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A Note on Obtaining Temperatures From Thermocouple EMF Measurements

Six methods are given for converting to temperatures the measured values of emf generated by thermocouples. Two are based on the use of thermocouple reference tables alone. Two are based on the use of calibration data alone. And two are based on a combination of calibration data with reference table functions. It is shown that one method is preferred if graphical presentation is required, while another is clearly the best for computer conversions of thermocouple emf to temperatures.

Introduction

A very common problem which engineers face concerns the interpretation of thermocouple output in terms of temperature. That is, given an emf generated by a thermocouple, what is the corresponding temperature? In this paper, we avoid discussion of installation effects, velocity effects, heat transfer effects, and so on, and concentrate on methods for converting measurements of thermocouple emf to temperature equivalents by computer analysis.

Method 1

One method which immediately suggests itself is based on use of the NBS (National Bureau of Standards) thermocouple Reference Tables [1, 2].[1] These in turn are based on $E = f(T)$ equations which represent the nominal emf—temperature characteristics of thermocouples, and which are accepted by international agreement. For example, for the Type J thermocouple (which signifies an iron (+) and constantan (−) combination of materials), the specified equation is [1, 3]

$$E = (+5.0372753027 \times 10 \times T + 3.0425491284 \times 10^{-2} \times T^2 - 8.5669750464 \times 10^{-5} \times T^3 + 1.3348825735 \times 10^{-7} \times T^4 - 1.7022405966 \times 10^{-10} \times T^5 + 1.9416091001 \times 10^{-13} \times T^6 - 9.6391844859 \times 10^{-17} \times T^7) \times 10^{-3} \quad (1)$$

for the temperature range 0–760°C, where the voltage (E) is in millivolts, and the temperature (T) is in degrees Celsius.

A difficulty is encountered since T is desired rather than E, but this problem can be solved by iteration. That is, a guess of T is made and E is forthcoming. This process is repeated until a T is guessed which yields E (as close as is desired to the measured E) from equation (1). With digital computers, this is a rapid, routine method of solution. However, according to the thermocouple standards themselves [2, 4], the emf-temperature relationship of an individual thermocouple may differ from that given by the nominal reference table values by as much as 7.5°F (4.2°C) at the 1000°F (538°C) level. This recognized difference which depends on the type of materials involved and on temperature level is tabulated for various material combinations in a Limit of Error Table [2, 4].

To summarize: Use of the NBS thermocouple reference table alone is not satisfactory for obtaining a precise temperature from thermocouple emf because an iteration scheme is involved, and the method does not account for individual thermocouple characteristics which can differ markedly from the nominal reference table values.

Method 2

A second method removes one of the difficulties encountered in Method 1, namely the iteration problem. This is accomplished by making use of the direct approximation functions developed by NBS of the form $T = f(E)$. For example, for the Type J thermocouple, the NBS recommended equations are [1, 3]

$$T = 1.9750953 \times 10 \times E - 1.8542600 \times 10^{-1} \times E^2 + 8.3683958 \times 10^{-3} \times E^3 - 1.3280568 \times 10^{-4} \times E^4 \quad (2)$$

for the temperature range 0–400°C, where the voltage is in millivolts, and

$$T = 9.2808351 \times 10 + 5.4463817 \times E + 6.5254537 \times 10^{-1} \times E^2 - 1.3987013 \times 10^{-2} \times E^3 + 9.9364476 \times 10^{-5} \times E^4 \quad (3)$$

[1] Numbers in brackets designate References at end of paper.

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for the temperature range 400–760°C. Temperatures again are in degrees Celsius.

It should be emphasized that equations (2) and (3) provide only approximations to the functional relationship between E and T that is given by equation (1). That is, equation (1) and equations (2) and (3) do not lead to exactly the same solution. The difference is small, however, and amounts to about ±0.2°F (±0.1°C) for the Type J thermocouple in the range 32–1000°F (0–538°C). This discrepancy is negligible when compared to the obvious, and potentially larger, error caused by overlooking the individual thermocouple characteristics.

