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IMPACT OF SIMPLIFIED STATIONARY CAVITY BEAM LOADING ON THE LONGITUDINAL FEEDBACK SYSTEM FOR SIS100*

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Abstract

The main synchrotron SIS100 of the Facility for Antiproton and Ion Research (FAIR) will be equipped with a bunch-by-bunch feedback system to damp longitudinal beam oscillations. In the basic layout, one three-tap finite impulse response (FIR) filter will be used for each single bunch and oscillation mode. The detected oscillations are used to generate a correction voltage in dedicated broadband radio frequency (RF) cavities. The digital filter is completely described by two parameters, the feedback gain and the passband center frequency, which have to be defined depending on the longitudinal beam dynamics. In earlier works [1, 2], the performance of the closed loop control with such an FIR-filter was analyzed and compared to simulations and measurements with respect to the damping of coherent dipole and quadrupole modes, the first modes of oscillation.
This contribution analyzes the influence of cavity beam loading on the closed loop performance and the choice of the feedback gain and passband center frequency to verify future high current operation at FAIR.

BEAM LOADING

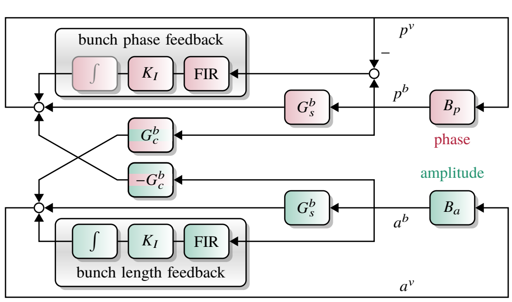

Figure 1: Block diagram for small modulation with the notation of [3] (p: phase, a: amplitude, v: voltage, b: beam and G_s^b, G_c^b, B_p, B_a: transfer functions, see also Eqs. (1-5)).

To eliminate the coupling term $\tan\Phi_s$, usually used in the Pedersen model [3, 4], shown as part of Fig. 1, the gap voltage modulations,

$$v(x) = (1 + \tilde{a}_v)\sin(x + \tilde{p}_v) \qquad (1)$$

were chosen in quadrature with the first harmonic of the beam current

$$i_b(x) = Y I_0 (1 + a_b)\sin(x - \Phi_s + p_b) \qquad (2)$$

where $Y I_0$ denotes the steady state amplitude, Y the relative beam loading and Φ_s the synchronous phase angle, by setting

$$a_v = \tilde{a}_v - \tan\Phi_s \tilde{p}_v \quad \text{and} \quad p_v = \tilde{p}_v + \tan\Phi_s \tilde{a}_v \qquad (3)$$

as sketched in Fig. 2.

The single particle dynamics are then described by $\ddot{\Delta\varphi} = \omega_{syn,0}^2 \left[v(\Phi_s + \Delta\varphi) - \sin\Phi_s \right]$ with the gap voltage amplitude included in the stationary synchrotron frequency $\omega_{syn,0}$. This choice of the gap voltage modulations from

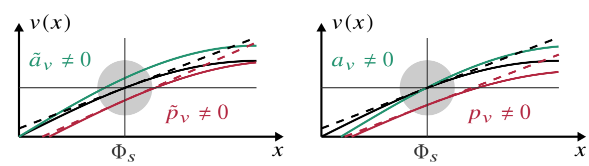

Figure 2: Convenient choice of gap voltage modulations.

Eqs. (3), in addition, reduces the cavity transfer functions Eqs. (5) for the beam current (with an equivalent meaning of the tilde) to a simpler form. Nevertheless, the following considerations, other than e.g. Eq. (6), are shown for $\Phi_s = 0$.

The SIS100 accelerating system will be based on the SIS18 cavities with a loaded Q in the order of 10. Thus it is a reasonable assumption to consider only the fundamental harmonic of the beam induced voltage.

