Study on Dynamic Strength Evaluation Method of Mechanical Members Based on Energy Balance

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In Japan, mechanical structures installed in nuclear power plants, such as piping and equipment, are usually designed statically in an elastic region. Although these mechanical structures have sufficient seismic safety margin, understanding the ultimate fatigue endurance is very important in order to improve the seismic safety reliability for unexpected severe earthquakes. Moreover, clarifying a margin of seismic resistance of mechanical structures that suffered a severe earthquake is being required. In this study, the energy balance equation that is one of valid methods for structural calculation is applied to the above-mentioned issues. The main feature of the energy balance equation is that it explains accumulated information of motion. Therefore the energy balance is adequate for the investigation of the influence of cumulative load such as seismic response. The investigation is implemented by forced vibration experiments. The experiment models are simple single–degree-of-freedom models that are made of stainless steel and carbon steel. In the experiment, random waves having predominant frequency similar to natural frequency of the experimental model are input in order to obtain adequate response not only in the elastic region but also in the plastic region. As a result, experimental models vibrate under resonance condition, so response acceleration is approximately seven times as big as the input. The excitation is continued until the experimental models fracture, and is carried out with various input levels. In the experiment, models suffered cracks at the bottom end, and fractured finally. As a result, input energy for failure increases as time for failure. In other words, more input energy for failure is needed in case of small input. Moreover the correlation between increment in input energy and input energy for failure is investigated. It was confirmed that input energy for failure is inversely proportional to increment in input energy per unit time. Additionally energy for failure of stainless steel is about twice as big as carbon steel. The correlation between fatigue failure and energy is confirmed from the vibration experiment. Therefore it is expected that time for fatigue failure can be evaluated by the energy balance equation.
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Keywords: energy balance, fatigue failure, seismic strength

1 Introduction

In Japan, mechanical structures installed in nuclear power plants, such as piping and equipment, are generally designed statically in an elastic region. Moreover, a static seismic force that is 3.6 times stronger than seismic force for an ordinary building must be considered in accordance with the regulatory guide called "Regulatory Guide for Reviewing Seismic Design of Nuclear Power Reactor Facilities" [1]. Therefore mechanical structures in nuclear power plants in Japan have a sufficient seismic margin. On the other hand, unexpected severe earthquakes such as The Great Kobe Earthquake (1995) and The Niigataken Chuetsu-oki Earthquake (2007) have occurred. In particular, earthquake waves observed at Kashiwazaki-Kariwa nuclear power plant in the Niigataken Chuetsu-oki Earthquake exceeded the design earthquake ground motion, although no damage to cause a leak of radioactivity occurred. Therefore it is very important to grasp vibration characteristics in a plastic region for unexpected severe earthquakes. In addition, to grasp residual fatigue lives quantitatively is also required for a safety operation of nuclear power plants after a severe earthquake.

On the other hand, it has been reported that failure of ordinary piping in earthquakes is produced not by momentary large load but by cumulative fatigue damage [2]. In response to this, many methods for evaluation of seismic resistance have been proposed. One of them is the energy balance equation that was developed by Housner [3,4] and Akiyama and co-worker [5,6]. The energy balance equation is one of valid methods for structural calculation, and it is easily derived from the equation of motion. Since energy indicates cumulative information of motion, the energy balance equation is suitable to evaluate seismic response. For prediction of low cyclic fatigue lives, some studies exist that adopt hysteresis energy obtained from an area of a hysteresis loop. These studies using a hysteresis energy are usually investigated by the typical both ends fixed tensile-compression fatigue test. However vibration response is more complicated, and nonlinear vibration response should be considered. Hence it is difficult to adopted results of these studies for evaluation of seismic resistance without further investigations. On the other hand, there are few studies that investigate low cyclic fatigue lives from the viewpoint of the

