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Aperiodically Poled Nonlinear Crystals as Sources of Multi-Frequency Laser Radiation

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The new multi-frequency process, which consists of three coupled nonlinear optical interactions: two parametric down-conversions and one up-conversion, in aperiodically poled nonlinear crystal is investigated. The spatial dynamics of wave intensities is studied in detail. The possibility of secondary simplification of coupled equations for correct describing the dynamics of wave interactions is demonstrated. The optimal conditions for parametrical instability of the initial stage of wave interactions are found.

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1. Introduction

Recently the quasi-phase matched (QPM) wave interactions are widely used in nonlinear optics due to the development of the fabrication technique of periodically poled nonlinear crystals (PPNCs). It is possible to simultaneously realize two coupled processes of wave interactions due to selection of modulation period of nonlinear coefficient in PPNCs [1]. In this case the reciprocal vector of nonlinear coefficient must compensate the phase mismatches of two processes at once. It should be noted that the compensation may be achieved only for limited numbers of optical frequencies [2]. The same difficulties occur at simultaneous implementation of more than two wave interactions.

The simultaneous realization of many nonlinear processes can be achieved in aperiodically poled nonlinear crystals (APNCs), in which nonlinear coupling coefficient changes according to aperiodic law (see, for example, [3–8]). It allows compensating phase mismatches of several nonlinear processes. However, the present-day APNCs in the same way as PPNCs also allow to simpler realize only

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one three-frequency process of wave interactions and have difficulties in implementation of several nonlinear processes due to dispersion of crystals.

In this paper we study a new type of multi-frequency process in APNC, which is designed by using new simple aperiodic law.

2. The method of APNCs design for multi-frequency processes

The idea of fabrication of APNCs is based on the generalization of modulation rule, which is used for fabrication of PPNC with modulation period Λ:

$$g(z) = \mathrm{sign}[\sin(2\pi z/\Lambda)], \tag{1}$$

where $g(z)$ is the modulation function of nonlinear coefficient of PPNC; $\Lambda = 2\pi m/\Delta k$; Δk is the phase mismatch of wave vectors for three-frequency nonlinear process, which can be effectively realized in PPNC.

The generalization of Eq. (1) for the case of N nonlinear process has the following form:

$$g(z) = \mathrm{sign}\left[\sum_{j=1}^{N} a_j \sin(2\pi z/\Lambda_j)\right], \tag{2}$$

where $\Lambda_j = 2\pi m/k_j$; Δk_j is the phase mismatch of wave vectors for the j-th three-frequency process; N is the number of simultaneous implemented nonlinear processes; a_j is the numerical coefficient. Modulation function in Eq. (2) is

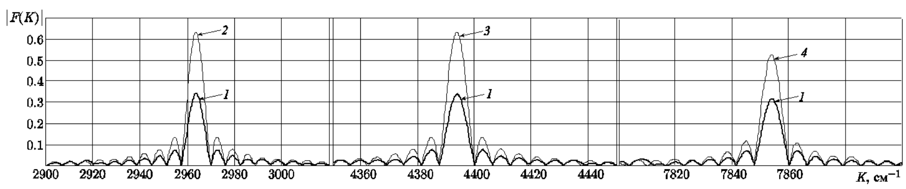

Fig. 1. Module of Fourier spectrum of function $g(z)$ of PPNC and APNC. *1* — $g(z) = \mathrm{sign}\left[\sum_{j=1}^{3}\sin(2\pi z/\Lambda_j)\right]$, *2* — $g(z) = \mathrm{sign}[\sin(2\pi z/\Lambda_1)]$, *3* — $g(z) = \mathrm{sign}[\sin(2\pi z/\Lambda_2)]$, *4* — $g(z) = \mathrm{sign}[\sin(2\pi z/\Lambda_3)]$.