Figure 3 shows the *frequency diagrams* coming along with the well-known Robinson *stability diagram* [3, 5] calculated without the common approximations for a rigid bunch and a high loaded quality factor of the cavity. To be able to cope with long bunches ($\sigma = 33°$ as standard deviation at the bottom of Fig. 3) the beam dynamics were taken from [5], replacing the beam transfer functions for short bunches (here e.g. $\sigma = 6°$) [6]:

$$B_p(s) = \frac{\omega_{\text{syn},0}^2}{s^2 + \omega_{\text{syn},0}^2} \quad \text{and} \quad B_a(s) \sim \frac{-2\omega_{\text{syn},0}^2}{s^2 + 4\omega_{\text{syn},0}^2} \qquad (4)$$

The white curve indicates the optimum (concerning power consumption) detuning $\Phi_L = 0$ (generator current and gap voltage in phase) for the loading angle Φ_L and stationary beam loading compensation. The diagrams were calculated for $Q = 10$ and a synchrotron tune of 10^{-3} but are valid for all cavities that react on modulations of the beam current within one period of a synchrotron oscillation. This, however, primarily depends on the real part of the dominant pole of the cavity impedance with the resonant frequency ω_0, which is the half cavity bandwidth $\alpha = \frac{\omega_0}{2Q}$ for $Q \geq \frac{1}{2}$

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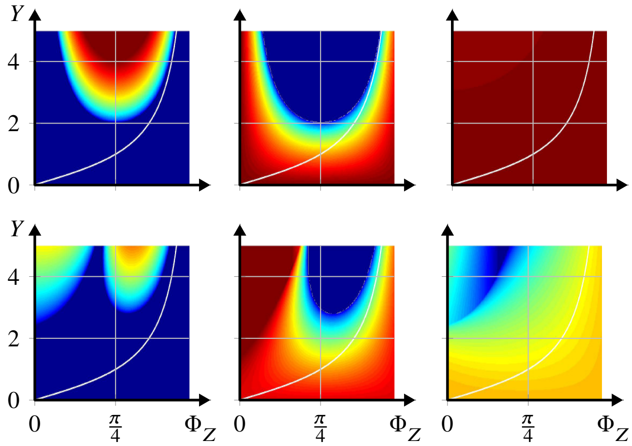

Figure 3: Damping, coherent dipole and quadrupole frequency (left to right) for short (top) and long (bottom) bunches; with [colour bar] from $\omega_{\text{syn},0}$ to $2\omega_{\text{syn},0}$ for the quadrupole frequency and from 0 to $\omega_{\text{syn},0}$ otherwise.

and $\frac{\omega_0}{2Q}\left(1-\sqrt{1-4Q^2}\right)$ for $Q \leq \frac{1}{2}$. To support the following simplification, a more detailed evaluation regarding the dependence of the extended Robinson stability diagrams (Fig. 3) on the synchrotron tune and the loaded Q-factor was done based on the transfer functions given in the Appendix. Supposing that the cavity reacts on changes in the beam current virtually immediately, the transfer functions can be taken as quasi-static, with $G_s = 1$ and $G_c = 0$ as transfer functions for the total current, and hence for the beam current

$$\tilde{G}_s^b = Y \cos\Phi_Z (\sin\Phi_Z \cos\Phi_s - \cos\Phi_Z \sin\Phi_s) \quad (5a)$$

$$\tilde{G}_c^b = Y \cos\Phi_Z (\cos\Phi_Z \cos\Phi_s + \sin\Phi_Z \sin\Phi_s) \quad (5b)$$

$$G_s^b = Y \sin\Phi_Z \cos\Phi_Z / \cos\Phi_s \quad (5c)$$

$$G_c^b = Y \cos^2\Phi_Z / \cos\Phi_s \quad (5d)$$

is obtained, where Φ_Z is the cavity detuning angle. This gives the high current limit from the stability considerations by Robinson [3]

$$Y = \frac{2\cos\Phi_s}{\sin(2\Phi_Z)}$$

which is due to the loss of phase focusing as the phase angle between the generator induced voltage $\Phi_Z - \Phi_L$ and the synchronous phase Φ_s is $90° = \Phi_s + \Phi_Z - \Phi_L$ with the steady state condition [3]

$$\tan\Phi_L = \frac{Y\cos\Phi_s - \tan\Phi_Z}{Y\sin\Phi_s + 1}$$

to keep the total gap voltage independent of Y and Φ_Z. Thus a single particle arriving too late in the cavity is no longer accelerated with respect to the synchronous particle.
From $p_b = B_p(s)p_v$ and $p_v = G_s^b p_b$ using Eqs. (4) and (5) the coherent synchrotron frequency with beam loading, corresponding to the dipole frequency shift along the white curve in Fig. 3, is found to be [7]:

$$\omega_{syn,bl}^2 = \omega_{syn,0}^2 \left(\cos\Phi_s - Y\frac{1}{2}\sin(2\Phi_Z)\right) \quad (6)$$

It is shown in the following that Eq. (6) is important to tune the passband center frequency of the bunch phase feedback.

