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Synthesis of Bimodal Function Generating Mechanisms Without Branch Defect

A theory is proposed for the synthesis of bimodal function generators which are free from the branch defect. It is applicable to planar as well as spatial linkages. Precision type synthesis has been considered for planar four-bar and spatial R-S-S-R linkages. Applications of the theory are illustrated by means of numerical examples.

Introduction

In this paper we present a method for eliminating branch defect in the precision point synthesis of bimodal function generating mechanisms. The term bimodal is used to denote those planar and spatial linkages which can be assembled in two modes. Recent noteworthy works on the elimination of branch defect in planar linkages using graphical methods are due to Filemon [6, 7], Waldron [8] and Waldron et al. [9]. The algebraic-geometrical techniques for precision synthesis of four-bar mechanisms with completely rotatable driving links which are also free from branch defect are developed in [11].

In this paper we present an algebraic-geometrical approach to eliminate branching in the precision synthesis of planar and spatial bimodal linkages. The main advantage of this approach is that it is not dependent upon linkage type (or structure). In multimodal linkages (with more than two modes of assembly), this approach may not guarantee the elimination of branch defect but increases the likelihood of eliminating it.

Theory

The input and output displacement equation of a one degree of freedom mechanism can be written in the form

$$F(\theta, \phi)=0 \tag{1}$$

where θ and ϕ are, respectively, the input and output angles of the mechanism. The function $\phi=f(\theta)$ implicit in equation (1) is a multivalued function of θ. The multiplicity of $f(\theta)$ is equal to the number of possible ways in which the mechanism can be assembled at a position corresponding to the input angle θ. Each branch of the function $f(\theta)$ corresponds to a mode of assembly or a "branch" of the mechanism. The curve $F(\theta, \phi) = 0$ can have points (θ, ϕ) where it has vertical tangents (i.e., parallel to ϕ-axis) and those points which are singular (or critical) points. Such points, where two or more values of ϕ coalesce, are called branch points of the function $\phi = f(\theta)$ [4].

If $F_\phi(\theta, \phi) \neq 0$ for any value of (θ, ϕ), then the branches of the multivalued function $\phi = f(\theta)$ are distinct. If there are points where $F_\phi = 0$, then the branches of $\phi = f(\theta)$ are defined between the points which have vertical tangents (i.e., $F_\phi = 0$, $F_\theta \neq 0$). Such situations occur, for example, when the curve $F(\theta, \phi) = 0$ has a closed circuit, either a simple circuit or a figure of eight. It should be noted that at the singular points of $F(\theta, \phi) = 0$, where $F_\theta = F_\phi = 0$, the branches of $\phi = f(\theta)$ continue such that smoothness is maintained. The mixed situations where some branches of $\phi = f(\theta)$ are distinct while others touch each other are quite common in multimodal mechanisms such as Watt's six link or spatial R-C-R-C-R linkage.

So far we have used the term branch point as it is used in context of multivalued functions. Unfortunately, it includes points where $F(\theta, \phi) = 0$ has vertical tangents and the singular points. The confusion arises from the fact that a branch of $\phi = f(\theta)$, as identified in the preceding, and as it corresponds to linkages, goes from one point with vertical tangent to another such point and it continues through singular points, if any. That is, the branches of $\phi = f(\theta)$ do not necessarily go from a branch point to another branch point. In the context of linkages, we have therefore coined the word mode point for those points where the curve $F(\theta, \phi) = 0$ has vertical tangents. The linkage is at a locking (or dead center) configuration at such points. The singular points of $F(\theta, \phi) = 0$ have been named crossover points, change points or uncertainty configurations in the literature [2, 3]. We now have the following simple description of a linkage mode or a linkage branch. In the $\theta - \phi$ plane, the linkage branches are either distinct (i.e., not touching) or they go from one mode point to another. The change points are traversed as dictated by velocity continuity (or inertia considerations).

Each branch of the multivalued function $\phi = f(\theta)$ behaves like a single-valued smooth function unless a mode point or change point is reached. From the implicit function theorem of calculus [5], it then follows that $F_\phi \neq 0$, or equivalently that F_ϕ does not change its sign, on any individual branch unless a singular point (change point) is traversed. In fact, it can be shown that F_ϕ changes its sign only when a singular point of even multiplicity is traversed by the branch. This is done easily by considering the contours of F ($F = 0^+$ and F

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$= 0^-$) in the vicinity of the singular point and the gradient vector ∇F with components F_θ and F_ϕ along the positive θ and ϕ axes. It may also be noted that F_ϕ changes its sign when a mode point is traversed, but then we have moved to another branch. This is because the mode points, if they exist, are terminal or end point of a branch while change point, if they exist, are interior points.

The branch defect in the precision point synthesis of linkages occurs when the precision points do not all belong to a single branch of the designed linkage. In the context of generating single-valued, smooth and monotonic functions, such a linkage must be considered a defective linkage even if all the precision points are "eventually" reachable without disassembling the mechanism. In our interpretation of branching, the defect arises simply because the precision points lie on more than one branch.

