Entropy Generation Minimization of Fully Developed Internal Flow With Constant Heat Flux

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This paper uses the entropy generation minimization (EGM) method to optimize a single-phase, convective, fully developed flow with uniform and constant heat flux. For fixed mass flow rate and fixed total heat transfer rate, and the assumption of uniform and constant heat flux, an optimal Reynolds number for laminar and turbulent flow is obtained. The study also compares optimal Reynolds number and minimum entropy generation for cross sections: square, equilateral triangle, and rectangle with aspect ratios of two and eight. The rectangle with aspect ratio of eight had the smallest optimal Reynolds number, the smallest entropy generation number, and the smallest flow length. [DOI: 10.1115/1.1777585]

Introduction

This paper presents the thermodynamic optimum for fully developed internal convective flow, i.e., flow through a tube with constant and uniform heat flux. The optimum is obtained by minimizing the sum of viscous momentum transfer losses and heat transfer losses. The viscous momentum transfer losses are due to fluid friction between the wall and the fluid and within the fluid. Heat transfer losses are due to heat transfer across finite temperature differences between the wall and the fluid. The losses must be quantified in equal units in order to compare them and to minimize the sum of losses. One method of comparing losses is based on the second law of thermodynamics, entropy generation minimization (EGM).

Bejan [1–6] presents analyses of a tube flow using EGM. He has found thermodynamic optimums of the ratio of film coefficient to pumping power and the dimensionless temperature difference with constant mass flow rate and heat transfer rate per unit length. Note constant heat transfer rate per unit length is different from uniform and constant heat flux. Reference [5] presents laminar and turbulent flow through a tube with circular cross-section. Entropy generation rate per unit length was minimized at a fixed heat transfer rate per unit length and mass flow rate, and the optimal Reynolds number (optimal tube diameter) was obtained. For laminar flow, the optimal Reynolds number is zero. For turbulent flow, the optimal Reynolds number is a function of Prandtl number and duty parameter. The duty parameter is a function of fluid properties, heat transfer rate per unit length, and mass flow rate.

Sahin [7–9] investigated laminar and turbulent flow through a tube with uniform and constant heat flux. He investigated the effect of temperature-dependent viscosity on the entropy generation rate as well as the ratio of pumping power to heat transfer [7,8]. He presented the optimum cross-section shape for laminar flow and constant heat flux while comparing ducts of the same cross-sectional area and length [9]. He determined that the circular cross-section was superior and the equilateral triangular and rectangular cross-sections were inferior.

Nag and Mukherjee [10] investigated the tradeoff losses within a heat exchanger's fluid passage minimizing entropy generation rate for fixed wall temperature and mass flow rate. They obtained the optimal wall-fluid temperature difference and the optimal ratio of film coefficient to pumping power. Sahin [11] also investigated the entropy generation at fixed wall temperature. He presented particularly the effect of temperature-dependent viscosity on entropy generation and pumping power.

The rationale for this study is to complement the work by others in regards to convective flow through a tube. This paper is different by considering a different boundary condition, uniform and constant heat flux without fixing the duct geometry. This paper describes the optimal passage geometry (cross-section shape, tube length, and tube hydraulic diameter) for fully-developed laminar and turbulent flows with fixed total heat transfer rate, fixed mass flow rate, and uniform and constant heat flux. Also the paper presents the solution's dependence on the heat transfer and friction factor correlations.

Model Development

Figure 1 is a description of single-phase, steady, and fully developed flow through a tube subjected to heat transfer as well as wall shear forces. A small differential section of the flow is shown, where its properties change across dx due to the interactions. Bejan [6] developed the entropy generation equation per unit length of tube.

$$\dot{S}'_{\mathrm{gen}}=\frac{\dot{q}''\mathscr{P}(T_w-T)}{T^2}+\frac{\dot{m}^3 f}{2\rho T D_h A_c^2} \tag{1}$$

where $\dot{q}''$, $\mathscr{P}$, T_w, T, $\dot{m}$, f, ρ, D_h, and A_c are the heat flux, tube perimeter, wall temperature, bulk fluid temperature, mass flow rate, Darcy friction factor, fluid density, hydraulic diameter, and cross-sectional area, respectively.