LONGITUDINAL FEEDBACK SYSTEM

The closed loop bunch-by-bunch feedback system basically consists of

- broadband measurement of the beam current
- de-multiplexing of the beam signal
- one DSP-system per bunch with
  - IF pre-processing of the pulsed signals
  - phase / amplitude detection (actuating variables, see Eq. (2))
  - one digital FIR-filter for each oscillation mode
  - post-processing of the filter output (control variables, see Eqs. (1) and (3))
- multiplexing of the control signals
- two broadband kicker cavities

The scope of the longitudinal feedback system is to reduce dilution in longitudinal phase space due to filamentation. Different single bunch modes are damped, in the first instance dipolar ($m = 1$) and quadrupolar ($m = 2$) oscillations, whereas all coupled bunch modes ($0 \leq n \leq h-1$) are covered by the bunch-by-bunch processing. The modularity and scalability of the digital control system allow considerable flexibility regarding the oscillation modes and frequencies. Fig. 4 (top) shows the basic stability diagrams of the FIR-filters (feedback gain and passband center frequency) used to damp undesired beam phase [1] and bunch length [2] oscillations. The large stability regions indicate robustness against parameter uncertainties. The dynamics of the digital FIR-filter with

$$k = \frac{f_{\text{sample}}}{2\, f_{\text{pass}}}$$

are implemented as:

$$y(t) = -\frac{1}{4}x(t) + \frac{1}{2}x(t-kT_{\text{sample}}) - \frac{1}{4}x(t-2kT_{\text{sample}}) \quad (7)$$

It is followed by an integrator with the gain K_I as shown in Fig. 1. Please note that the bunch phase feedback works with both inputs $p_b - \tilde{p}_v$ and $p_b - p_v$ if Φ_s changes adiabatically as constant offsets are suppressed by the FIR-filter (7). Figure 4 verifies the strong dependence of the passband center frequency f_{pass} of the bunch phase feedback (left columns) on the relative beam loading Y according to Eq. (6). The predicted optimum for a short bunch marked with a white "x" on the k-axis in the left column of Fig. 4 is remarkably close to the optimum k found from the eigenvalue analysis of the full order closed loop system.

In addition to the references [1, 2] the analysis includes the coupling of the two feedback loops. The slower (due to more filter taps) bunch phase feedback dominates the dynamics, resulting in the feignedly extended optimum of the bunch length feedback, with respect to reference [2]. The parameters of the other loop are each set to the value marked with a white "o" for respective scenario (Y, σ). For the scaling of the K_I-axes please refer to [1] and [2, Eq. (24)].

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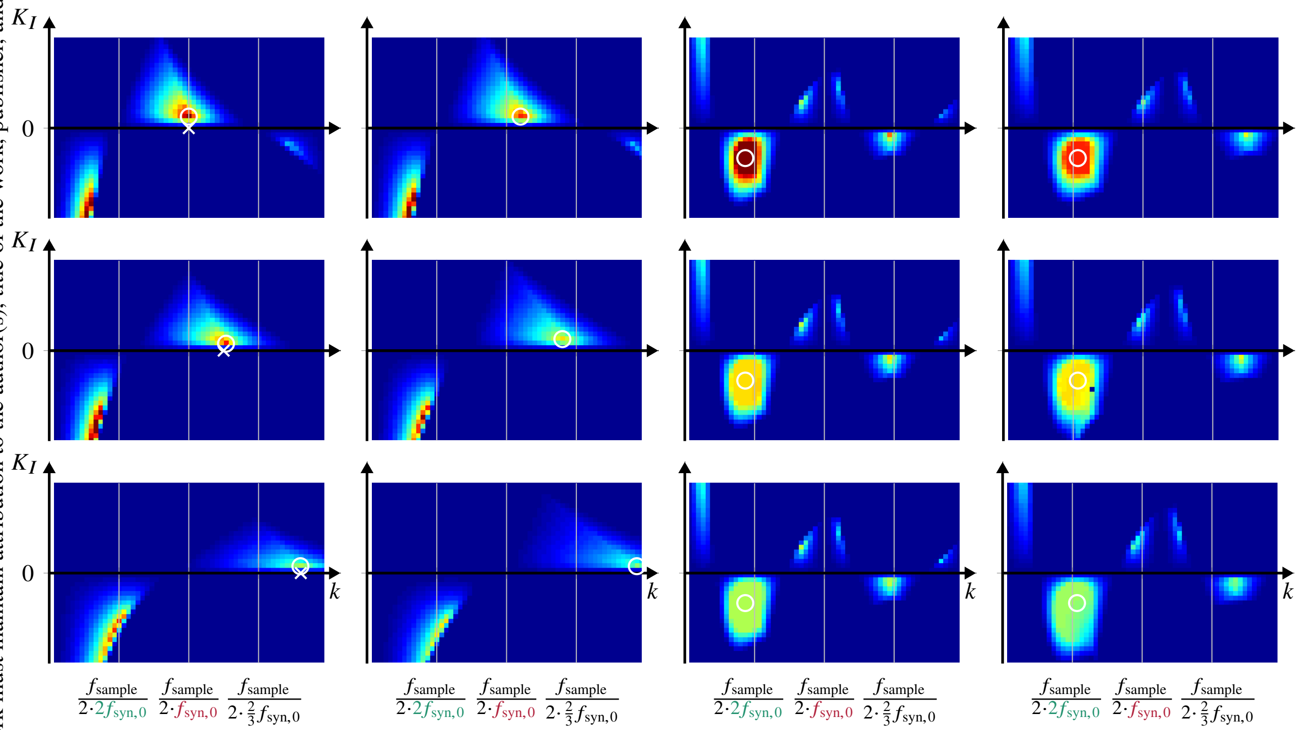