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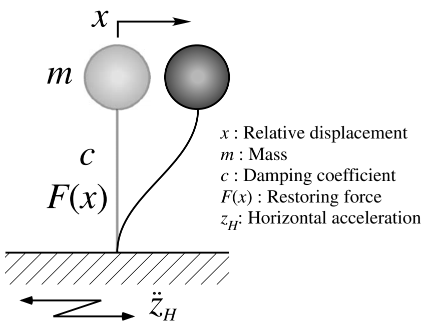

Fig. 1 Single degree of freedom model

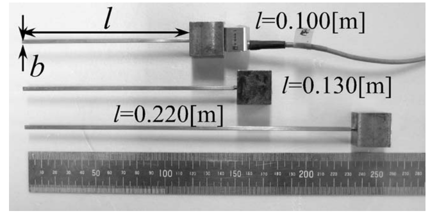

Fig. 2 Experimental models

energy balance equation instead of the hysteresis energy. Additionally almost all studies using the energy balance equation are conducted by simulation. In other words, relationship between the energy balance equation and failure by earthquake has not been clarified yet. Therefore investigations into the relationship are required.

This paper deals with an evaluation of seismic resistance using the energy balance equation. The aim of this study is to investigate the relationship between the energy balance equation and fatigue failure. The relationship is investigated by vibration experiment that leads experimental models to fatigue failure.

2 Energy Balance

In this study, energy balance method is adopted for evaluation of vibration and fatigue accumulation. This method is one of the valid methods for building structures in architectural engineering field. A notification called "Earthquake-Resistant Calculation Method based on Energy Balance" has been established since Sept. 2005 in Japan [7].

The energy balance equation is derived from the equation of motion as follows. Therefore the energy balance equation is able to explain vibration characteristics dynamically. Equation (1) shows the equation of motion of single-degree-of-freedom model, such as that Fig. 1

$$m\ddot{x} + c\dot{x} + F(x) = -m\ddot{z}_H \quad (1)$$

where m is the mass, c is the damping coefficient, and $\ddot{z}_H$ is the horizontal acceleration of ground motion. These operate in the system as inertia force $m\ddot{x}$, damping force $c\dot{x}$, restoring force $F(x)$, and vibration disturbance $-m\ddot{z}_H$. Multiplying Eq. (1) by the displacement increment dx ($=\dot{x}dt$) leads to the work of the time dt, as shown in Eq. (2)

$$m\ddot{x}\dot{x}dt + c\dot{x}^2dt + F(x)\dot{x}dt = -m\ddot{z}_H\dot{x}dt \quad (2)$$

Then the energy balance equation is obtained by the time integral of Eq. (2), as shown in Eq. (3)

$$m\int_0^t \ddot{x}\dot{x}dt + c\int_0^t \dot{x}^2dt + \int_0^t F(x)\dot{x}dt = -m\int_0^t \ddot{z}_H\dot{x}dt \quad (3)$$

where $m\int_0^t\ddot{x}\dot{x}dt$ is the kinetic energy, $c\int_0^t\dot{x}^2dt$ is the dissipation energy by viscous damping, $\int_0^tF(x)\dot{x}dt$ is the sum of the elastic strain energy and the cumulative plastic energy, and $-m\int_0^t\ddot{z}_H\dot{x}dt$ is the input energy.

The energy balance equation, Eq. (3), shows a sum of the work until the time t, although the equation of motion shows the momentary condition at the time t. Therefore energy balance is adequate to investigate influence of the cumulative load because it includes cumulative information.

In the energy balance equation, only dissipation energy by viscous damping and plastic energy are cumulated. Furthermore kinetic energy and elastic energy converge after vibration of the system finished. Therefore the sum of all cumulated energy, the dissipation energy, and the plastic energy is equal to the input energy after vibration of system finished. Input energy consists of parameters that are easy to measure, although the dissipation energy and plastic energy consist of parameters difficult to measure, such as a damping coefficient, a yielding stress, and so on. Additionally input energy is basically stable against errors. For these reasons, input energy is very important information, so input energy is focused on in this study.

3 Fatigue Failure Vibration Experiment

Forced vibration experiment using random waves is implemented. In this experiment, models are damaged by fatigue failure that vibration response under resonance condition causes. Input energy obtained by this experiment was utilized for evaluation of correlation between fatigue failure and energy in Sec. 4.