aperiodic in general case, and its Fourier spectrum has spectral components for compensating phase mismatches for several processes. For example, Fig. 1 illustrates the Fourier spectrum

$$F(K) = \frac{1}{L}\int_0^L g(z)\exp(-\mathrm{i}Kz)\mathrm{d}z \tag{3}$$

of function $g(z)$ in APNC for implementation of three nonlinear processes ($N = 3$) in comparison with spectrum of function $g(z)$ in PPNC (see Eq. (1)). Here $\Lambda_1 = 21.2$ μm, $\Lambda_2 = 14.3$ μm, $\Lambda_3 = 8$ μm, which are necessary for each of multi--frequency process (see below). One can see that spectral components in APNC, which are referring to different periods, are approximately equal to 0.3, and the maximal absolute value of the Fourier spectral component for PPNCs is 0.63. Comparison of spectra of modulation functions in APNC and PPNC also shows that its spectral widths are approximately equal. This comparison is promising from the viewpoint of good efficiency of several simultaneous nonlinear processes in APNCs, which are created using the method under consideration.

3. The dynamics of three coupled wave process with five waves

We study the five-frequency process, which consists of simultaneous three nonlinear processes: (1) parametric down-conversion $\omega_1 \to \omega_2 + \omega_3$, (2) parametric up-conversion (sum-frequency generation) $\omega_1 + \omega_2 \to \omega_4$ and (3) parametric down--conversion $\omega_5 = 2\omega_1 \to \omega_3 + \omega_4$. The five waves take part in this process, and two of their frequencies are divisible. It is well known that correlated photons with different frequencies are generating in non-degenerated parametric processes [9]. Thus the process under consideration is of interest for obtaining multipartite entanglement photon states [10].

For waves with wavelengths $\lambda_1 = 1.064$ μm, $\lambda_2 = 2.129$ μm, $\lambda_3 = 2.127$ μm, $\lambda_4 = 0.709$ μm, and $\lambda_5 = 0.532$ μm the five-frequency process can be implemented in APNC, in which $g(z)$ is defined by Eq. (2) at $\Lambda_1 = 21.2$ μm, $\Lambda_2 = 14.3$ μm, $\Lambda_3 = 8$ μm. The process is described by the system of equations for amplitudes of interacting waves

$$\mathrm{d}A_1/\mathrm{d}z = \mathrm{i}g(z)\left(\beta_{11}A_2A_3\mathrm{e}^{\mathrm{i}\Delta k_1 z} + \beta_{21}A_2^*A_4\mathrm{e}^{-\mathrm{i}\Delta k_2 z}\right), \tag{4}$$

$$\mathrm{d}A_2/\mathrm{d}z = \mathrm{i}g(z)\left(\beta_{12}A_1A_3^*\mathrm{e}^{-\mathrm{i}\Delta k_1 z} + \beta_{22}A_4A_1^*\mathrm{e}^{-\mathrm{i}\Delta k_2 z}\right), \tag{5}$$

$$\mathrm{d}A_3/\mathrm{d}z = \mathrm{i}g(z)\left(\beta_{13}A_1A_2^*\mathrm{e}^{-\mathrm{i}\Delta k_1 z} + \beta_{33}A_5A_4^*\mathrm{e}^{-\mathrm{i}\Delta k_3 z}\right), \tag{6}$$

$$\mathrm{d}A_4/\mathrm{d}z = \mathrm{i}g(z)\left(\beta_{24}A_1A_2\mathrm{e}^{\mathrm{i}\Delta k_2 z} + \beta_{34}A_5A_3^*\mathrm{e}^{-\mathrm{i}\Delta k_3 z}\right), \tag{7}$$

$$\mathrm{d}A_5/\mathrm{d}z = \mathrm{i}g(z)\beta_{35}A_3A_4\mathrm{e}^{\mathrm{i}\Delta k_3 z}, \tag{8}$$