From the foregoing discussion of linkage branches, their properties and the branch defect, we can conclude that the function $G(\theta, \phi) = (\partial F/\partial \phi)$ can be used to separate the linkage branches. Let us consider N precision point synthesis of multimodal linkages. The N precision points are given as (θ_i, ϕ_i), $i = 1, N$. Then the linkages can be sorted into two groups. Those in one group have the property that $G(\theta_i, \phi_i) \geq 0$ at all precision points $i = 1, N$, while those in the other group have the property that $G(\theta_i, \phi_i) \leq 0$ $i = 1, N$. This process amounts to separation of branches into two groups with only one exception. If there are any branches which pass through a singular point (change point) of even multiplicity, then these branches are incorrectly sorted. Considering that the linkages which pass through a change point position between the precision points (e.g., folding position in a planar four-bar linkage) will most likely be discarded, this flaw appears to be a minor one.

For bimodal linkages, the identification and separation of the branches is complete at this stage. Planar and spherical four-bar linkages and spatial R-S-S-R, R-C-S-P linkages are some examples of such bimodal linkages. For multimodal linkages with more than two branches, while these considerations do not completely eliminate the branch defect, they seem to increase the likelihood of avoiding the branch defect by partial separation of the branches.

Let us now consider the parameter space of the linkage. In this space, each of the relations $G(\theta_i, \phi_i) = 0$, $i = 1, N$ represents a hypersurface. From these N hypersurfaces one can, in principle, determine the regions where all $G_i(\theta_i, \phi_i) \geq 0$ and all $G_i(\theta_i, \phi_i) \leq 0$. In the actual design of linkages, it is difficult to deal with these regions which are bounded by N (or less) hypersurfaces in the abstract parameter space. In the following we therefore develop an approximate single hypersurface representation of these regions where all the G_i, $i = 1, \ldots, N$ have the same sign.

If all G_i's > 0, then the average

$$G_{av} = \sum_{i=1}^{N} G_i/N > 0$$

A sufficient condition for positive G_i's is

$$G_{av} - \max_i |G_i - G_{av}| > 0 \tag{2}$$

On the other hand, if all G_i's<0, $G_{av}<0$ and a sufficient condition for negative G_i's is

$$G_{av} + \max_i |G_i - G_{av}| < 0 \tag{3}$$

Combining (2) and (3), all G_i's have the same sign only if

$$\max_i |G_i - G_{av}| < |G_{av}| \tag{4}$$

Let us introduce the following upperbound estimate.

$$\max_i |G_i - G_{av}|^2 \leq \sum_{i=1}^{N} (G_i - G_{av})^2 \tag{5}$$

The condition (4) is now replaced by condition (6)

$$\sum_{i=1}^{N} (G_i - G_{av})^2 < G_{av}^2 \tag{6}$$

It is important to note that condition (6) guarantees that all G_i's have the same sign. However, the converse is not true. Geometrically, condition (6) defines a region in the parameter space which is bounded by a single hypersurface. This region is completely within the aforementioned complex region bounded by N hypersurfaces $G_i = 0$ in a special way. Computationally the inequality (6) is more attractive than the inequalities (4), in that it provides us with a single condition as opposed to the N conditions implied in (4). Another useful form of (6) is as follows:

$$H = \sum_{i=1}^{N} G_i^2 - (N+1)G_{av}^2 < 0 \tag{7}$$

Condition (7) can be used in conjunction with precision point equations $F(\theta_i, \phi_i) = 0$ $i = 1, N$ in various ways to avoid branch defects. It is desirable to formulate the synthesis problem in such a way that $F(\theta_i, \phi_i) = 0$ is algebraic in the design parameters (usually related to link lengths, reference angles, etc.). $H = 0$ (equation (7)) then becomes a polynomial in these parameters. Furthermore, it is often possible to eliminate some of the design parameters in $H = 0$ with the aid of precision point equations. If all but one, two or three design parameters is eliminated from $H = 0$, the branch defect can be avoided by investigating the algebraic H-polynomial, H-curve or H-surface. An advantage of this approach is that a large number of designs without branch defect become available in a graphic (or visual) form. A suitable design can be selected from these to meet other quality requirements. We now consider the application of the foregoing procedure in the synthesis of planar four-bar and spatial R-S-S-R linkages.

Applications

Planar Four-Bar Linkages

For the four-bar mechanism shown in Fig. 1, the displacement equations is of the form [10]

$$U_i A_3 + V_i A_4 + A_5 - W_i = 0 \tag{8}$$

Functions U_i, V_i, W_i are polynomials of degree one in the design parameters A_1 and A_2 and they are defined as follows

$$\left.\begin{aligned} U_i &= -A_1\cos(\Delta\theta_i - \Delta\phi_i) - A_2\sin(\Delta\theta_i - \Delta\phi_i) - \cos\theta_i \\ V_i &= A_1\sin(\Delta\theta_i - \Delta\phi_i) - A_2\cos(\Delta\theta_i - \Delta\phi_i) + \sin\theta_i \\ W_i &= -(A_1\cos\Delta\phi_i - A_2\sin\Delta\phi_i) \end{aligned}\right\} \tag{9}$$

