to smooth and transitional smooth-to-rough turbulent flow. An inverted boundary layer similar to that at the seabed is applicable under the sea ice. The water drag coefficient at the ice surface and the direction of the surface shear stress are presented for rough, smooth and transitional turbulent flows.

Introduction

Knowledge of the water drag is important for accurate determination of the dynamics of sea ice. Although the wind is the primary force that drives the ice, a numerical model of ice motion must also include the influence of water drag against the ice. Other contributions to the force exerted on the ice are from waves, sea surface tilt and internal ice stress. The modeling of the vertical structure of the current under drifting pack ice is analogous to the modeling of the vertical structure of the bottom boundary layer on the continental shelves. This is in the most general and complex case dominated by several interacting physical effects. Among these effects are the earth's rotation, tidal effects, stratification due to salinity and temperature gradients and suspended sediments, internal friction in the fluid and topographical effects.

One important feature of the vertical structure of the current boundary layer under drifting sea ice is determined by the influence of planetary rotation on steady, horizontally uniform, unbounded and unstratified flow. In this case there is a balance between the driving pressure gradient, the momentum flux gradient and the Coriolis force. This idealized boundary layer flow may occur away from any coasts and in the region below the sea ice and above currents which are steady over times comparable with the inertial period, $2\pi/f$, where $f = 2\Omega \sin \psi$ is the Coriolis parameter. Here Ω is the earth's angular frequency of rotation and ψ the latitude.

In most cases the boundary layer flow is in the rough turbulent flow regime; but it also appears in practice that the flow conditions might be in the smooth and the transitional smooth-to-rough turbulent flow regimes. Water drag data on sea ice reported by Untersteiner and Badgley [2] demonstrate that this can occur. Obviously most of the data were in the rough and the transitional smooth-to-rough turbulent flow regimes.

This note represents an expansion of and a supplement to Myrhaug [1], where a simple analytical theory which describes the motion in a turbulent planetary boundary layer near a rough seabed by using a two-layer eddy viscosity approach was presented. An inverted boundary layer similar to that at the seabed is applicable under the sea ice. The approach for rough turbulent flow is extended to smooth and transitional smooth-to-rough turbulent flow.

The Planetary Boundary Layer Model for Transitional Turbulent Flow

The intention here is, as it was in Myrhaug [1], to eventually invert an ocean bottom boundary layer in order to model the boundary layer under drifting sea ice. The formulation of the problem, i.e., the equation of motion, boundary conditions and eddy viscosity approach, are given in Myrhaug [1] and follows that for an ocean bottom boundary layer. Consequently, the details will not be represented here. However, the boundary condition at the seabed is specified at $z = z_0$, which for transitional smooth-to-rough turbulent flow is given by

$$z_0 = \frac{k}{30}\left(1 - e^{-(ku_*/27\nu)}\right) + \frac{\nu}{9u_*} \tag{1}$$

according to Christoffersen and Jonsson [3]. Equation (1) is obtained from a fit to the laboratory data points in Schlichting [4, Fig. 20.21, p. 620]. Here, ν is the kinematic viscosity of the fluid and k the Nikuradse's equivalent sand roughness for the surface, that is, the characteristic dimension of the physical roughness of the surface. However, k may be very different from what the physical roughness of the surface would suggest. Soulsby [5] has given a detailed discussion on how k is determined for various seabed conditions. $u_* = (\tau_0/\rho)^{1/2}$ is the friction velocity at the seabed. τ_0 is the magnitude of the shear stress at the seabed and ρ is the density of the fluid.

Table 1 gives the turbulent flow regimes according to Schlichting [4] and the given and dimensionless quantities associated with the three flow regimes. Here, Ro is the Rossby number, and Re is the Reynolds number based on the magnitude of the geostrophic velocity $|R_\infty|$ and the characteristic length $|R_\infty|/f$. Another length scale which is commonly used in the scaling of planetary boundary layer flow is u_*/f, which is a measure of the boundary layer thickness. The surface Rossby number and the surface Reynolds number associated with this length scale are defined as $\mathrm{Ro}_* = 30u_*/fk$ and $\mathrm{Re}_* = u_*^2/f\nu$, respectively. However, here the geostrophic Rossby

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Table 1 Turbulent flow regimes with given and dimensionless quantities

Turbulent flow regimes		Given quantities	Dimensionless quantities
Smooth:	$0 < ku_*/\nu < 5$	$\lvert R_\infty \rvert, f, \nu$	$\mathrm{Re} = \lvert R_\infty \rvert^2/f\nu$
Transition:	$5 < ku_*/\nu < 70$	$\lvert R_\infty \rvert, f, \nu, k$	Re, Ro
Rough:	$ku_*/\nu > 70$	$\lvert R_\infty \rvert, f, k$	$\mathrm{Ro} = 30\lvert R_\infty \rvert/fk$

number and geostrophic Reynolds number as defined in Table 1 are used. The reason for this will be given subsequently.

