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Differential Transform Method for Solving the Two-dimensional Fredholm Integral Equations

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Abstract

In this paper, we develop the Differential Transform (DT) method in a new scheme to solve the two-dimensional Fredholm integral equations (2D-FIEs) of the second kind. The differential transform method is a procedure to obtain the coefficients of the Taylor expansion of the solution of differential and integral equations. So, one can obtain the Taylor expansion of the solution of arbitrary order and hence the solution of the given equation can be obtained with required accuracy. Here, we first give some basic definitions and properties about DT from references, and then we prove some theorems to extend the DT method for solving the 2D-FIEs. Then by using the DT, the 2D-FIE is converted to a system of linear algebraic equations whose unknowns are the coefficients of the Taylor expansion of the solution. Solving the system gives us an approximate solution. Finally, we give some examples to show the accuracy and efficiency of the presented method.

Keywords: Differential Transform Method, Two-Dimensional Fredholm Integral Equation

MSC 2010: 65R20

1. Introduction

The differential transform was first introduced by Zhou (1986), and up to now, the DT method has been developed for solving various kinds of differential and integral equations in many literatures. For example, Chen (1999) has developed the DT method for solving partial differential equations and Ayaz (2004) has applied this method to differential algebraic equations. Arikoglu and Ozkol (2005) have solved the integro-differential equations with boundary value conditions by the DT method. Odibat (2008) has used the DT method for solving Volterra integral equations with separable kernels. Tari and Shahmorad (2011) have solved the system of the two-dimensional nonlinear Volterra integro-differential equations by the DT method. The systems of integral and integro-differential equations, the multi-order fractional differential equations, the systems of fractional differential equations, the singularly perturbed Volterra integral equations and the time-fraction diffusion equation have been solved by the DT method in [Arikoglu and Ozkol (2008); Erturk, Momani and Odibat (2008); Erturk and Momani (2008); Dogan, Erturk, Momani, Akin and Yildirim (2011); Cetinkaya and Kimaz (2013)]. Also, the DT method has been applied to nonlinear parabolic-hyperbolic partial differential equations and a modified approach of DT has been developed to nonlinear partial differential equations [Biazar, Eslami and Islam (2010); Alquran (2012)]. But, up to now, the DT method has not been developed to solve equations of the form

$$u(x,t) = f(x,t) + \int_0^b \int_0^a k(x,t,y,z)u(y,z)dydz, \quad x \in [0,a], \quad t \in [0,b], \tag{1}$$

where $f(x,t)$ and $k(x,t,y,z)$ are known sufficiently differentiable functions and $u(x,t)$ is an unknown function, called the solution of the equation. In this paper, we develop the DT method to solve (1).

2. Some results about differential transform

The two-dimensional differential transform of the $(m,n)^{th}$ derivative of function $f(x,t)$ at (x_0,t_0) is defined as:

$$F(m,n) = \frac{1}{m!n!}\left[\frac{\partial^{m+n} f(x,t)}{\partial x^m \partial t^n}\right]_{x=x_0,t=t_0} \tag{2}$$

and the inverse transform of $F(m,n)$ is defined as

$$f(x,t) = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} F(m,n)(x-x_0)^m (t-t_0)^n, \tag{3}$$

which is the Taylor series of the function $f(x,t)$. Similarly, the differential transform of a function with four variables, such as $h(x,t,y,z)$, can be defined.

In what follows, we give some basic properties of DT.

Theorem 1. [Tari and Shahmorad (2011)]

If $F(m,n)$, $U(m,n)$ and $V(m,n)$ are differential transforms of the functions $f(x,t)$, $u(x,t)$ and $v(x,t)$ at $(0,0)$ respectively, then:

(a) If $f(x,t)=u(x,t)\pm v(x,t)$ then,

$$F(m,n)=U(m,n)\pm V(m,n).$$

(b) If $f(x,t)=au(x,t)$ then,

$$F(m,n)=aU(m,n).$$

(c) If $f(x,t)=u(x,t)v(x,t)$ then,

$$F(m,n)=\sum_{k=0}^{m}\sum_{l=0}^{n}U(k,l)V(m-k,n-l).$$

(d) If $f(x,t)=x^k t^l$ then,

$$F(m,n)=\delta_{m,k}\delta_{n,l}.$$

(e) If $f(x,t)=x^k\sin(at+b)$ then,

$$F(m,n)=\frac{a^n}{n!}\delta_{m,k}sin(\frac{n\pi}{2}+b).$$

(g) If $f(x,t)=x^k\cos(at+b)$ then,

$$F(m,n)=\frac{a^n}{n!}\delta_{m,k}cos(\frac{n\pi}{2}+b).$$

Theorem 2. [Tari and Shahmorad (2011)]