3.1 Experimental Model. Experimental models are designed to satisfy following requirements: easy natural frequency changeability, plastic ability, uniformity among each model, and verifiability between experiment and simulation. Figure 2 shows experimental models. The experimental model consists of a mass and a pole. The experimental model is such an easy structure that it can be considered a single-degree-of-freedom model. Moreover the experimental model has few differences between individual pieces. Also the experimental model has linearity in the elastic region, and a plastic deformation occurs even in small response acceleration. Therefore the experimental models are suitable for this study.

The mass is a cube that has breadth of 0.023 m on a side, it is made of carbon steel Japanese Industrial Standard (JIS) SS400 and weighs 0.114 kg, including an accelerometer. The pole is made of stainless steel JIS SUS304 that has been often used for piping or equipment in nuclear power plants, and carbon steel JIS SS400 that has been often used in a general mechanical structures. Table 1 shows mechanical properties of JIS SUS304 and JIS SS400. The length of poles is shown in Table 2 and cross section of poles is square, having 0.003 m on a side. Changing length of the pole changes the natural frequency of the experimental model. The natural frequency and the damping ratio are measured by impulse experiment before each excitation. Nominal natural frequencies and nominal damping ratios are shown in Table 2 as well.

3.2 Experimental Setup. A shaking table with hydraulic actuator having maximum excitation force of 10,000 N, a velocity of 0.5 m/s, and an acceleration of 30 $\mathrm{m/s^2}$ is utilized for this experiment. Figure 3 shows the shaking table with the experimental

Table 1 Mechanical properties of materials

	Young's modulus (GPa)	Yield stress (MPa)	Ultimate stress (MPa)
Stainless steel JIS SUS304	197	205	520
Carbon steel JIS SS400	206	240	450

Table 2 Natural frequency and damping ratio of experimental model

Length l (m)	SUS304		SS400	
	Natural freq. (Hz)	Damping ratio (%)	Natural freq. (Hz)	Damping ratio (%)
0.100	21.3 (21.0–21.9)	3.34 (1.89–4.28)	21.6 (21.4–21.8)	2.44 (1.90–3.42)
0.130	15.1 (14.8–15.4)	1.43 (0.75–2.12)	15.4 (15.2–15.7)	2.30 (1.71–4.84)
0.220	7.44 (7.35–7.60)	1.59 (0.70–2.28)	7.39 (7.21–7.64)	2.49 (1.33–4.90)

Upper: nominal.
Lower: range of measured data.

model 0.100 m in length and made of stainless steel JIS SUS304

A little phase difference between the acceleration, the velocity, and the displacement in vibration experiment induces much error in computation phase of the energy balance equation because an error is also cumulated by the integration of Eq. (3). Therefore only response acceleration at the top of the model and input acceleration at the shaking table are measured throughout this experiment. Response displacement and velocity that are required in the computation phase of energy are calculated from the response acceleration by Simpson's integral formula. Also Simpson's integral formula is utilized for the integration in Eq. (3).

3.3 Experimental Procedure. Experiment that leads experimental model to fatigue failure by continuous vibration disturbance is carried out. Random waves having predominant frequency similar to the natural frequency of the experimental model are input, so experimental models vibrate in the resonance condition. Table 3 shows characteristics of input random waves. 10 waves for the 0.220 m model, 12 waves for 0.130 m models, and 6 waves for 0.100 m models are made. The random waves are made of normally distributed random number filtered by digital filter. In case of random waves input, it is expected that experimental models can vibrate sufficiently even after natural frequency decline by reason of yield, as well as in an elastic region. Moreover it is expected by using a random wave that there is little influence of work hardening and the Bauschinger effect on response. Two types of random waves for 30 s are prepared for each experimental model, and input to the experimental model by several amplitudes, as shown in Table 3. An excitation is continued until the experimental model is damaged, and is carried out with various input levels. The experimental data are measured for 30 s on every 300 s interval.