Figure 4: Impact of (relative) beam loading, $Y \in \{0, 0.75, 1.5\}$ (top to bottom), on the closed loop performance for damping of dipole (left) and quadrupole (right) oscillations for short and long bunches, respectively.

CONCLUSION

It was shown that the simple estimation of the coherent synchrotron frequency from Eq. (6) is sufficient for the tuning of the longitudinal feedback system when beam loading becomes relevant during future high current operation at SIS100. This result may be limited to heavy ion synchrotrons as the coherent synchrotron frequency decreases with beam loading only if the loaded quality factor of the cavity is small compared to the inverse of the synchrotron tune times the harmonic number.

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APPENDIX

To find the scope of application of Eq. (6) it is important to use the full 4th order cavity transfer functions

$$G_s(s) = \frac{2\alpha\left(s^3+(2\alpha-\omega t_Z)s^2+2\omega(\alpha t_Z+\omega)s+2\alpha\omega^2(t_Z^2+1)\right)}{s^4+4\alpha s^3+4(\alpha\omega t_Z+\alpha^2+\omega^2)s^2+8\alpha\omega(\alpha t_Z+\omega)s+4\alpha^2\omega^2(t_Z^2+1)}$$

$$G_c(s) = \frac{2\alpha s\left(t_Z s^2+(2\alpha t_Z+\omega)s+2\omega t_Z(\alpha t_Z+\omega)\right)}{s^4+4\alpha s^3+4(\alpha\omega t_Z+\alpha^2+\omega^2)s^2+8\alpha\omega(\alpha t_Z+\omega)s+4\alpha^2\omega^2(t_Z^2+1)}$$

with the angular RF frequency ω and the abbreviation $t_Z := \tan\Phi_Z = \frac{\omega_0^2-\omega^2}{2\alpha\omega}$.

Usually 1st or 2nd order transfer function are applied when dealing with beam loading due to high Q-factors [3–5].

REFERENCES

[1] H. Klingbeil et al., “A digital beam-phase control system for heavy-ion synchrotrons”, IEEE Trans. Nucl. Sci. **54**, 2604 (2007).

[2] D. Lens, H. Klingbeil, “Stability of longitudinal bunch length feedback for heavy-ion synchrotron”, Phys. Rev. ST Accel. Beams **16**, 032801 (2013).

[3] F. Pedersen, “Beam loading effects in the CERN PS booster”, IEEE Trans. Nucl. Sci. **22**, 1906 (1975).

[4] D. Boussard, “Design of a ring RF system”, CERN Accelerator School, CERN-SL-91-02-RFS-REV (1991).

[5] K.W. Robinson, “Radiofrequency acceleration II”, CEA(MIT-Harvard)-11 (1956).

[6] K. Groß, D. Lens, “Modeling longitudinal bunched beam dynamics in hadron synchrotrons using scaled Fourier-Hermite expansions”, Proc. of IPAC’13, WEPEA010, Shanghai, China.

[7] S.R. Koscielniak, “Coherent and incoherent bucket for a beam loaded RF system”, Part. Accel. **48**, TRI-PP-94-8 (1994).