Let $F(m,n)$, $U(m,n)$ and $V(m,n)$ be differential transforms of the functions $f(x,t)$, $u(x,t)$ and $v(x,t)$ around $(0,0)$ respectively, then

(a) If $f(x,t)=\dfrac{\partial u(x,t)}{\partial x}$ then,

$$F(m,n)=(m+1)U(m+1,n).$$

(b) If $f(x,t)=\dfrac{\partial u(x,t)}{\partial t}$ then,

$$F(m,n)=(n+1)U(m,n+1).$$

(c) If $f(x,t)=\dfrac{\partial^{r+s}u(x,t)}{\partial x^r\partial t^s}$ then,

$$F(m,n)=(m+1)(m+2)\cdots(m+r)(n+1)(n+2)\cdots(n+s)U(m+r,n+s).$$

Now we prove some theorems to develop the DT method to the two-dimentional Fredholm integral equations.

Theorem 3.

If

$$g(x,t)=\int_0^b\int_0^a h(x,t,y,z)dydz$$

then its differential transform at $(0,0)$ is:

$$G(i,j)=\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}H(i,j,m,n)\frac{a^{m+1}b^{n+1}}{(m+1)(n+1)},$$

where $H(i,j,m,n)$ is differential transform of $h(x,t,y,z)$.

Proof:

Substituting the Taylor expansion of $h(x,t,y,z)$ at $(0,0,0,0)$ in

$$g(x,t)=\int_0^b\int_0^a h(x,t,y,z)dydz,$$

implies that

$$g(x,t)=\int_0^b\int_0^a\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}H(i,j,m,n)x^i t^j y^m z^n dydz$$

$$=\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}H(i,j,m,n)x^i t^j\int_0^b\int_0^a y^m z^n dydz$$

$$=\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\left[\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}H(i,j,m,n)\frac{a^{m+1}b^{n+1}}{(m+1)(n+1)}\right]x^i t^j.$$

So, by definition of differential transform, we have

$$G(i,j)=\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}H(i,j,m,n)\frac{a^{m+1}b^{n+1}}{(m+1)(n+1)},$$

which is the asserted relation. ■

Theorem 4.

If $g(x,t,y,z)=u(x,t,y,z)v(x,t,y,z)$ and if U, V and G are the differential transforms of functions u, v and g at $(0,0,0,0)$, respectively, then

$$G(i,j,m,n)=\sum_{c=0}^{i}\sum_{d=0}^{j}\sum_{k=0}^{m}\sum_{l=0}^{n}U(c,d,k,l)V(i-c,j-d,m-k,n-l).$$

Proof:

We have

$$\frac{\partial^{i+j+m+n}(u(x,t,y,z)v(x,t,y,z))}{\partial x^i\partial t^j\partial y^m\partial z^n}$$
$$=\frac{\partial^{i+j+m}}{\partial x^i\partial t^j\partial y^m}\left(\sum_{l=0}^{n}\binom{n}{l}\frac{\partial^l u(x,t,y,z)}{\partial z^l}\frac{\partial^{n-l}v(x,t,y,z)}{\partial z^{n-l}}\right).$$

In general

$$\frac{\partial^{i+j+m+n}(u(x,t,y,z)v(x,t,y,z))}{\partial x^i\partial t^j\partial y^{m^n}}=\sum_{c=0}^{i}\sum_{d=0}^{j}\sum_{k=0}^{m}\sum_{l=0}^{n}\binom{i}{c}\binom{j}{d}\binom{m}{k}\binom{n}{l}$$
$$\times\frac{\partial^{c+d+k+l}u(x,t,y,z)}{\partial x^c\partial t^d\partial y^k\partial z^l}\frac{\partial^{i-c+j-d+m-k+n-l}v(x,t,y,z)}{\partial x^{i-c}\partial t^{j-d}\partial y^{m-k}\partial z^{n-l}}.$$

Therefore,

$$G(i,j,m,n)=\frac{1}{i!\,j!m!n!}\left[\frac{\partial^{i+j+m+n}g(x,t,y,z)}{\partial x^i\partial t^j\partial y^m\partial z^n}\right]_{x=t=y=z=0}$$
$$=\frac{1}{i!\,j!m!n!}\sum_{c=0}^{i}\sum_{d=0}^{j}\sum_{k=0}^{m}\sum_{l=0}^{n}\binom{i}{c}\binom{j}{d}\binom{m}{k}\binom{n}{l}$$
$$\times c!d!k!l!U(c,d,k,l)(i-c)!(j-d)!$$
$$\times(m-k)!(n-l)!V(i-c,j-d,m-k,n-l),$$
$$=\sum_{c=0}^{i}\sum_{d=0}^{j}\sum_{k=0}^{m}\sum_{l=0}^{n}U(c,d,k,l)V(i-c,j-d,m-k,n-l),$$

and so the proof is complete. ■

Corollary 5.