For large and small values of ku_*/ν, equation (1) reduces to $z_0 = k/30$ and $z_0 = \nu/9u_*$ for rough and smooth turbulent flow, respectively. It should be noted that, according to Soulsby [5], there is not yet sufficient evidence to confirm the use of the factors 30 and 9 in these expressions for the sea. However, since no definite conclusions have been made about this, these factors will be used herein. These results are based on the assumption of the validity of the logarithmic velocity profile close to the surface. Thus, the boundary condition is taken at a fixed level $z = z_0$ above the seabed rather than at $z = 0$. The boundary condition at the seabed is taken in analogous form to that for steady, unidirectional fully turbulent flow. As long as fully turbulent flow conditions are considered, the Reynolds shear stresses are of greater magnitude than the viscous stresses close to the seabed. That is, the kinematic eddy viscosity predominates close to the seabed. Thus, the total shear stress is given by its components τ_{xz} and τ_{yz} along horizontal orthogonal axes x and y, respectively, as

$$\tau_{xz} = \rho\epsilon\frac{\partial U}{\partial z}$$

and

$$\tau_{yz} = \rho\epsilon\frac{\partial V}{\partial z}$$

where U and V are velocities along x and y, and the kinematic eddy viscosity ϵ is assumed to be the same in both horizontal directions. ϵ is as specified in Myrhaug [1], equations (13) and (14).

Measurements by Sternberg [6, 7] have indicated the value $ku_*/\nu = 165$ for the critical value for rough turbulent flow (see also, Soulsby [5]). However, this value is based on limited data. Thus, consistent with equation (1), the criteria in Table 1 will be used in this approach.

A friction or drag coefficient associated with the friction velocity at the seabed can be defined by

$$C_D = \frac{\tau_0}{\rho\lvert R_\infty \rvert^2} = \left(\frac{u_*}{\lvert R_\infty \rvert}\right)^2 \qquad (2)$$

It should be noted that the present approach covers flow over a surface with a given constant roughness and not flow near a surface with a step change in the roughness.

The velocity and shear stress profiles can now be calculated according to the solution given in Myrhaug [1] by using equation (1). However, in order to do these calculations, the friction velocity at the seabed has to be known. u_* is determined from

$$u_* = \tfrac{1}{2}\kappa(1 - \xi_0^2)\cdot[\chi(\xi_0; u_*)\cdot\hat{\chi}(\xi_0; u_*)]^{1/2} \qquad (3)$$

where $\xi_0 = 20z_0f/u_* - 1$ and $\kappa = 0.4$ is von Karman's constant. $\hat{\chi}$ denotes the complex conjugate of χ, which is a complex quantity given by equation (30) in Myrhaug [1] having dimension of velocity. Equation (3) is an implicit equation for determination of u_*. For a given surface roughness condition, free stream current velocity and Coriolis parameter, u_* can be determined from equation (3) by iteration. Then C_D is given from equation (2).

Results and Discussion

It should be noted that when the present results are inverted to the boundary layer under drifting sea ice, the velocities solved are for the current speeds relative to the ice velocity. Thus, the geostrophic velocity is the geostrophic velocity relative to the ice velocity, which should be used in the definition of the drag coefficient in equation (2).

Figures 1 and 2 show the water drag coefficient at the ice surface (C_D) and the direction of the surface shear stress (ϕ_0), respectively, versus Re for smooth turbulent flow. In the same figures, C_D and ϕ_0 versus Ro for rough turbulent flow and C_D

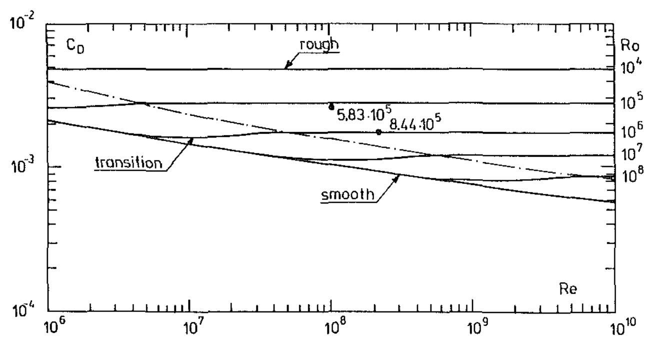

Fig. 1 Water drag coefficient on sea ice (C_D) versus Reynolds number ($\mathrm{Re} = \lvert R_\infty \rvert^2/f\nu$) and Rossby number ($\mathrm{Ro} = 30\lvert R_\infty \rvert/fk$): •McPhee and Smith's [8] data, numbers refer to Ro-values; —·—· lower limit for rough turbulent flow, see equation (4). The transition zone is represented by only the portions of the curves between the smooth curve and the lower limit for rough turbulent flow. Note that $\lvert R_\infty \rvert$ is the magnitude of the geostrophic velocity relative to the drifting ice velocity.