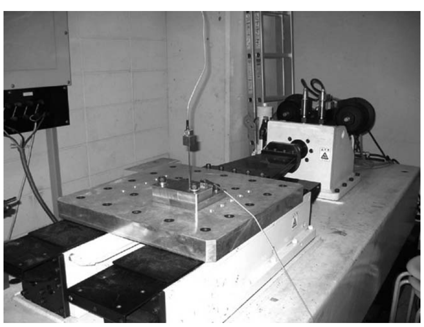

Fig. 3 Shaking table with experimental model

Table 3 Characteristics of input random wave

Length l (m)	Base wave	Time length (s)	Passed freq. (Hz)	Input wave No.	Input wave Max. acc. $(\mathrm{m/s^2})$
0.220	Normally distributed random number	30	6–8	1	20.0, 24.5, 27.7 29.1, 31.3
				2	22.6, 25.9, 30.2 32.3, 34.4
0.130			13–16	1	16.0, 19.3, 22.6 25.7, 29.4, 32.4
				2	16.7, 21.2, 24.2 28.8, 29.8 35.4
0.100			19–22	1	25.1, 28.8, 31.6
				2	25.5, 29.8, 32.5

3.4 Experimental Result. In this experiment, fatigue failure occurred in all experimental models. At first, the bottom end of the experimental model suffered a crack, and then the crack grew with the gradually decreasing natural frequency. Finally the crack grew completely, so that experimental model fractured. Figure 4 shows time histories for stainless steel JIS SUS304 0.130 m in length as an example. Upper row shows results of the first excitation, second row shows results of the final excitation among recorded results, and lower row shows power spectra obtained by fast Fourier transforms, respectively. From Fig. 4, response acceleration is approximately seven times as big as the input, so that effectiveness of random waves is confirmed. In addition it is confirmed that experimental models have maintained stable response until just before fatigue failure. Therefore prepared random waves satisfy the requirement of this experiment, and are suitable for this experiment. From power spectra, it is confirmed that spectra of response have peaks from 14 Hz to 16 Hz, although the spectrum of input acceleration has peaks from 10 Hz to 20 Hz. The reason for this is that experimental models filter only its natural frequency. In addition, predominant frequency of response acceleration and displacement of final excitation slightly decline compared with initial excitation. This results from decline of natural frequency of experimental model due to occurrence of crack.

Figure 5 shows changes in responses of experimental models made of stainless steel JIS SUS304. Also Fig. 6 shows changes in responses of experimental models made of carbon steel JIS SS400. These figures consist of (a) changes in maximum response acceleration and (b) changes in input energy of each recorded data. From graphs of changes in maximum response acceleration, it is confirmed that experimental models respond steadily until just before fatigue failure. Moreover it is expected for cracks to occur when the maximum response acceleration decreases rapidly. In case of large input, experimental models are damaged earlier than those with small input. This tendency agrees with the typical fatigue fracture test. Furthermore experimental models made of stainless steel JIS SUS304 are damaged earlier than carbon steel JIS SS400. For example, stainless steel broke at 2400 s when the input acceleration was 35.1 $\mathrm{m/s^2}$, and carbon steel broke at 5500 s when input acceleration was 35.4 $\mathrm{m/s^2}$. From graphs of changes in input energy, it is confirmed that input energy for failure decreases with an increase in the input acceleration. Although this tendency is also confirmed from results of carbon steel JIS SS400, energy for failure is twice as big as stainless steel JIS SUS304. Therefore the material dependency between fatigue failure and energy was confirmed. We attempt to investigate regarding these tendencies in detail in Sec. 4.

4 Correlation Between Fatigue Failure and Input Energy

Experimental results obtained in Sec. 3 are arranged synthetically in this chapter. The arrangement is implemented from the viewpoint of both response and input.