If

$$g(x,t,y,z) = h(x,t,y,z)u(y,z)$$

then,

$$G(i,j,m,n) = \sum_{k=0}^{m}\sum_{l=0}^{n} H(i,j,k,l)U(m-k,n-l).$$

Finally, Theorem 3 and Corollary 5 imply the following result.

Theorem 6.

If

$$g(x,t) = \int_0^b\int_0^a h(x,t,y,z)u(y,z)dydz$$

then,

$$G(i,j) = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\sum_{k=0}^{m}\sum_{l=0}^{n} H(i,j,k,l)U(m-k,n-l)\frac{a^{m+1}b^{n+1}}{(m+1)(n+1)}.$$

3. Description of the method

In this section, we describe the method. As mentioned previously, we convert the equation (1) to a syatem of linear algebraic equations, whose unknowns are the differential transforms of the solution of equation (1) ($U(i,j)$), by using the DT. To this purpose, by using the differential transform to equation (1), it can be written as

$$U(i,j) = F(i,j) + \sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\sum_{k=0}^{m}\sum_{l=0}^{n} K(i,j,k,l)U(m-k,n-l)\frac{a^{m+1}b^{n+1}}{(m+1)(n+1)}, \tag{4}$$

by Theorem 6. Or, in truncated form

$$U(i,j) = F(i,j) + \sum_{m=0}^{M}\sum_{n=0}^{N}\sum_{k=0}^{m}\sum_{l=0}^{n} K(i,j,k,l)U(m-k,n-l)\frac{a^{m+1}b^{n+1}}{(m+1)(n+1)}, \tag{5}$$

for $i = 0,1,\cdots,M$, $j = 0,1,\cdots,N$, which is a $(M+1)(N+1)\times(M+1)(N+1)$ linear system for unknown values $U(i,j)$.

For the convenience of the reader the system (5) can be written in a simple form as:

$$\left(\sum_{k=0}^{M}\sum_{l=0}^{N} K(i,j,k,l)\frac{b^{k+1}a^{l+1}}{(k+1)(l+1)}\right)U(0,0)$$

$$+\left(\sum_{k=0}^{M-1}\sum_{l=0}^{N}K(i,j,k,l)\frac{b^{k+2}a^{l+1}}{(k+2)(l+1)}\right)U(1,0)$$

$$+\left(\sum_{k=0}^{M}\sum_{l=0}^{N-1}K(i,j,k,l)\frac{b^{k+1}a^{l+2}}{(k+1)(l+2)}\right)U(0,1)$$

$$+\left(\sum_{k=0}^{M-1}\sum_{l=0}^{N-1}K(i,j,k,l)\frac{b^{k+2}a^{l+2}}{(k+2)(l+2)}\right)U(1,1)$$

$$\vdots$$

$$+\left(\sum_{k=0}^{M-i}\sum_{l=0}^{N-j}K(i,j,k,l)\frac{b^{k+i+1}a^{l+j+1}}{(k+i+1)(l+j+1)}-1\right)U(i,j)$$

$$\vdots$$

$$+\left(K(i,j,0,0)\frac{b^{M+1}a^{N+1}}{(M+1)(N+1)}\right)U(M,N)=F(i,j).$$

The structure of this system is such that it can be solved easily for high order, which is an important advantage of the DT method with respect to other methods as expansion method [Biazar, Eslami and Islam (2010)] and the method of [Alquran (2012)].

By solving the above system, the approximate solution of the equation (1) can be obtained as

$$u_{M,N}(x,t)=\sum_{i=0}^{N}\sum_{j=0}^{N}U(i,j)x^i y^j. \tag{6}$$

To solve the system (5) we use the Maple software.

4. Numerical examples

In this section, we give some examples to show effeciency and accuracy of the presented method.