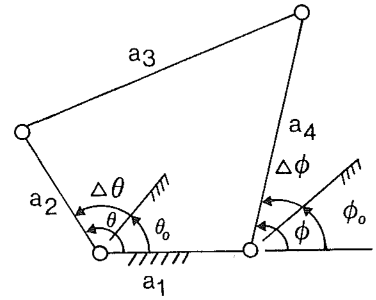

Fig. 1 Planar four-bar linkage and its parameters

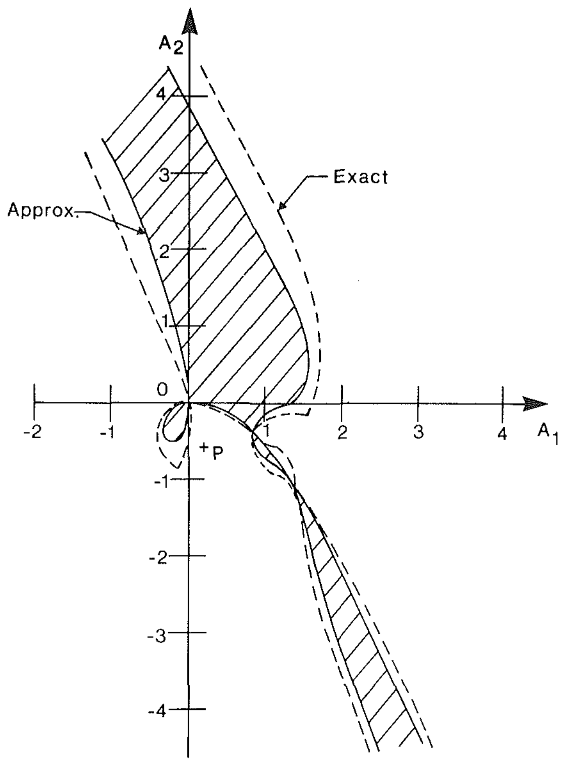

Fig. 2 Exact feasible region is bounded by dashed lines. Approximate feasible region is bounded by solid lines and is hatched. Point *P* yields linkage in reference [1].

The A's are given as:

$$A_1 = a_4\cos\phi_0 \quad A_2 = a_4\sin\phi_0 \quad A_3 = a_2\cos\theta_0$$

$$A_4 = a_2\sin\theta_0 \quad A_5 = ½(1+a_2^2+a_4^2-a_3^2)$$

The G_i's are obtained from equation (8).

$$\begin{aligned} G_i = & \{-A_1\sin(\Delta\theta_i-\Delta\phi_i)+A_2\cos(\Delta\theta_i-\Delta\phi_i)\}A_3 \\ & -\{A_1\cos(\Delta\theta_i-\Delta\phi_i)+A_2\sin(\Delta\theta_i-\Delta\phi_i)\}A_4 \\ & -(A_1\sin\Delta\phi_i+A_2\cos\Delta\phi_i) \qquad i=1,\ldots,N \end{aligned} \tag{10}$$

In three precision point synthesis, $(\Delta\theta_2, \Delta\phi_2)$ and $(\Delta\theta_3, \Delta\phi_3)$ are known $(\Delta\theta_1 = \Delta\phi_1 = 0)$. The mechanism closure equation (8) is written three times, once for each precision point to obtain the following system of equations

$$\begin{bmatrix} U_1 & V_1 & 1 \\ U_2 & V_2 & 1 \\ U_3 & V_3 & 1 \end{bmatrix} \begin{bmatrix} A_3 \\ A_4 \\ A_5 \end{bmatrix} = \begin{bmatrix} W_1 \\ W_2 \\ W_3 \end{bmatrix} \tag{11}$$

Equation (11) is algebraically solved for A_3, A_4, A_5 in terms of A_1 and A_2. When the result is substituted in (7), resulting inequality is a six-order polynomial in A_1 and A_2. These operations were done by means of a digital computer. The region $H_6(A_1, A_2) \le 0$ defines the feasible region in $A_1 - A_2$ plane for mechanisms which are free from branch defect. The boundary curve $H_6 = 0$ has double points at the origin and at the relative poles. It is in general an open curve but may contain several closed loops. As mentioned earlier, it provides only an approximate feasible region because some of the total feasible region area is lost due to the sufficient nature of (7).

The approximate feasible region for the three precision points $(\Delta\theta_i, \Delta\phi_i)$ = (0 deg, 0 deg), (26 deg, 54 deg), (52 deg, 80 deg) is shown in Fig. 2. These specifications were taken from reference [1] where they were used to illustrate branch defect in linkage synthesis. Figure 2 also shows the exact feasible region bounded by dashed lines for these precision points. This exact region was obtained by plotting the individual G_i, $i = 1, 2, 3$, on the same page and the determining the regions where all the G_i's have the same signs. It can be seen from the figure that the approximate region (see inequality (7)) has features nearly similar to the exact region (see inequality (4)).

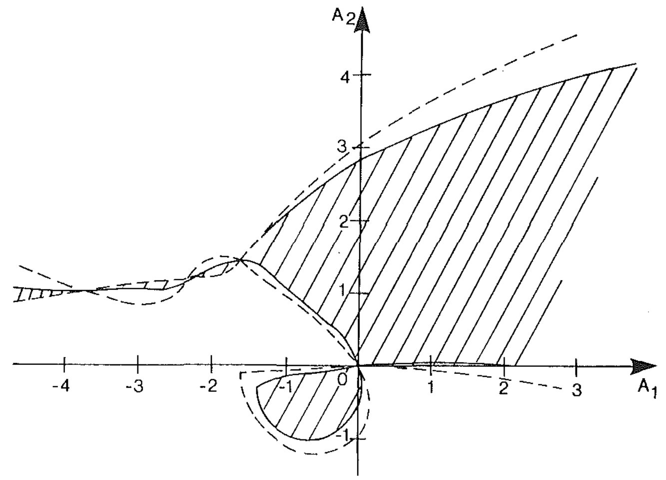

Fig. 3 Exact and approximate regions for four precision point synthesis. The latter is hatched.