To summarize: Use of the inverse NBS relation of $T = f(E)$ is not satisfactory for obtaining temperature from thermocouple emf because the method does not account for individual thermocouple characteristics which can differ markedly from the inverse solution values.

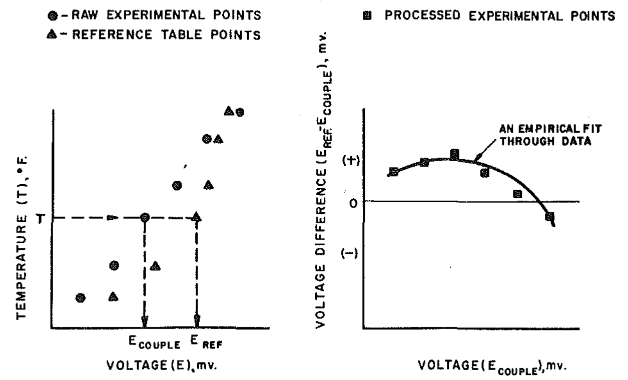

Fig. 1 Construction and use of voltage difference curves for representing thermocouple characteristics

Methods 3 and 4

Another general method for obtaining temperature from thermocouple emf combines individual thermocouple characteristics with an acceptable thermocouple reference table to obtain a mathematical relation between $\Delta E (= E_{REF} - E_{COUPLE})$ and E_{COUPLE}. This method has the advantage of displaying individual thermocouple characteristics on a magnified basis since it utilizes the difference from a reference table of nominal thermocouple characteristics. Such differences will usually be small and change slowly and smoothly with temperature level. This method is illustrated graphically in Fig. 1.

In calibration, one obtains ΔE by subtracting E_{COUPLE} from E_{REF} at each calibration temperature. A low degree least squares fit is then made to the corresponding values of ΔE and E_{COUPLE} and serves as the calibration characteristic of the individual thermocouple.

In use, one adds E_{COUPLE} to ΔE (obtained from the individual thermocouple characteristic) to obtain E_{REF}, and then solves the reference function (similar to equation (1)) for T iteratively, as described in Method 1.

In Method 3, the obvious choice of a reference table is used, namely; the NBS table as given by equation (1). Fig. 2 shows the ΔE_{NBS} versus E_{COUPLE} characteristics of three arbitrarily chosen Type J thermocouples. This illustrates that the emf output of an individual thermocouple can be either greater or less than the NBS table value. It can be seen that the maximum difference between the emfs of these three thermocouples and the NBS Table Value is on the order of ±1°F (about ±0.5°C). Of course, this 1°F is not to be interpreted as the maximum difference to be expected in other thermocouples.

In Method 4, a reference function other than the NBS function of equation (1) is used. One such function that suggests itself is a lower degree least squares fit of the NBS reference table. The obvious advantage would be the simplicity of the resulting function. For example, for the Type J thermocouple, the second degree least squares fit of equation (1) yields:

$$E = -9.34991530 \times 10^{-1} + 2.92152891 \times 10^{-2} \times T + 1.35877570 \times 10^{-6} \times T^2 \quad (4)$$

for the temperature range 32–1400°F (0–760°C), where the voltage is in millivolts and the temperature is in degrees Fahrenheit. However, if the higher degree NBS function of equation (1) more closely matches the thermocouple characteristics, then use of a lower degree fit (such as equation (4)) may result in wiggles in the difference plot. Fig. 3 shows the characteristics of the three thermocouples of Fig. 2 with respect to the second degree reference function of equation (4). For purposes of comparison, the NBS reference function is also shown. The wiggles introduced by use of the second degree reference function are readily apparent. The maximum differences between the second degree reference function and the higher degree NBS function are on the order of ±4°F (±2°C). It is clear that the NBS reference function matches all three thermocouples (arbitrarily chosen) better than does the second degree reference function. As these results show, the NBS did a good job matching representative thermocouple outputs with the nominal function of equation (1).