Example 1. [Tari and Shahmorad (2008)**]**

Use the differential transform method to solve the Fredholm integral equation

$$u(x,t)=x\sin t+\left(\frac{2}{3}x+\frac{1}{2}t\right)\pi^3+\int_0^{\pi}\int_0^{\pi}(yx+tz)u(y,z)dydz, \tag{7}$$

which has the exact solution as $u(x,t) = x\sin t$.

The differential transform method transforms Equation (7) to

$$U(i,j) - \sum_{m=0}^{M}\sum_{n=0}^{N}\sum_{k=0}^{m}\sum_{l=0}^{n}\left[\delta_{i,1}\delta_{k,1} + \delta_{j,1}\delta_{l,1}\right]U(m-k,n-l)\frac{\pi^{m+1}\pi^{n+1}}{(m+1)(n+1)}$$
$$= \delta_{i,1}\frac{1}{j!}\sin(j\frac{\pi}{2}) + \left(\frac{2}{3}\delta_{i,1} + \frac{1}{2}\delta_{j,1}\right)\pi^3, \quad i = 0,1,\ldots,M, \quad j = 0,1,\ldots,N.$$

By solving the above system for unknowns $U(i,j)$, the approximte solution is obtained from (6). The numerical results, which are values of the error function $e(x,t) = u(x,t) - u_{M,N}(x,t)$ at some points, are reported in Table 1.

Table 1. Numerical results of Example 1

(x,t)	$M=2, N=25$	$M=2, N=30$	$M=2, N=40$
(0,0)	0	0	0
(0.5,0.5)	$0.2387e-14$	$0.6057e-20$	0
(1,1)	$0.4774e-14$	$0.1211e-19$	$0.1e-29$
(1.5,1.5)	$0.7161e-14$	$0.1817e-19$	$0.1e-28$
(2,2)	$0.9549e-14$	$0.2423e-19$	0
(2.5,2.5)	$0.1194e-13$	$0.2963e-19$	$0.2e-28$
(3,3)	$0.1640e-13$	$0.1870e-18$	$0.6e-29$

For comparison, the numerical results of **[**Tari and Shahmorad (2008)**]** are given in Table 2. In mentioned reference, the equations of the form (1) have been solved by an expansion method. The expansion method need more computations than the DT method, so the DT method gives solutions that are more accurate than the expansion method.

Table 2. Numerical results of **[**Tari and Shahmorad (2008)**]** for Example 1

(x,t)	(0,0)	(0.5,0.5)	(1,1)	(1.5,1.5)	(2,2)	(2.5,2.5)	(3,3)
$e(x,t)$	$0.611e-13$	$0.898e-10$	$0.918e-11$	$0.196e-9$	$0.298e-9$	$0.295e-10$	$0.123e-9$

Example 2. [Haleema and Lamyaa (2012)**]**

Consider the two-dimensional Fredholm integral equation:

$$u(x,t) = xt - \frac{1}{4}(x+t) - \frac{1}{3} + \int_0^1\int_0^1 (x+t+y+z)u(y,z)dydz, \tag{8}$$

with exact solution $u(x,t) = xt$.

The differential transform method transforms Equation (8) to

$$U(i,j) - \sum_{m=0}^{M}\sum_{n=0}^{N}\sum_{k=0}^{m}\sum_{l=0}^{n}\left[\delta_{i,1} + \delta_{j,1} + \delta_{k,1} + \delta_{l,1}\right]U(m-k, n-l)\frac{1}{(m+1)(n+1)}$$
$$= \delta_{i,1}\delta_{j,1} - \frac{1}{4}(\delta_{i,1}\delta_{j,0} + \delta_{i,0}\delta_{j,1}) - \frac{1}{3}\delta_{i,0}\delta_{j,0}, \quad i = 0,1,...,M\,,\ j = 0,1,...,N\,.$$

Similar to Example 1, by solving the above system for $M = 2$ and $N = 2$ we obtain:

$$U_{i,j}(x,t) = \begin{cases} 1, & i = j = 1 \\ 0, & \text{otherwise.} \end{cases} \tag{9}$$

So the approximate solution is $u_{2,2}(x,t) = xt,$ which is the exact solution.

In **[**Haleema and Lamyaa (2012)**]** the solution was reported at some points. For comparison, we report some of the numerical results of **[**Haleema and Lamyaa (2012)**]** as follows:

Table 3: Numerical results of **[**Haleema and Lamyaa (2012)**]** for Example 2

(x,t)	Exact	Approximation $u(x,t)$ $n=m=2$	Approximation $u(x,t)$ $n=m=3$
(0, 0)	0	0	0
(0.5, 0.5)	0.2500	0.2500	0.2500
(1, 1)	1	1	1

We have obtained the exact solution analytically.