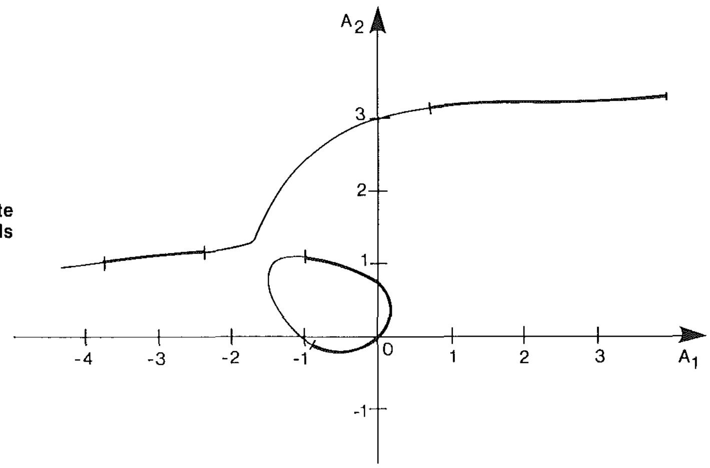

Fig. 4 Cubic curve for four precision point synthesis. Feasible segments are shown by heavy lines.

If a four precision point design is required, it is necessary to determine points (A_1, A_2) which lie on the locus given by the following 4×4 determinant [10]

$$|U_i\ V_i\ 1\ W_i| = 0 \qquad (i=1,\ldots,4) \tag{12}$$

It is well-known that this locus is a circular cubic curve in the $A_1 - A_2$ plane and it passes through the relative poles R_{12}, $R_{13}R_{23}'$ (image pole). The region $H_6 \le 0$ in $A_1 - A_2$ plane is obtained as before from inequality (7) and equations (11). Points on the cubic locus (12) which are also located in the region $H_6 \le 0$ yield mechanisms which are free from branch defect.

As an illustration, Fig. 3 is obtained by using the four precision points $(\Delta\theta_i, \Delta\phi_i)$: (0 deg, 0 deg), (40 deg, 30 deg), (80 deg, 52 deg), (120 deg, 60 deg). It shows approximate region $H_6 \le 0$ and the slightly larger region obtained from the individual plots of $G_i = 0$, $i = 1, \ldots, 4$. In Fig. 4, points on the cubic (12) which also lie in the region $H_6 \le 0$ and thus yield mechanisms which are free from branch defects are indicated by heavy lines.

R-S-S-R Mechanism. Figure 5 shows R-S-S-R mechanism with the associated design parameters. The angle between the input and output rotation axes is α. Other parameters are the link length ratios a_2/a_1, a_3/a_1, a_4/a_1, the axial distances S_2, S_4 of the input and output cranks and the two reference angles θ_0 and ϕ_0 at the input and output axes, respectively. The input and output incremental rotations $\Delta\theta_i$ and $\Delta\phi_i$ are measured from the reference lines at the input and output axes. In the R-S-S-R function generator, rotations $\Delta\theta_i$ and $\Delta\phi_i$ are to be coordinated for $i = 1, 2, \ldots, N$. The displacement equation of the R-S-S-R mechanism [1] shown in Fig. 5 is given in equation (13) where we have set $a_1 = 1$.

$$\begin{aligned} F(\Delta\theta_i, \Delta\phi_i) = {} & (1^2 + a_2^2 - a_3^2 + a_4^2 + S_2^2 + S_4^2 + 2S_2S_4\cos\alpha) \\ & + 2a_2\cos(\theta_0 + \Delta\theta_i) + 2a_2S_4\sin\alpha\sin(\theta_0 + \Delta\theta_i) \\ & - \cos(\phi_0 + \Delta\phi_i)[2a_4 + 2a_2a_4\cos(\theta_0 + \Delta\theta_i)] \\ & - \sin(\phi_0 + \Delta\phi_i)\{2a_2a_4\cos\alpha\sin(\theta_0 + \Delta\theta_i) \\ & - 2S_2a_4\sin\alpha\} \qquad \{i = 1, 2, \ldots, N\} \end{aligned} \tag{13}$$

Let us define a new set of design parameters A_i's as follows:

$$\left.\begin{aligned} & A_1 = a_4\cos\phi_0, \quad A_2 = a_4\sin\phi_0, \quad A_3 = S_4 \\ & A_4 = a_2\cos\theta_0, \quad A_5 = a_2\sin\theta_0, \quad A_6 = S_2 \\ & A_7 = (1^2 + a_2^2 - a_3^2 + a_4^2 + S_2^2 + S_4^2 + 2S_2S_4\cos\alpha) \end{aligned}\right\} \tag{14}$$

The mechanism parameters are obtained from the A_i's as follows

$$\left.\begin{aligned} & a_1 = 1, \quad a_2 = (A_5^2 + A_4^2)^{1/2} \\ & a_4 = (A_1^2 + A_2^2)^{1/2}, \quad a_3 = \{1 + a_2^2 + a_4^2 + S_2^2 + S_4^2 + 2S_2S_4\cos\alpha - A_7\}^{1/2} \\ & \theta_0 = \tan^{-1}(A_5/A_4), \quad \phi_0 = \tan^{-1}(A_2/A_1) \end{aligned}\right\} \tag{15}$$