4.1 Investigation Focused on the Response. In this section, correlation between fatigue failure and input energy is investigated from the viewpoint of (a) a relationship between time for failure and input energy for failure, and (b) a relationship between an increment in input energy per unit time and input energy for

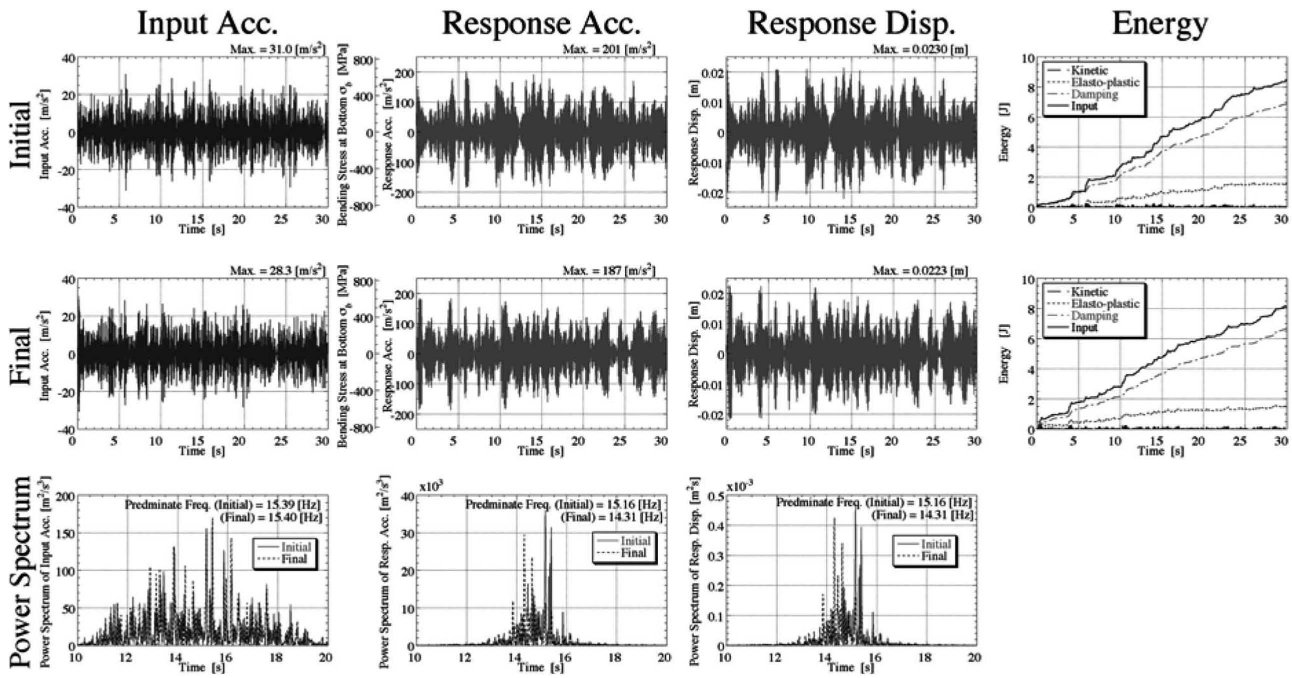

Fig. 4 Time histories of fatigue failure vibration experiment (stainless steel, 0.130 m, 15.2 Hz, and Wave 2)

failure. An increment in input energy per unit time is simply derived from the ratio of time for failure to input energy for failure. In other words, this is same as slope of the graph about the relationship between time for failure and input energy for failure.

Figure 7 shows the correlation between response and input energy for failure of experimental model made of stainless steel JIS SUS304. Also Fig. 8 shows the correlation of experimental model made of carbon steel JIS SS400. From these graphs, it is confirmed that input energy for failure is proportional to time for failure, and is inversely proportional to increment in input energy per unit time. Although there are a few differences, the slope of plots is similar in the case of the same material.