Example 3.

In this example, consider the equation:

$$u(x,t) = \cos x \sin t - xt\left(\frac{1}{3} + \frac{e^{\frac{\pi}{2}}}{3}\right) + \int_0^1\int_0^1 xt\sin 2y e^{z} u(y,z)dydz,$$

which has the exact solution $u(x,t) = \cos x \sin t$.

The differential transform method of the above equation implies:

$$U(i,j) - \sum_{m=0}^{M}\sum_{n=0}^{N}\sum_{k=0}^{m}\sum_{l=0}^{n}\delta_{i,1}\delta_{j,1}\frac{2^k}{k!}\sin(k\frac{\pi}{2})\frac{1}{l!}U(m-k,n-l)\frac{1}{(m+1)(n+1)}$$
$$= \frac{1}{i!}\cos(i\frac{\pi}{2})\frac{1}{j!}\sin(j\frac{\pi}{2}) - \frac{1}{3}(1+e^{\frac{\pi}{2}})\delta_{i,1}\delta_{j,1}, \quad i = 0,1,\ldots,M, \quad j = 0,1,\ldots,N.$$

By solving the above system for different values of M and N, we obtain the approximate solution. Numerical results are reported in Table 3.

Table 3. Numerical results of Example 3

(x,t)	$e(x,t)(M=25, N=25)$	$e(x,t)(M=35, N=35)$
(0.2,0.2)	$0.5581e-12$	$0.2000e-19$
(0.4,0.4)	$0.2232e-11$	$0.1000e-19$
(0.6,0.6)	$0.5023e-11$	$0.2000e-19$
(0.8,0.8)	$0.8929e-11$	$0.2000e-19$
(1,1)	$0.1395e-10$	$0.6668e-19$

Example 4.

Use the differential transform method to solve the Fredholm integro-differential equation

$$\frac{\partial^{2+1}u(x,t)}{\partial x^2 \partial t} = \left(\frac{2}{3} - \frac{\pi}{3}\right)x^2 - 2\sin(t) + \int_0^{\frac{\pi}{2}}\int_0^{\frac{\pi}{3}} x^2 \cos 2z \sin y\, u(y,z)\,dydz,$$

with conditions:

$$\left[\frac{\partial^{c+d}u(x,t)}{\partial x^c \partial t^d}\right]_{x=0,t=0} = 0, \qquad c = 0,1, \quad d = 0,1,$$

$$\left[\frac{\partial^{1}u(x,t)}{\partial x^2}\right]_{x=0,t=0} = 2,$$

which has the exact solution $u(x,t) = x^2 \cos t$.

We proceed as in previous examples to obtain:

$$(i+1)(i+2)(j+1)U(i+2,j+1)=\left(\frac{2}{3}-\frac{\pi}{3}\right)\delta_{i,2}-2\frac{1}{j!}\sin(j\frac{\pi}{2})$$
$$+\sum_{m=0}^{M}\sum_{n=0}^{N}\sum_{l=0}^{m}\sum_{k=0}^{n}\delta_{i,2}\frac{1}{m!}\sin\left(m\frac{\pi}{2}\right)\frac{2^n}{n!}\cos\left(n\frac{\pi}{2}\right)U(m-k,n-l)\frac{(\frac{\pi}{2})^{m+1}(\frac{\pi}{2})^{n+1}}{(m+1)(n+1)},$$
$$i=0,1,...,M,\quad j=0,1,...,N.$$

Table 4 shows the numerical results $e(x,t)=u(x,t)-u_{M,N}(x,t)$ at some points.

Table 4. Numerical results of Example 4

(x,t)	$e(x,t)(M=2,N=25)$	$e(x,t)(M=2,N=45)$
(0,0)	0	0
$(0.1\pi,0.1\pi)$	0	0
$(0.2\pi,0.2\pi)$	$0.1e-19$	$0.1e-19$
$(0.3\pi,0.3\pi)$	0	0
$(0.4\pi,0.4\pi)$	$0.9e-19$	$0.9e-19$
$(0.5\pi,0.5\pi)$	$0.3177e-19$	$0.3253e-19$

5. Conclusion

In this paper, we applied the differential transform method to solve the two-dimensional Fredholm integral equations of the second kind. It was done by converting the integral equation into a system of linear algebraic equations in terms of the coefficients of the approximate solution. As the numerical examples show, the presented method is simple and has high accuracy. It seems that this method can be applied to solve nonlinear Fredholm integral equations.

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