Ratts and Atul [12] integrated the equation over the tube length L. They assumed that fluid properties were constant. Their result was

$$\dot{S}_{\mathrm{gen}}\cong\frac{(\dot{q}'')^2\mathscr{P}D_h L}{\mathrm{Nu}\cdot k T_1 T_2}+\frac{8\dot{m}^3 f L}{\rho^2 T_{\mathrm{ave}} D_h^3 \mathscr{P}^2} \tag{2}$$

where Nu is the Nusselt number, k is the thermal conductivity, T_1 and T_2 are the inlet and outlet fluid temperatures, and

$$T_{\mathrm{ave}}\equiv\frac{(T_1-T_2)}{\ln(T_1/T_2)} \tag{3}$$

Integration was based on the boundary condition of uniform and constant heat flux. The first term on the right hand side of Eq. (2) is the entropy generation rate due to heat transfer dissipation and the second term is the entropy generation rate due to viscous dissipation.

The entropy generation rate in it its final form is

$$N_S=N_{S,\Delta T}\left(1+\frac{1}{\phi}\mathrm{Re}^{7-\gamma+\alpha}\,\mathrm{Pr}^{\beta}\right) \tag{4}$$

where the variables are: the entropy generation number

$$N_S\equiv\frac{\dot{S}_{\mathrm{gen}}}{\dot{Q}/T_{\mathrm{ave}}} \tag{5}$$

the entropy generation due to heat transfer dissipation

$$N_{S,\Delta T}=\frac{4}{\chi C_h}\left[\frac{\dot{q}''\dot{m}}{\mu k T_m}\right]\mathrm{Re}^{\alpha+1}\,\mathrm{Pr}^{-\beta} \tag{6}$$

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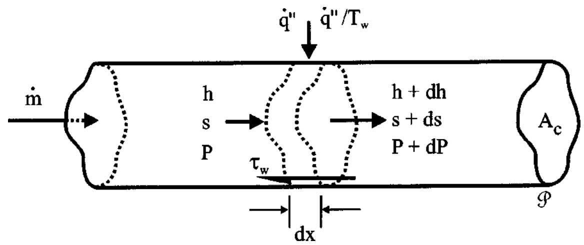


Fig. 1 Fully developed internal flow

the mean fluid temperature

$$T_m \equiv \frac{T_1 T_2}{T_{\mathrm{ave}}} \tag{7}$$

the ratio of cross-section perimeter to hydraulic diameter (a constant for a given cross-section shape and referred to as the shape ratio)

$$\chi = \frac{\mathscr{P}}{D_h} \tag{8}$$

and

$$\phi = \left[\frac{(4)^7}{8\chi^4 C_h C_f}\right]\left[\frac{\dot{q}''\rho\dot{m}^2}{\mu^{3.5}\sqrt{kT_m}}\right]^2 \tag{9}$$

The second term in brackets in Eq. (9) is the design criteria and is referred to as the duty parameter [5]. The constants C_h, C_f, and exponents α, β, and γ are from the Nusselt correlation

$$\mathrm{Nu} = C_h \,\mathrm{Re}^{\alpha}\,\mathrm{Pr}^{\beta} \tag{10}$$

and the friction factor correlation

$$f = C_f \,\mathrm{Re}^{-\gamma} \tag{11}$$

There is an optimal Reynolds number that minimizes the entropy generation. The minimum is found by taking the derivative of Eq. (4) with respect to Reynolds number, setting the derivative equal to zero, and solving for the Reynolds number. The optimal Reynolds number is

$$\mathrm{Re}_{\mathrm{opt}} = \left[\phi\left(\frac{\alpha+1}{6-\gamma}\right)\mathrm{Pr}^{-\beta}\right]^{1/(7-\gamma+\alpha)} \tag{12}$$

