Journal of Tribology

Technical Briefs

Lubricant Film Flow in Pin-Disk Testing

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The spread of lubricant across the wear track of a pin-disk machine may be adequately modeled assuming axisymmetric, viscous flow with zero shear stress at the lubricant-air interface. Waves can develop according to the shape of the initial profile.

Introduction

The pin-disk configuration is widely used for testing lubricant—material combinations. In a typical experimental procedure a few drops of lubricant are spread over the disk and the test is started. The lubricant migrates radially due to centripetal acceleration. Most of the lubricant initially within the wear track will cross the track, but the flow rate decreases with time.

This present work arose when designing tests of extreme pressure additives in a ball-disk configuration. No verified method was available to calculate the supply rate of lubricant to the track. In the boundary lubrication regime a finite rate of replenishment of the additive is necessary, particularly for additives that bond chemically. If the lubricant is not fed continuously, the rate of replenishment of additive to the wear track must drop asymptotically to zero. A method of calculating the delivered rate was sought and is described in this paper.

Theory

There have been many analyses of the spread of viscous liquid on a rotating disk. Several modes of instability can occur, as shown (for example) by Charwat et al. (1972). The earlier analysis of Emslie et al. (1958) used straightforward assumptions, that the flow is axisymmetric with negligible axial and tangential velocity, the pressure is uniform everywhere, the radial velocity u is a function of r, z and is driven by centripetal acceleration. This leads to

$$\eta \frac{d^2u}{dz^2} = -\rho\omega^2 r \tag{1}$$

The boundary conditions are $u = 0$ at $z = 0$ and $du/dz = 0$ at $z = h$, implying zero stress at the lubricant-air interface. The integration of (1) gives the velocity profile:

$$u = \frac{r\omega^2 h^2}{2\nu}\left[2\left(\frac{z}{h}\right) - \left(\frac{z}{h}\right)^2\right] \tag{2}$$

Continuity requires that:

$$\frac{dh}{dt} = -\frac{1}{r}\frac{d}{dr}(rh\bar{u}), \tag{3}$$

where the mean velocity $\bar{u} = r\omega^2h^2/3\nu$, so that the flow across any radius r is:

$$Q = 2\pi r^2\omega^2 h^3/3\nu \tag{4}$$

The application of (3) gives

$$\frac{dh}{dt} = -\frac{\omega^2}{3\nu r}\left[2rh^3 + 3r^2h^2\frac{dh}{dr}\right] \tag{5}$$

Subsequent profiles of h can be found from the integration of (5). Emslie et al. (1958) give examples of successive profiles that develop from various initial shapes. Hwang and Ma (1989) describe a numerical scheme to integrate (5). An interesting result of (5) is that if the layer is initially uniform, it remains uniform during thinning. Perturbation analyses have shown that some initial shapes will decay to a uniform film and others of the same amplitude can develop ripples which propagate radially in the time domain. If one assumes a shape of the form

$$h = Ar^a(t + t_0)^b \tag{6}$$

then substitution into (5) gives $a = 0$, $b = -1/2$, indicating that this film shape remains uniform for all time. Meyerhofer (1978) showed that once a uniform film developed, the degree of uniformity could be such that films would show successive uniform interference colors as thinning occurred. The integration of (5) for a uniform film of initial thickness $h(t = 0) = h_0$ gives $t_0 = 3\nu/4\omega^2h_0^2$ and produces the solution:

$$\frac{h}{h_0} = \left[1 + \frac{t}{t_0}\right]^{-1/2} = \left[1 + \frac{4t\omega^2h_0^2}{3\nu}\right]^{-1/2} \tag{7}$$

This result was compared against our experiments.

Experimental

The "disk" was a 5cm × 5cm steel square, ground and buffed to $R_a = 0.22$ μm, with a random lay. A sample of fluid (approximately 0.05 mL) was placed as concentrically as possible and a capacitance probe was set concentric with the spin axis, 508 μm (nominal) above the disk surface. The linearity of the system was verified using numerous combinations of plastic shim material of various thicknesses placed in the air gap (when dry). Calibration was performed with each of the

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Contributed by the Tribology Division of the THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS. Manuscript received by the Tribology Division March 16, 1993; revised manuscript received July 12, 1993. Associate Technical Editor: I. Etsion.

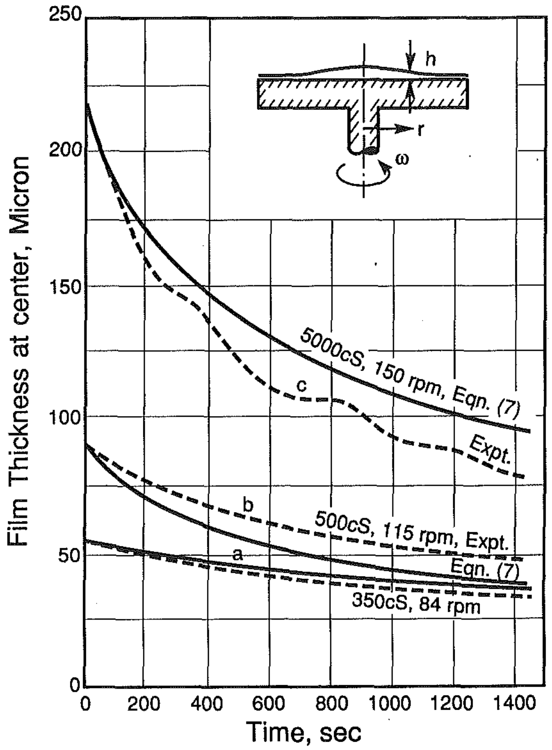

Fig. 1 Examples of thinning at the center of the film for three fluids. (*a*) 350cS fluid at 84 rpm; (*b*) 500cS fluid at 115 rpm; (*c*) 5000cS fluid at 150 rpm. Curve (*a*) was started after 11 minutes of rotation and curve (*b*) after 5 minutes.