To summarize: Use of a reference table together with individual thermocouple data yields temperatures which are quite satisfactory from an accuracy viewpoint, but which involve the complication of an iteration scheme. The NBS reference table used in Method 3 more closely follows today's thermocouple materials than does the second degree least squares reference table of Method 4. However, for these examples with the given number of points, the accuracy of converting thermocouple emfs to temperature essentially is independent of the reference table chosen.

Methods 5 and 6

Having obtained the calibration data required of Methods 3 and 4, one can go directly to curve fits to represent the individual thermocouple characteristics, independent of the use of any reference table.

Two possibilities, as in Methods 1 and 2, are apparent. In Method 5, we fit $E_{COUPLE} = f(T)$, while for Method 6 we fit $T = f(E_{COUPLE})$.

As with Method 1, Method 5 involves an iteration scheme; however, unlike Method 1, the individual thermocouple characteristics are accounted for in Method 5.

As with Method 2, Method 6 avoids any iteration; however, unlike Method 2, the individual thermocouple characteristics are accounted for in Method 6.

To summarize: Use of the direct fits of $E_{COUPLE} = f(T)$ and $T = f(E_{COUPLE})$ is convenient in that use of thermocouple reference tables is avoided. These direct fits, over any extended range, invariably are of higher degree equations than the $\Delta E = f(E_{COUPLE})$

Nomenclature

E = electromotive force

f = function

J = iron-constantan thermocouple

T = temperature

Δ = finite difference

NBS = National Bureau of Standards

Subscripts

COUPLE = pertaining to thermocouple

REF = pertaining to reference table

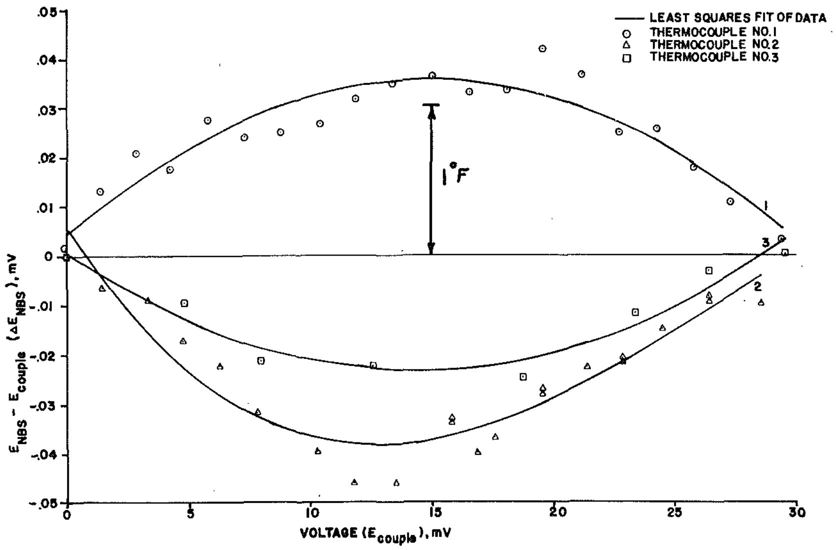

Fig. 2 Voltage difference curves for three Type *J* thermocouples with respect to the NBS Table Values (E_{NBS})