The displacement equation (15) in terms of the new design parameters becomes

$$P_iA_4 + U_iA_5 + V_iA_6 + A_7 - W_i = 0 \tag{16}$$

where

$$\begin{aligned} P_i &= 2\{(\sin\Delta\phi_i\cos\Delta\theta_i - \cos\alpha\sin\Delta\theta_i\cos\Delta\phi_i)A_2 \\ &\quad - (\cos\Delta\theta_i\cos\Delta\phi_i + \cos\alpha\sin\Delta\theta_i\sin\Delta\phi_i)A_1 \\ &\quad + \sin\alpha\sin\Delta\theta_iA_3 + \cos\Delta\theta_i\} \\ U_i &= 2\{(\sin\Delta\theta_i\cos\Delta\phi_i - \cos\alpha\cos\Delta\theta_i\sin\Delta\phi_i)A_1 \\ &\quad - (\sin\Delta\phi_i\sin\Delta\theta_i + \cos\alpha\cos\Delta\theta_i\cos\Delta\phi_i)A_2 \\ &\quad + \sin\alpha\cos\Delta\theta_iA_3 - \sin\Delta\theta_i\} \\ V_i &= 2\sin\alpha\{A_1\sin\Delta\phi_i + A_2\cos\Delta\phi_i\} \\ W_i &= -2\{A_2\sin\Delta\phi_i - A_1\cos\Delta\phi_i\} \end{aligned} \tag{17}$$

Equation (16) is linear in A_4, A_5, A_6, and A_7. If α is specified, then the functions P_i, U_i, V_i, W_i are polynomials of degree one in the remaining design parameters A_1, A_2, and A_3.

The G_i's are obtained from equations (16) and (17), that is,

$$G_i = (\partial P_i/\partial\phi)A_4 + (\partial U_i/\partial\phi)A_5 + (\partial V_i/\partial\phi)A_6 - (\partial W_i/\partial\phi) \tag{18}$$

The partial derivatives in (18) are linear in parameters A_1, A_2, A_3 (see equation (17)). When substituted in equation (7), these G_i's yield an algebraic condition in the design parameters.

We now discuss the elimination of branching in four precision point synthesis of R-S-S-R linkage. Parameters α and A_3 are selected arbitrarily. Among the remaining parameters A_1 and A_2 will be selected in the following way so that the branch defect is eliminated.

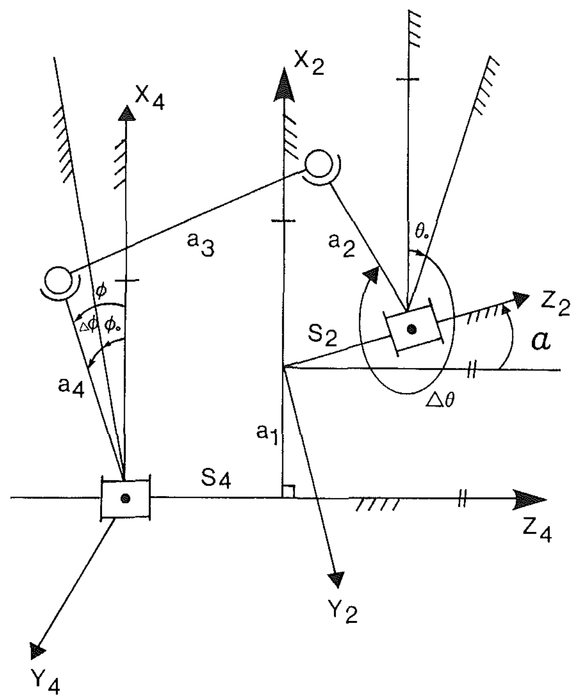

Fig. 5 Spatial R-S-S-R linkage and its parameters

The mechanism closure equation (16) can be written four times, once for each precision point, to obtain the following system of equations

$$\begin{bmatrix} P_1 & U_1 & V_1 & 1 \\ P_2 & U_2 & V_2 & 1 \\ P_3 & U_3 & V_3 & 1 \\ P_4 & U_4 & V_4 & 1 \end{bmatrix} \begin{bmatrix} A_4 \\ A_5 \\ A_6 \\ A_7 \end{bmatrix} = \begin{bmatrix} W_1 \\ W_2 \\ W_3 \\ W_4 \end{bmatrix} \tag{19}$$

The unknown parameters A_4, A_5, A_6, A_7 can be determined in terms of A_1 and A_2 implicitly contained in (P_i, U_i, V_i, W_i) as follows

$$\text{and} \quad \left.\begin{aligned} & A_4 = D_4/D, \quad A_5 = D_5/D, \quad A_6 = D_6/D \\ & A_7 = D_7/D \end{aligned}\right\} \tag{20}$$

The 4×4 determinants D_4, D_5, D_6, D_7, D are defined in equation (21)

$$\left.\begin{aligned} D &= |P_i\ U_i\ V_i\ 1| \\ D_4 &= |W_i\ U_i\ V_i\ 1| \\ &\ \ \vdots \\ D_7 &= |P_i\ U_i\ V_i\ W_i| \end{aligned}\right\} \tag{21}$$