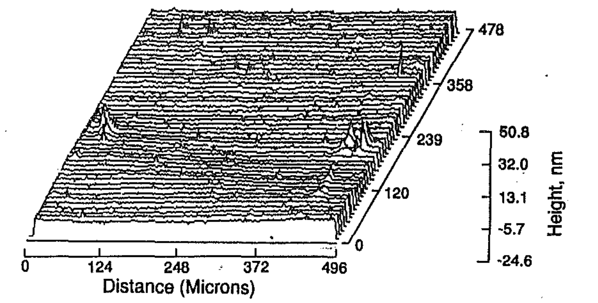

Fig. 2 Surface profile of a 500 μm × 500 μm area on a spreading water droplet

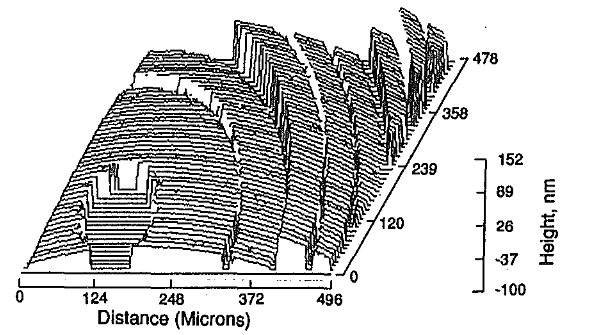

Fig. 3 Surface profile of a 500 μm × 500 μm area on a static droplet of 350cS fluid

test fluids filling the 508 μm gap. The accuracy was estimated as better than ± 2 μm. Seven silicone fluids were used with viscosities in the range 200–60,000 cS (see Appendix) and speeds in the range of 35–230 rpm, according to viscosity. The relatively high viscosities were expected to give "thick" films with gradual thinning, thereby improving the accuracy of measurement.

Results

Figure 1 shows raw data for three runs with viscosities of 350, 500 and 5000 cS. Usually the rate of thinning was greater than given by Eq. (7), for example curve (*a*). Log plots of experimental data in the form of (7) gave acceptable linearity, but with the exponent $b \approx -0.60$ compared to the analytical result $b = -0.50$. Curve (*b*) shows an exception and curve (*c*) shows oscillations. It is interesting that for water fed continuously to the center of the disk, Espig and Hoyle (1965) and Charwat et al. (1972) also show rates of thinning in the range 30–50 percent greater than their models. Charwat attributes the lack of agreement in part to neglect of the azimuthal (tangential) velocity originating from the induced swirl of air, that would give a stress $\tau_{\theta z}$ at the interface. We believe this to be negligible in our experiments.

Surface tension T could tend to reduce the rate of thinning. An order of magnitude analysis may be made using the convenient case of a droplet of spherical radius R. This would give a uniform disjoining pressure of

$$p = \frac{2T}{R} \approx 2T\frac{d^2h}{dr^2} \qquad (8)$$

The addition of dp/dr to the right-hand side of (1) allows the relative effect α of thinning due to surface tension to that from centripetal acceleration to be derived:

$$\alpha = 8T^* \cdot \mathrm{Re}^{-1} \cdot (h/d)^3 \qquad (9)$$

where

$$T^* = 2T/\eta\omega d$$

$$\mathrm{Re} = \rho\omega h^2/\eta$$

$T =$ surface tension

$d =$ disk diameter

In our experiments, α is typically of the order 10^{-3}.

Discussion

The capacitance electrode was of 350 μm diameter and soon after starting the majority of droplets assumed a slightly concave shape at the center. Comparing the two terms on the right-hand side of (5) indicates that the concavity (which would increase the rate of thinning) could account for all or part of the discrepancy. The undulations in curve (*c*) show the alternating development of concavity and convexity, and the occurrence of waves. Asymmetry of the droplets on the disk and run-out of the disk in the "swash" mode could also cause an increased rate of thinning.

Flow Under Gravity Alone

An optical profilometer was used to map the surface of spreading droplets of water and the 350 cS fluid. Figure 2 shows a result for water, with several peaks of the order 30 nm, which could act as discontinuities and increase the rate of thinning. Maps of the silicone surface consistently showed concentric waves of up to 100 nm depth. Figure 3 shows a result for a 350 cS droplet left for several hours to stabilize. The center of the drop and distribution of axisymmetric ripples are clearly apparent. The predictions of Benjamin (1957) are interesting in this respect. Various authors using perturbation analyses have derived critical Reynolds numbers for the onset of ripples. Benjamin proposes that ripples can develop at any Reynolds number, although the amplitude may be small.

Conclusions

The rate of flow of lubricant across a wear track can be calculated with acceptable accuracy assuming viscous, axisymmetric, radial flow. Waves may develop according (apparently) to the initial profile of the droplet.

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APPENDIX

Silicone Fluids (Manufacturer's Data)

Viscosity:	Figure 1 gives nominal viscosity at 25°C (lab temperature) A typical Viscosity Index is indicated by (for 500 cS fluid) 1000cS @ −7°C 500cS @ 25°C 100cS @ 145°C
Specific gravity	0.972 (200cS) − 0.977 (60,000cS)
Surface tension, T	21.1 dynes/cm (200cS) − 21.3 dynes/cm (60,000cS)
Volatility	% Wt loss 24 hrs at 150°C 0.5 (200cS fluid) − 2.0 (60,000cS fluid)
Dielectric constant	2.75 (25°C, 10^2 − 10^6 Hz)
Resistivity	10^{14} ohms