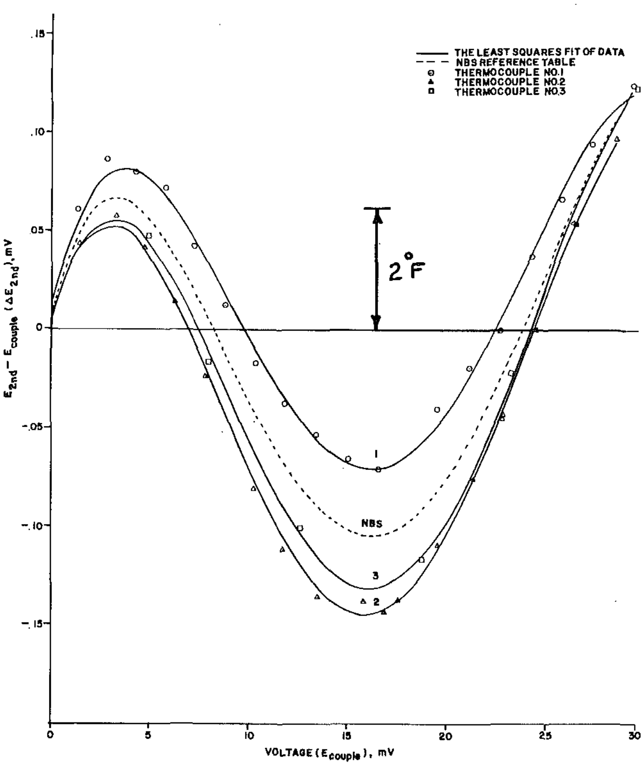

Fig. 3 Voltage difference curves for the three Type *J* thermocouples of Fig. 2 and for the NBS reference table with respect to a second degree reference voltage function

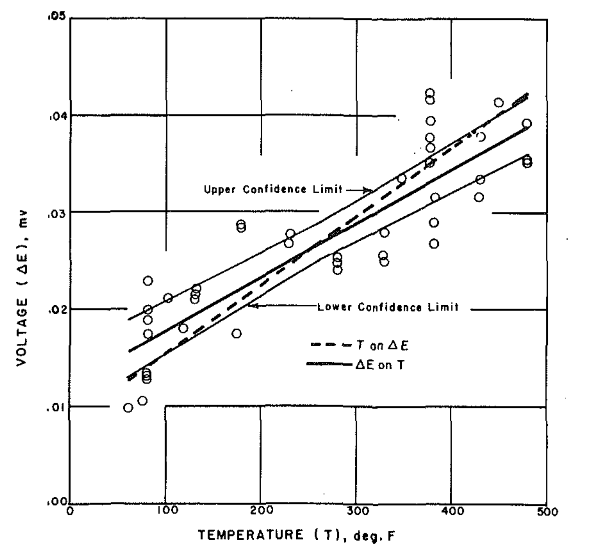

Fig. 4 Comparison of two regressions (*T* on ΔE and ΔE on *T*) based on the same thermocouple calibration data.

equations. However, using computers to reduce the data, this is not considered a problem. Finally, there could be an uncertainty introduced (as between Methods 1 and 2), in that the results of Methods 5 and 6 may differ because neither yields the true functional relationship between E and T. Fig. 4 illustrates this possibility. Here, is shown a fit of $\Delta E = f(T)$ and $T = g(\Delta E)$. Only when these two fits coincide are we assured that the true functional relationship has been found. As we shall see in the results, the uncertainty indicated in Fig. 4 usually does not materialize, the Methods 5 and 6 are each satisfactory for obtaining temperatures from thermocouple emfs.

Results

Temperatures, obtained by the six methods, are compared with the actual calibration temperatures in Fig. 5, for one of the three Type J thermocouples used in this study. The same information, in digital from, is presented in Tables 1, 2, and 3 for all three of the Type J thermocouples.

It is clear from these results that all six methods can be used to obtain temperatures from thermocouple emfs. However, it can be seen that Methods 1 and 2, which neglect individual thermocouple characteristics, are the least preferable methods. The difference methods (3 and 4), and the direct methods (5 and 6), are insignificantly different in the final results. Any choice between them must be made on the basis of the difficulty involved in obtaining temperature, and/or on the intended presentation of the final information.