An increment in input energy per unit time is a more useful parameter in this investigation because an increment in input en-

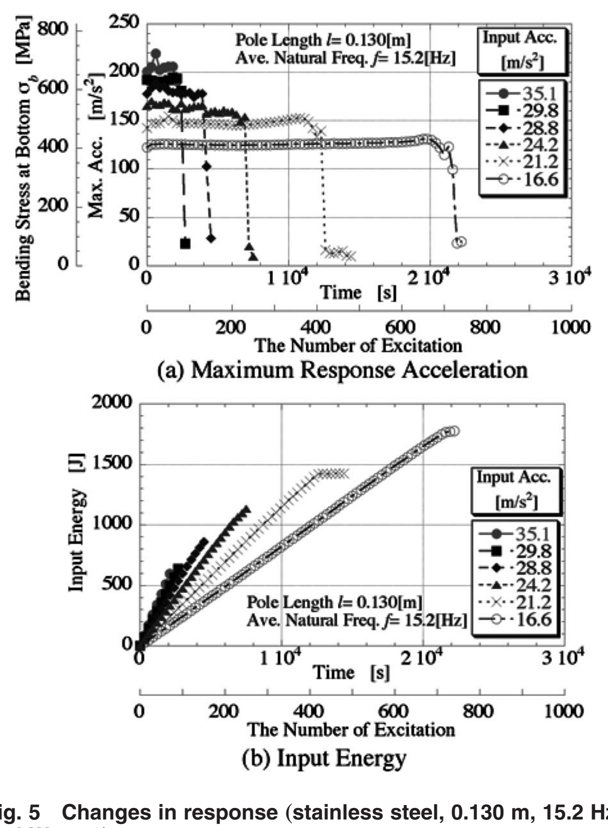

Fig. 5 Changes in response (stainless steel, 0.130 m, 15.2 Hz, and Wave 2)

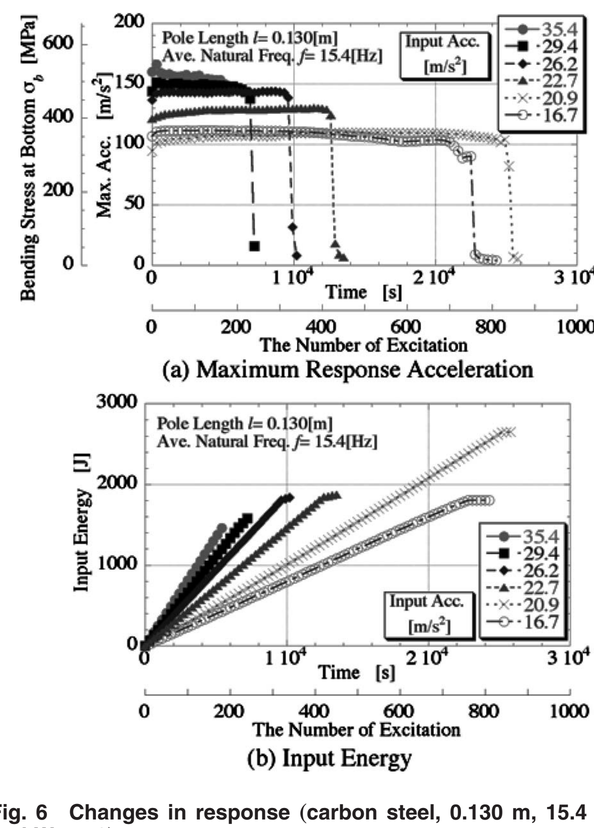

Fig. 6 Changes in response (carbon steel, 0.130 m, 15.4 Hz, and Wave 2)