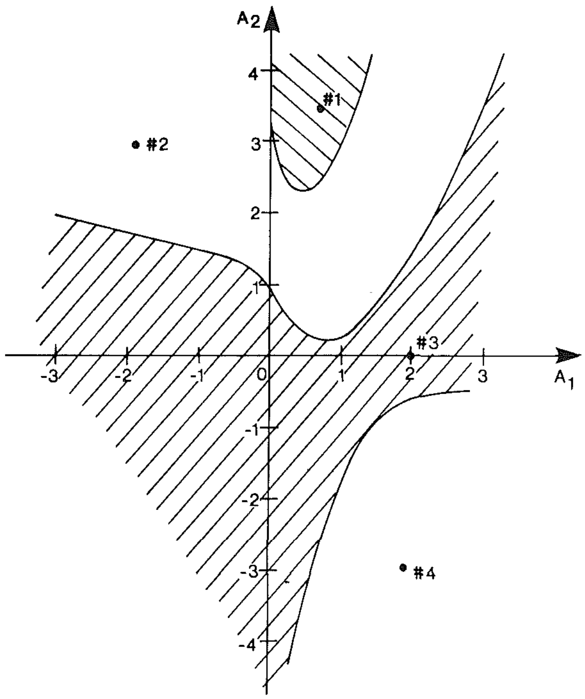

Fig. 6 Approximate feasible region (hatched) when $\alpha = 90$ deg, $S_4 = +1.0$

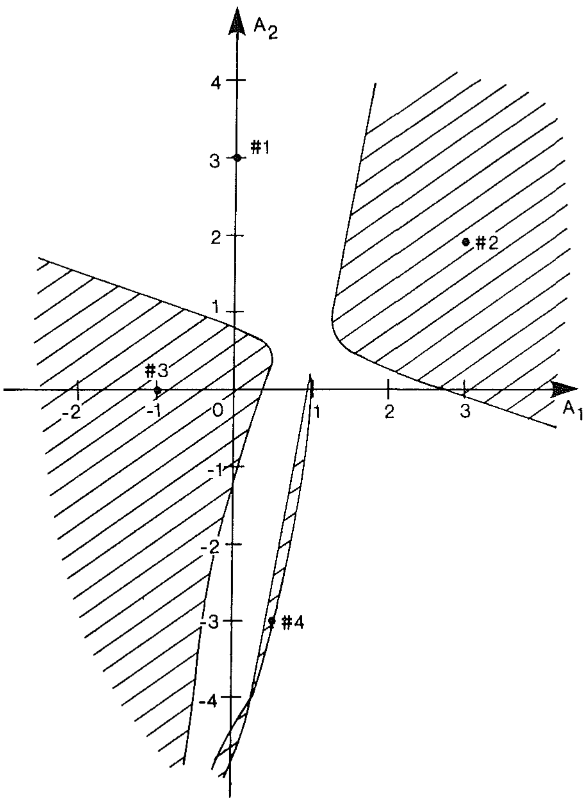

Fig. 7 Approximate feasible region (hatched) when $\alpha = 90$ deg, $S_4 = 0.0$

From the nature of the polynomials P_i, U_i, V_i, W_i, it is easily determined that in the $A_1 - A_2$ plane $D = 0$ and $D_6 = 0$ represent general cubic curves. $D_4 = 0$ and $D_5 = 0$ are cubic curves, each consisting of a real linear factor and a quadratic factor of the form $(A_1^2 + A_2^2)$ which has no real roots. $D_7 = 0$ is a quartic curve which degenerates into real quadratic curve because it has a quadratic factor $(A_1^2 + A_2^2)$.

If equation (18) is written in terms of the D_i's in equation (21), G_i's become

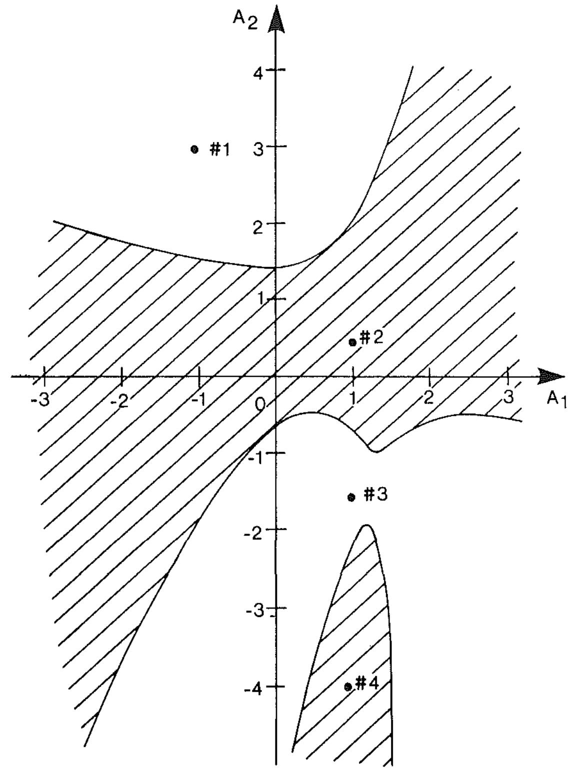

Fig. 8 Approximate feasible region (hatched) when $\alpha = 90$ deg, $S_4 = -1.0$

$$G_i = (\partial P_i/\partial\phi)D_4/D + (\partial U_i/\partial\phi)D_5/D + (\partial V_i/\partial\phi)D_6/D - (\partial W_i/\partial\phi) \qquad (22)$$

When the G_i's given in (22) are substituted into inequality (7), we obtain an eighth degree polynomial H_8 in terms of A_1 and A_2. The boundary curve $H_8(A_1, A_2) = 0$ degenerates into two quartic curves, one of which is the factor $(A_1^2 + A_2^2)^2 = 0$ which has no real points. The other quartic (i.e., $H_4(A_1, A_2) = 0$) is a real curve which defines the approximate feasible region $H_4(A_1, A_2) \leq 0$ for R-S-S-R mechanisms which are free from branch defect. The approximate feasible regions $H_4(A_1, A_2) \leq 0$ in Figs. 6, 7, and 8 were obtained using the following four precision points $(\Delta\theta_i, \Delta\phi_i)$: (0 deg, 0 deg), (19.4 deg, -5.125 deg), (53.03 deg, -30.165 deg), (91.85 deg, -69.695 deg) and $\alpha = 90$ deg. In Figs. 6, 7, and 8 we have taken follower offset $S_4 = 1$, 0, and -1, respectively. Shaded areas in the figures yield R-S-S-R mechanisms which are free from branch defect. It should be noted that because of the condition (7) some points in the unshaded areas close to the boundary may yield mechanisms which are not defective. R-S-S-R function generators corresponding to some of the points in Figs. 6, 7, 8 are given in Table 1.