If a graphical presentation of thermocouple characteristics is required, the difference method has the advantage because differences change only slowly and smoothly with level, and magnifica-

Table 1 Differences between actual thermocouple temperatures and temperatures generated by Methods 1 through 6 for thermocouple No. 1

Thermocouple No. 1

E_{COUPLE} (mV)	T_{COUPLE} (^oF)	Values of $T_{couple} - T_{method}$ (ΔT_{couple}), ^oF METHOD 1	METHOD 2	METHOD 3	METHOD 4	METHOD 5	METHOD 6
-.0017	32.00	.06	.06	-.09	-.28	-.02	-.07
1.3597	80.33	.45	.57	.12	.17	.02	.08
2.8058	130.23	.71	.79	.20	.30	.07	.11
4.2320	178.14	.59	.60	-.06	-.01	-.15	-.16
5.7876	230.11	.95	.88	.13	.14	.13	.09
7.2500	278.02	.79	.74	-.11	-.12	-.04	-.07
8.7856	328.15	.82	.78	-.17	-.19	-.05	-.06
10.3950	380.50	.87	.87	-.20	-.19	-.05	-.04
11.8519	427.90	1.03	1.06	-.08	-.06	.05	.07
13.3807	477.56	1.14	1.19	-.01	.04	.08	.11
15.0103	530.47	1.18	1.22	.02	.08	.04	.06
16.5340	579.88	1.08	1.09	-.07	-.02	-.12	-.11
18.0770	630.12	1.00	1.07	-.02	.00	-.13	-.14
19.5425	678.18	1.37	1.33	.31	.30	.17	.15
21.1289	729.80	1.20	1.19	.22	.17	.08	.06
22.6597	779.37	.81	.84	-.07	-.16	-.17	-.18
24.2104	829.94	.84	.89	.08	-.04	.05	.05
25.7582	879.94	.53	.59	-.03	-.15	.01	.03
27.2990	929.47	.34	.32	-.09	-.14	.01	.03
29.3840	995.92	.10	.06	-.08	.16	-.01	-.02

T_{method} is obtained as follows:

METHOD 1 - NBS Reference Temp. for given E_{couple}.
METHOD 2 - NBS Inverse Equation Temp. for given E_{couple}.
METHOD 3 - Temp. from conventional calibration equation $(E_{REF} - E_{COUPLE}) = f\ (E_{COUPLE})$
METHOD 4 - Temp. from same method as 3, but replacing NBS as reference with 2nd degree L.S. fit of NBS tables.
METHOD 5 - Temp. from 5th degree L. S. equation $E_{COUPLE} = f\ (T_{COUPLE})$.
METHOD 6 - Temp. from 5th degree L. S. equation $T_{COUPLE} = f\ (E_{COUPLE})$.

Table 2 Same as Table 1 except for thermocouple No. 2

Thermocouple No. 2

E_{COUPLE} (mV)	T_{COUPLE} (^oF)	Values of $T_{couple} - T_{method}$ (ΔT_{couple}), ^{o}F METHOD 1	METHOD 2	METHOD 3	METHOD 4	METHOD 5	METHOD 6
0.0000	32.00	0.00	0.00	-.21	-.12	-.07	-.15
1.4660	83.34	-.22	-.11	-.06	.03	-.03	.05
3.3010	145.98	-.31	-.25	.22	.19	.14	.21
4.7430	194.01	-.58	-.59	.17	.06	.05	.07
6.2770	244.58	-.73	-.79	.20	.05	.08	.05
7.8220	294.89	-1.04	-1.09	.03	.09	-.06	-.11
10.2810	374.63	-1.30	-1.29	-.09	-.12	-.10	-.14
11.7340	421.55	-1.49	-1.46	-.25	-.21	-.21	-.23
13.4910	478.49	-1.50	-1.46	-.26	-.16	-.17	-.17
15.8030	553.94	-1.10	-1.07	.09	.21	.20	.22
15.8110	554.21	-1.08	-1.06	.10	.22	.21	.23
16.8670	588.33	-1.30	-1.30	-.15	-.05	-.06	-.03
17.5890	611.94	-1.19	-1.21	-.08	.01	-.00	.02
19.5550	676.33	-.89	-.33	.09	.11	.11	.12
19.5580	676.41	-.91	-.95	.07	.09	.09	.10
21.3640	735.53	-.74	-.74	.09	.03	.03	.03
22.8230	783.17	-.72	-.68	-.01	-.13	-.12	-.14
22.8250	783.27	-.68	-.64	.03	-.09	-.09	-.10
24.4660	836.93	-.48	-.44	.06	-.08	-.08	-.10
26.3970	899.76	-.28	-.28	.08	.01	.01	-.01
28.5080	967.60	-.31	-.35	-.18	.07	.06	.10