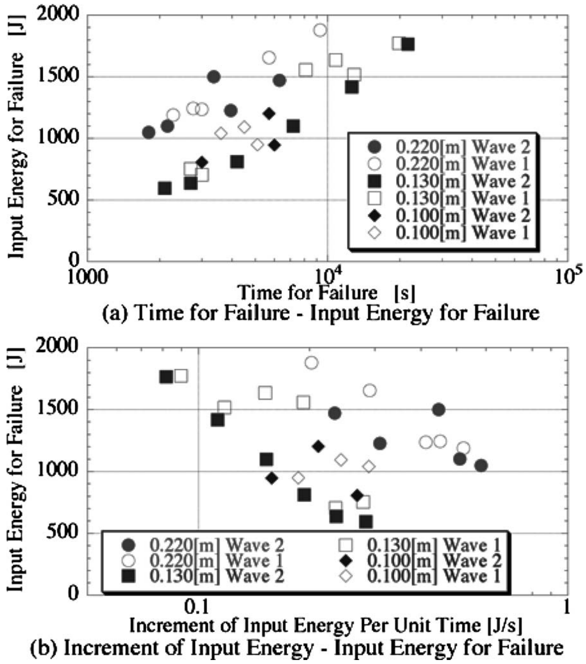

Fig. 7 Time and input energy for failure (stainless steel)

ergy can be calculated before a model fractures. That is to say, an increment in input energy per unit time can be adopted to fatigue life predictions. Therefore an increment in input energy per unit time is investigated in detail. Figures 9 and 10 show the correlation between an increment in input energy per unit time and input energy for failure of experimental models that have a length of 0.130 m. Figure 9 shows the correlation of stainless steel JIS SUS304, and Fig. 10 shows the correlation of carbon steel JIS SS400, and both graphs include regression lines. It is confirmed that input energy for failure decreases according to an increase in the increment in input energy per unit time. Moreover, regression lines of stainless steel JIS SUS304 have a slope that is about 1.6 times as big as carbon steel JIS SS400, and have an intercept that is equal to carbon steel JIS SS400. In addition, input energy for failure of stainless steel is twice as big as carbon steel if the increment in input energy per unit time is the same.

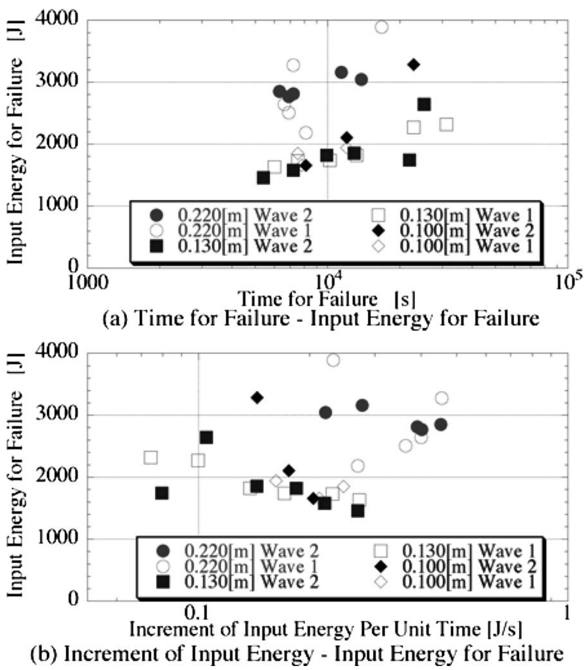

Fig. 8 Time and input energy for failure (carbon steel)

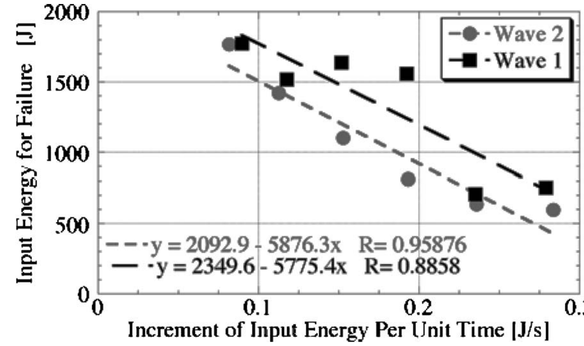

Fig. 9 Increment in input energy and input energy for failure (stainless steel, 0.130 m, and 15.2 Hz)

Consequently, correlation between fatigue failure and input energy is confirmed. In addition, the material dependency of the correlation between time for failure and input energy for failure is also confirmed.

4.2 Investigation Focused on the Input. In this section, the correlation between fatigue failure and input energy is investigated from the viewpoint of relationship between maximum input acceleration and input energy for failure. Although maximum input accelerations depend on input waves, it represents wave strength.