For five precision point synthesis of R-S-S-R, the region $H_4 \leq 0$ is obtained from (7) and using the first four precision points. In addition, we must consider the quartic curve defined by the following 5×5 determinant [12]

$$|P_i \; U_i \; V_i \; 1 \; W_i| = 0 \qquad i = 1, \ldots, 5 \qquad (23)$$

The quartic in (23) consists of two quadratics, one of which is real [13] and the other is $A_1^2 + A_2^2 = 0$. Points (A_1, A_2) which lie on the real quadratic and are within the approximate feasible region $H_4 \leq 0$ will yield R-S-S-R mechanisms without branch defect. The quadratics in Figs. 9, 10, 11 were obtained by using (125.47 deg, -94.735 deg) as the fifth precision point. The solid lines indicate the feasible segments of these quadratics. Due to the approximate nature of condition (7), there are points which do not lie on the indicated feasible segment that also yield mechanisms without branch

Table 1 Analysis of four precision point R-S-S-R mechanisms ($a_1 = 1.0$, $\alpha = \pi/2$)

	Point	A_1	A_2	a_2	a_3	a_4	S_2	θ_0 (rad)	ϕ_0 (rad)	Branch defect
$S_4 = 1.0$ Fig. 6	1	0.7500	3.5000	4.6123	3.7873	3.5794	−3.4637	−2.3880	1.3597	No
	2	−2.0000	3.0000	2.0949	1.1567	3.6055	−3.4452	−2.8105	2.1588	Yes
	3	2.0000	0.0000	1.2540	2.8126	2.0000	−0.8997	2.4565	0.0000	No
	4	2.0000	−3.0000	1.1116	5.2686	3.6055	−1.6400	2.6679	−0.9827	Yes
$S_4 = 0.0$ Fig. 7	1	0.0000	3.0000	0.8322	1.9784	3.0000	−1.0286	3.1317	1.5708	Yes
	2	3.0000	2.0000	4.0088	5.9847	3.6055	−2.3563	−2.9131	0.5880	No
	3	−1.0000	0.0000	1.2439	2.2440	1.0000	−2.1109	−3.0849	0.0000	No
	4	0.5000	−3.0000	3.3901	6.2820	0.3041	5.5617	0.5899	−1.4056	No
$S_4 = -1.0$ Fig. 8	1	−1.0000	3.0000	0.6996	2.5992	3.1622	−0.9178	2.8022	1.8925	Yes
	2	1.0000	0.5000	1.2969	1.6549	1.1180	1.1233	1.6803	0.4636	No
	3	1.0000	−1.5000	1.6828	0.9466	1.8027	0.9193	1.3354	−0.9827	Yes
	4	1.0000	−4.0000	2.8069	2.2438	4.1231	3.1124	0.9703	−1.3258	No

Table 2 Analysis of five precision point R-S-S-R mechanisms ($a_1 = 1.0$, $\alpha = \pi/2$)

	Point	A_1	A_2	a_2	a_3	a_4	S_2	θ_0 (rad)	ϕ_0 (rad)
$S_4 = 1.0$ Fig. 9	P_1	−1.3215	−1.0933	4.6657	8.4660	1.7152	−7.0910	−2.6340	−2.4504
$S_4 = 0.0$ Fig. 10	P_1	−0.9048	−0.5742	1.2967	2.7932	1.0717	−2.1504	−3.0831	−2.5761
$S_4 = -1.0$	P_1	−1.1269	−0.6195	0.7923	2.4495	1.2860	−1.2704	2.7256	−2.6389
Fig. 11	P_2	2.1060	0.8338	1.5765	1.9326	2.2651	1.0679	0.9528	0.3769

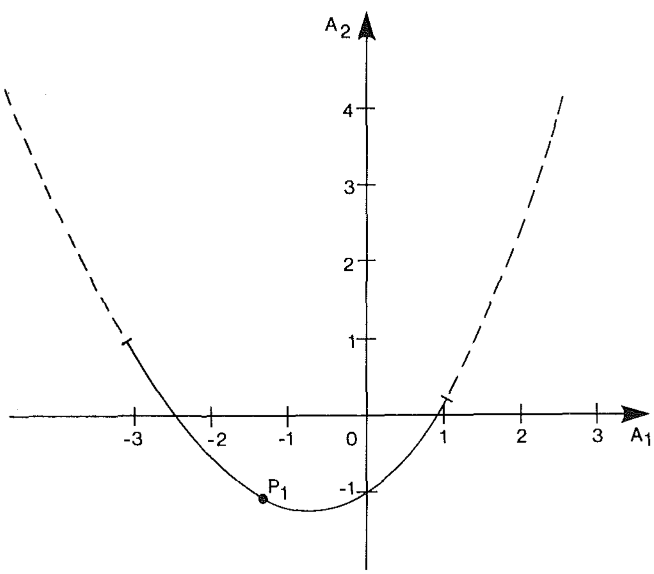

Fig. 9 Real second order curve for five precision point synthesis when α = 90 deg, S_4 = 1.0. The approximate feasible segment is shown with solid line.

defect. Table 2 gives R-S-S-R function generators corresponding to some points on the feasible segments of the quadratics in Figs. 9, 10, 11.