Table 3 Same as Table 1 except for thermocouple No. 3

Thermocouple No. 3

E_{COUPLE} (mV)	T_{COUPLE} (Deg.F)	Values of $T_{COUPLE} - T_{METHOD}$ (ΔT_{COUPLE}), ^oF METHOD 1	METHOD 2	METHOD 3	METHOD 4	METHOD 5	METHOD 6
0.0000	32.00	0.00	.00	-.03	-.02	-.01	-.02
4.9080	199.76	-.31	-.33	.12	.11	.09	.14
7.9640	299.85	-.71	-.76	-.10	-.17	-.14	-.18
12.5830	449.84	-.72	-.68	.03	.09	.09	.07
18.7400	649.83	-.80	-.85	-.11	-.03	-.04	-.00
23.3240	799.84	-.38	-.33	.08	.01	.02	.02
26.3980	899.97	-.10	-.11	.11	-.02	-.01	-.04
29.5020	999.59	.02	-.03	-.09	.01	.00	.02

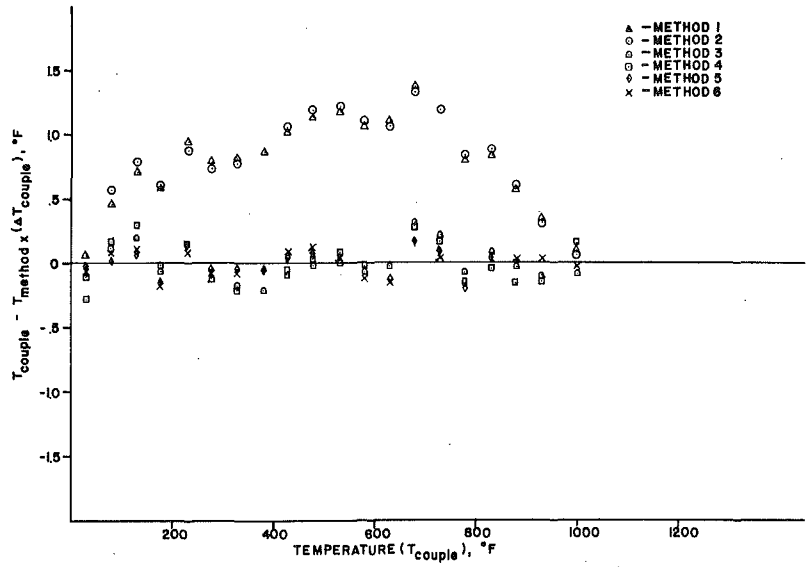

Fig. 5 Temperature difference curves for thermocouple 1 of Fig. 2, comparing the results of Methods 1–6

tion is possible. Of the two difference methods, the one based on the NBS reference table (i.e., Method 3) is believed the most desirable.

For the great bulk of the emf conversions, where computers are involved, the direct methods are clearly the most desirable. Of the two direct methods, the one based on $T = f(E_{COUPLE})$ (i.e., Method 6) is far and away the simplest and hence the preferred method.

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