where A_j is the amplitude of wave with frequency $\omega_j = 2\pi/\lambda_j$; $\Delta k_1 = k_1 - k_2 - k_3$, $\Delta k_2 = k_4 - k_1 - k_2$, $\Delta k_3 = k_5 - k_3 - k_4$ are the phase mismatches for processes (1), (2), and (3), respectively; $\Lambda_j = 2\pi m/\Delta k_j$; $\beta_{lj} = 4\pi\omega_j d_j^{(l)}/(cn_j)$ is the nonlinear coupling coefficient; n_j is the refractive index of crystal for wave at frequency ω_j; $d_j^{(l)}$ is the effective nonlinear coefficient for wave at frequency ω_j in the process l (l = 1, 2, 3, see above); c is the velocity of light in vacuum. The following conditions are fulfilled for nonlinear coupling coefficients

$$\beta_{11} = \beta_{12} + \beta_{13}, \quad \beta_{24} = \beta_{21} + \beta_{22}, \quad \beta_{35} = \beta_{33} + \beta_{34}. \tag{9}$$

The system of Eqs. (4)–(8) is satisfying the condition

$$\sum_{j=1}^{5} I_j(z) = \mathrm{const}, \tag{10}$$

where $I_j(z) = |A_j(z)|^2$.

Figure 2 presents the results of numerical solutions of system (4)–(8) at $\beta_{11} = \beta_{24} = \beta_{35}$, $\beta_{12} = \beta_{13}$, $\beta_{21} = 2\beta_{22}$, $\beta_{34} = 3\beta_{33}$, $a_1 = a_2 = a_3$. The calculations have shown that the presence of intensity waves (pump) with frequencies ω_1

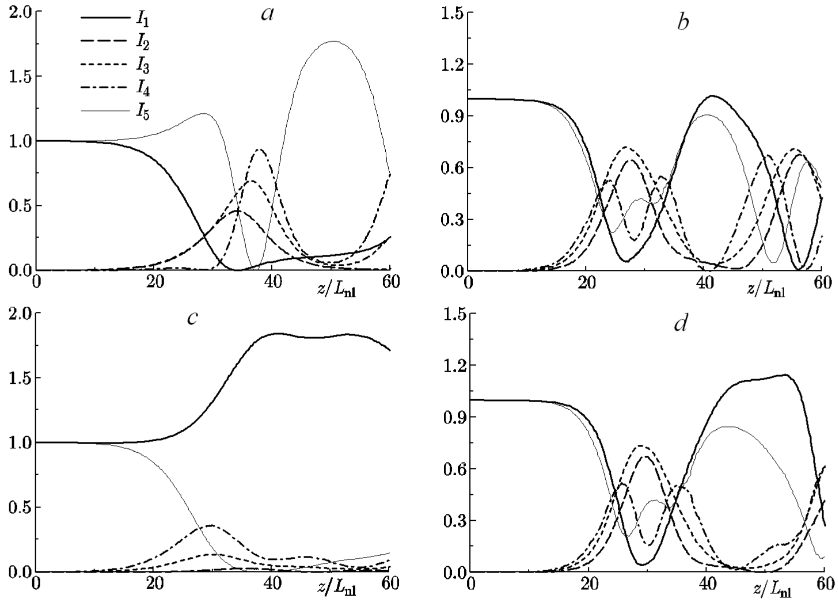

Fig. 2. Intensities I_j (normalized on $I_1(0)$) of wave with frequency ω_j as functions of crystal length: (a) $\mathrm{Re}A_1(0) = \mathrm{Re}A_5(0)$, $\mathrm{Re}A_{2,3,4}(0) = 10^{-2}\mathrm{Re}A_1(0)$, $\mathrm{Im}A_j(0) = 0$; (b) $\mathrm{Re}A_1(0) = \mathrm{Im}A_5(0)$, $\mathrm{Re}A_{2,3,4}(0) = 10^{-2}\mathrm{Re}A_1(0)$, $\mathrm{Im}A_{1,2,3,4}(0) = \mathrm{Re}A_5(0) = 0$; (c) $\mathrm{Im}A_1(0) = \mathrm{Re}A_5(0)$, $\mathrm{Re}A_{2,3,4}(0) = 10^{-2}\mathrm{Im}A_1(0)$, $\mathrm{Re}A_1(0) = \mathrm{Im}A_{2,3,4,5}(0) = 0$; (d) $\mathrm{Im}A_1(0) = \mathrm{Im}A_5(0)$, $\mathrm{Re}A_{2,3,4}(0) = 10^{-2}\mathrm{Im}A_1(0)$, $\mathrm{Re}A_1(0) = \mathrm{Im}A_{2,3,4}(0) = 0$.