Figure 11 shows the correlation between maximum input acceleration and input energy for failure of stainless steel JIS SUS304. Also Fig. 12 shows the correlation of carbon steel JIS SS400.

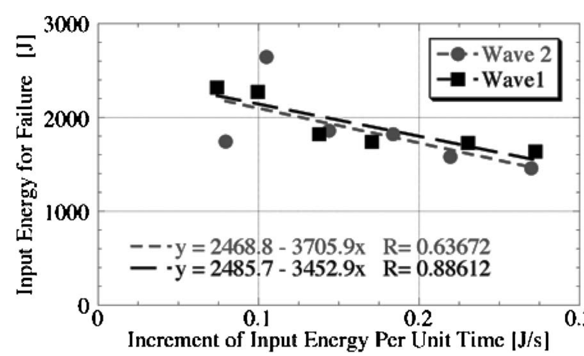

Fig. 10 Increment in input energy and input energy for failure (carbon steel, 0.130 m, and 15.4 Hz)

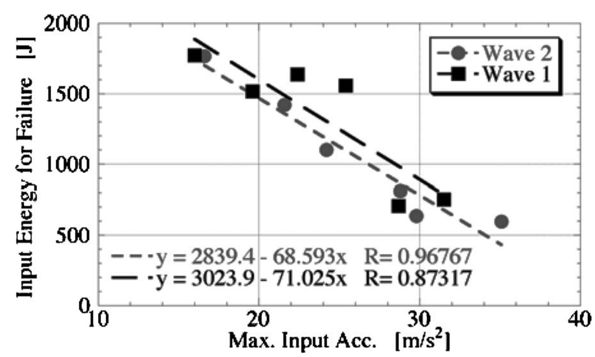

Fig. 11 Maximum input acceleration and input energy for failure (stainless steel, 0.130 m, and 15.2 Hz)

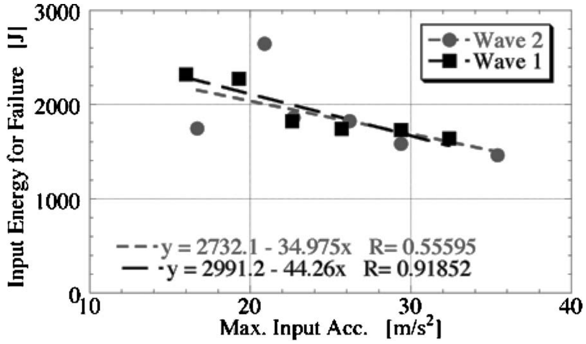

Fig. 12 Maximum input acceleration and input energy for failure (carbon steel, 0.130 m, and 15.4 Hz)

These graphs show the results of the experimental model 0.130 m in length as an example. From these graphs, it is confirmed that input energy for failure is negatively correlated with maximum input acceleration. In addition, regression lines of stainless steel JIS SUS304 have a slope that is about 1.8 times as big as carbon steel JIS SS400, and have an intercept that is equal to carbon steel JIS SS400. This relationship between materials is similar to the above-mentioned increment in input energy.

As a result, correlation between input energy for failure and the maximum input acceleration was confirmed. The maximum response accelerations depend on input wave as mentioned above, so investigations focused on a rms value of the input acceleration, a spectrum, and so on will be conducted for more synthetic evaluations in the future.

5 Conclusion

In this study, ultimate fatigue endurance of the simple single–degree-of-freedom model was investigated from the viewpoint of the energy balance equation. Results of this paper are summarized as follows.

(1) The correlation between fatigue failure and energy is confirmed from vibration experiment.
(2) Input energy for failure depends on the material.
(3) More input energy is needed for failure when time for failure is long.
(4) More input energy is needed for failure when increment in input energy is small.
(5) More input energy is needed for failure in case of small input acceleration.

In the future, an experiment using scale model of actual piping will be carried out, and then the correlation and the tendency between fatigue failure and energy of piping will be investigated.

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