In a six precision point synthesis, the pictorial representations become more difficult. In this case, A_3 cannot be selected as before. Thus the only parameter specified is α. The first four precision points can be used to obtain $H_4(A_1, A_2, A_3) \leq 0$ as before. It now represents a volume in $A_1 - A_2 - A_3$ space. The points (A_1, A_2, A_3) which lie in this volume and also lie on the curve of intersection of the following second order surfaces yield desired linkages.

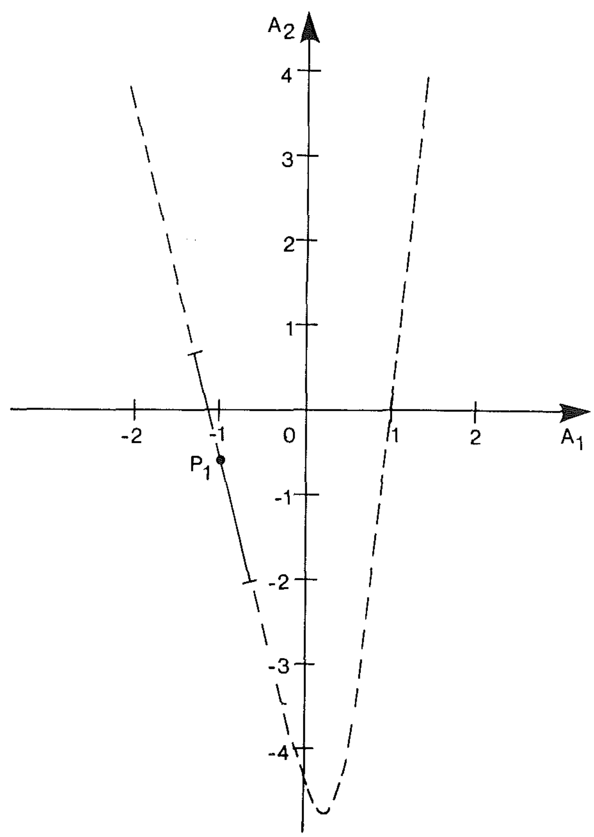

Fig. 10 Case α = 90 deg, S_4 = 0.0 (see Fig. 9 for description)

$$\left.\begin{array}{ll} |P_i \; U_i \; V_i \; 1 \; W_i| = 0 & i = 1, 2, 3, 4, 5 \\ |P_i \; U_i \; V_i \; 1 \; W_i| = 0 & i = 1, 2, 3, 4, 6 \end{array}\right\} \quad (24)$$

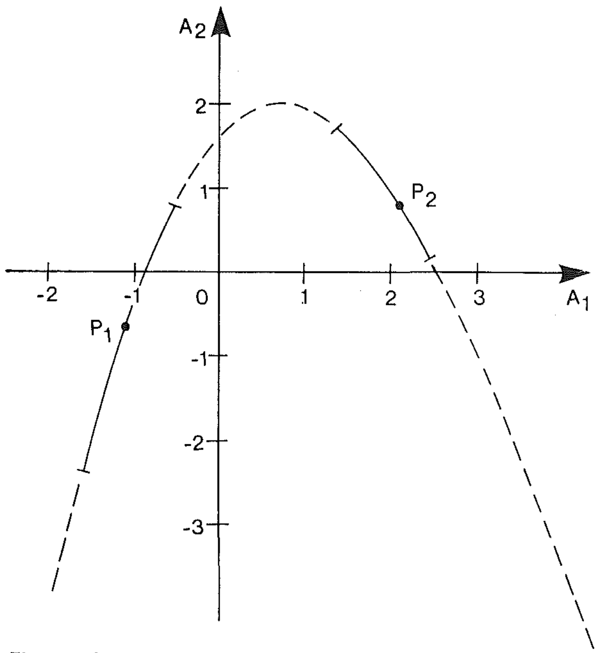

Fig. 11 Case α = 90 deg, S_4 = −1.0 (see Fig. 9 for description)

As shown in reference [13], the two surfaces in equation (24) intersect into two space curves. One of these is a space cubic [13] and points (A_1, A_2, A_3) on it can be used to design six precision point R-S-S-R linkages. Segments of the space cubic which also lie within the region $H_4(A_1, A_2, A_3) \leq 0$ yield R-S-S-R linkages without branch defect. The other curve of intersection of the surfaces in (24) is a spurious line [13].

A seven precision point R-S-S-R linkage design without branch defect would require that α be treated as a variable design parameter. The method outlined in the preceding paragraphs can, in principle, be extended for this case. However, it seems that in a practical situation which calls for an R-S-S-R function generator, the value of angle α would usually be specified.

Conclusions

We have shown how to design precision point bimodal function generators (planar and spatial) which meet the requirement of "no branch defects." The results of the present method can be combined with other quality requirements (e.g., crank rotatability, good transmission characteristics, etc.) to obtain suitable designs.

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