and ω_5 is necessary for the process implementation. In calculations we supposed $I_1(0) = I_5(0)$, $I_{2,3,4}(0) \ll I_{1,5}(0)$. Coordinate z and intensities I_j are normalized on $L_{\mathrm{nl}} = 1/[\beta_{11}(I_1(0))^{1/2}]$ and $I_1(0)$, respectively.

Figure 2 shows that the effective energy exchange between interacting waves takes place in the process under consideration. The dynamics of energy exchange has the complex character, which can be changed in dependence on ratio between real and image parts of amplitudes of pump waves at the crystal input. Our analysis also shown that the effective energy exchange takes no place at $a_2 > a_{1,3}$.

As a rule the condition $L_{\mathrm{nl}} \gg \max\{\Lambda_1,\ \Lambda_2,\ \Lambda_3\}$ is fulfilled for PPNCs. In this connection we have calculated the intensities of interacting waves using system (4)–(8), in which the fast changing multipliers $g(z)\exp(\mathrm{i}\Delta k_j z)$ and $g(z)\exp(-\mathrm{i}\Delta k_j z)$ were changed by its average values q_j and q_j^* respectively, where

$$q_j = \frac{1}{L}\int_0^L g(z)\mathrm{e}^{\mathrm{i}\Delta k_j z}\mathrm{d}z. \tag{11}$$

Thus the system (4)–(8) has the form

$$\mathrm{d}A_1/\mathrm{d}z = \mathrm{i}(q_1\beta_{11}A_2A_3 + q_2^*\beta_{21}A_2^*A_4), \tag{12}$$

$$\mathrm{d}A_2/\mathrm{d}z = \mathrm{i}(q_1^*\beta_{12}A_1A_3^* + q_2^*\beta_{22}A_4A_1^*), \tag{13}$$

$$\mathrm{d}A_3/\mathrm{d}z = \mathrm{i}(q_1^*\beta_{13}A_1A_2^* + q_3^*\beta_{33}A_5A_4^*), \tag{14}$$

$$\mathrm{d}A_4/\mathrm{d}z = \mathrm{i}(q_2\beta_{24}A_1A_2 + q_3^*\beta_{34}A_5A_3^*), \tag{15}$$

$$\mathrm{d}A_5/\mathrm{d}z = \mathrm{i}q_3\beta_{35}A_3A_4. \tag{16}$$

Our calculations have demonstrated that the solution of system (4)–(8) and system (12)–(16) are in a good agreement for $z \approx 50L_{\mathrm{nl}}$. We note that such simplification of equations allows to essentially decrease the time of numerical solution of equations and develops the quantum theory of the process under consideration (see [10]).

The system (12)–(16) has the simple solution in the case of $\mathrm{Re}\{q_2q_3A_1^2A_5^*\} = 0$. For example, the solution for $A_2(z)$ is the following:

$$A_2(z) = C_1\mathrm{ch}(\Gamma z) + C_2\mathrm{sh}(\Gamma z) + C_3, \tag{17}$$

where

$$\Gamma^2 = \left(q_1^2\beta_{12}\beta_{13} - |q_2|^2\beta_{22}\beta_{24}\right)|A_1(0)|^2 + |q_3|^2\beta_{33}\beta_{34}|A_5(0)|^2;$$

C_j is defined by initial condition. The parametric instability of initial state takes place at $\Gamma^2 > 0$. In this case the intensities of interacting waves are increased with crystal length. The dependences of Fig. 2 at small z correspond to parametric instability regime.

4. Conclusion

The dynamics of new multi-frequency process, which can be realized in APNCs, was investigated. The multi-frequency processes in the considered APNCs are very promising for compact frequency converter and applications of quantum optics.

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