Nomenclature

C_D = drag coefficient
$f = 2\Omega\sin\psi$ = Coriolis parameter
$i = (-1)^{1/2}$
k = Nikuradse's equivalent sand roughness parameter
$R = U + iV$ = complex velocity
$\lvert R_\infty \rvert$ = magnitude of geostrophic velocity
$\mathrm{Re} = \lvert R_\infty \rvert^2/f\nu$ = geostrophic Reynolds number
$\mathrm{Re}_* = u_*^2/f\nu$ = surface Reynolds number
$\mathrm{Ro} = 30\lvert R_\infty \rvert/fk$ = geostrophic Rossby number
$\mathrm{Ro}_* = 30u_*/fk$ = surface Rossby number
u_* = friction velocity
U, V = velocities along x and y, respectively
x, y, z = Cartesian coordinates
z_0 = roughness parameter, see equation (1)
ϵ = kinematic eddy viscosity
$\kappa = 0.4$ = von Karman's constant
ν = kinematic viscosity of fluid
$\xi_0 = 20z_0f/u_* - 1$
ρ = density of fluid
τ_{xz}, τ_{yz} = shear stress along x and y, respectively
τ_0 = magnitude of shear stress at surface
χ = complex function
ϕ_0 = direction of surface shear stress
ψ = latitude
Ω = the earth's angular frequency of rotation

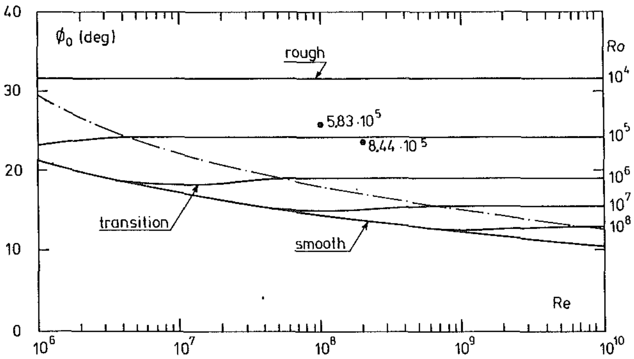

Fig. 2 Direction of surface shear stress (ϕ_0) versus Reynolds number (Re) and Rossby number (Ro): symbols and explanations as in Fig. 1

and ϕ_0 versus Re and Ro for transitional smooth-to-rough turbulent flow are shown, together with the data from McPhee and Smith [8] (here $\nu = 0.019$ cm^2/s has been used), which represent rough turbulent flow conditions. The rough turbulent flow results and the main parameters and results associated with McPhee and Smith's data were presented in Myrhaug [1]. By presenting C_D versus Ro and Re, C_D is given explicitly in terms of the given parameters. Otherwise, if C_D is given versus Ro_* and Re_*, C_D is given implicitly since u_* is part of the solution.

The lower limit of rough turbulent flow as given in Table 1 is also shown in the figures, which can be expressed as

$$\frac{ku_*}{30\nu} = \frac{\text{Re}}{\text{Ro}} \sqrt{C_D} = 2.3 \tag{4}$$

For rough turbulent flow $C_D = C_D$ (Ro) is known, and thus the lower limit of rough turbulent flow is known from equation (4). Similarly, the upper limit of smooth turbulent flow as given in Table 1 can be expressed as

$$\frac{ku_*}{30\nu} = \frac{\text{Re}}{\text{Ro}} \sqrt{C_D} = 0.17 \tag{5}$$

For smooth turbulent flow, $C_D = C_D$ (Re) is known, and thus the upper limit of smooth turbulent flow is known from equation (5).

For comparison, it should be noted that the laminar drag coefficient according to the classical solution by Ekman [9] is given by $C_D = \text{Re}^{-1/2}$ and with $\phi_0 = 45$ deg.

From Figs. 1 and 2, it appears that a continuation of the "rough" lines represents an adequate approximation to the "transitional" results. For engineering applications, this suggests that all surfaces which are not smooth could be considered rough.

Another simple approach to water drag on sea ice by using similarity theory has recently been presented and discussed in Myrhaug [10, 11]. This approach gives the same qualitative behavior for C_D and ϕ_0 as those obtained here.

Summary and Conclusions

This note represents an expansion of and a supplement to the approach in Myrhaug [1], where a simple analytical theory describing the motion in a turbulent planetary boundary layer near a rough seabed by using a two-layer eddy viscosity model was presented. An inverted boundary layer similar to that at the seabed is applicable under the sea ice. The approach for rough turbulent flow is extended to smooth and transitional smooth-to-rough turbulent flow. The water drag coefficient at the ice surface (C_D) and the direction of the surface shear stress (ϕ_0) are presented. For rough, smooth and transitional turbulent flows, C_D and ϕ_0 versus the Rossby number Ro, C_D and ϕ_0 versus the Reynolds number Re, and C_D and ϕ_0 versus Re and Ro, are presented. The present results suggest for engineering applications that all surfaces which are not smooth could be considered rough.

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