Note the optimal Reynolds number scales inversely to the shape ratio, the heat transfer correlation coefficient, and the friction factor correlation coefficient. Substituting the optimal Reynolds number into Eq. (4) results in the minimum entropy generation number.

$$N_{S,\min} = \frac{4}{\chi C_h}\left[\frac{\dot{q}''\dot{m}}{\mu k T_m}\right]\left[1+\frac{(\alpha+1)}{(6-\gamma)}\right]\mathrm{Re}_{\mathrm{opt}}^{-(\alpha+1)}\,\mathrm{Pr}^{-\beta} \tag{13}$$

The ratio of Eq. (4) to Eq. (11) is

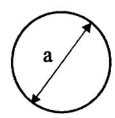
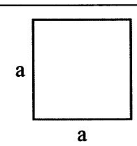
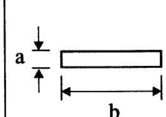
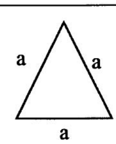
$$\frac{N_S}{N_{S,\min}} = \left(\frac{6-\gamma}{7-\gamma+\alpha}\right)\left(\frac{\mathrm{Re}}{\mathrm{Re}_{\mathrm{opt}}}\right)^{-(\alpha+1)} + \left(\frac{\alpha+1}{7-\gamma+\alpha}\right)\left(\frac{\mathrm{Re}}{\mathrm{Re}_{\mathrm{opt}}}\right)^{6-\gamma} \tag{14}$$

Note that this equation is independent of the Prandtl exponent, and therefore the solution is the same for heating and cooling.

Circular Cross-Section Tubes

The model was applied to a circular cross-section tube. For laminar flow, the heat transfer and friction factor correlations are given in Table 1. The constants for Eq. (10) are $C_h = 4.36$ and $\alpha = \beta = 0$. The constants for Eq. (11) are $C_f = 64$ and $\gamma = 1$. Substituting the constants into Eq. (12) provides the optimal Reynolds number as a function of the duty parameter. Figure 2 presents Eq. (12) for laminar flow. As the duty parameter increases, the optimal Reynolds number increases. Substituting the constants into Eq. (14) results in the entropy number ratio as a function of the Reynolds number ratio. Figure 3 presents Eq. (14) for laminar flow.

Table 1 Different Cross Section Tubes

Cross section	Diagram	Perimeter	Area	Hydraulic Diameter	Nu†	fRe†
Circular		πa	$\frac{\pi a^2}{4}$	a	4.36	64
Square		$4a$	a^2	a	3.61	57
Rectangle		$2a\left(1+\frac{b}{a}\right)$	$a^2\left(\frac{b}{a}\right)$	$\frac{2a}{\left(\frac{a}{b}+1\right)}$	$b/a = 2$: 4.12 $b/a = 8$: 6.49	$b/a = 2$: 62 $b/a = 8$: 82
Equilateral Triangle		$3a$	$\frac{\sqrt{3}}{4}a^2$	$\frac{\sqrt{3}}{3}a$	3.11	53

†Cengel [13]

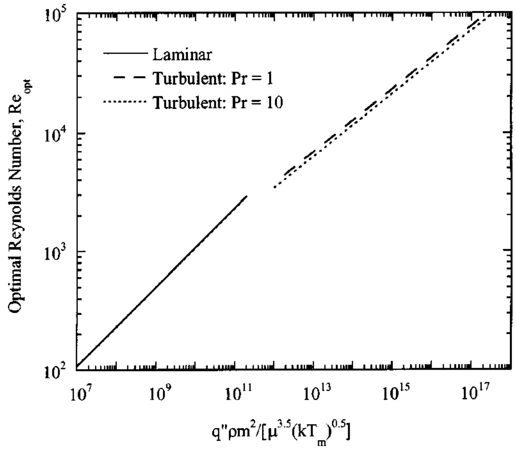


Fig. 2 Optimal Reynolds number for circular cross-section

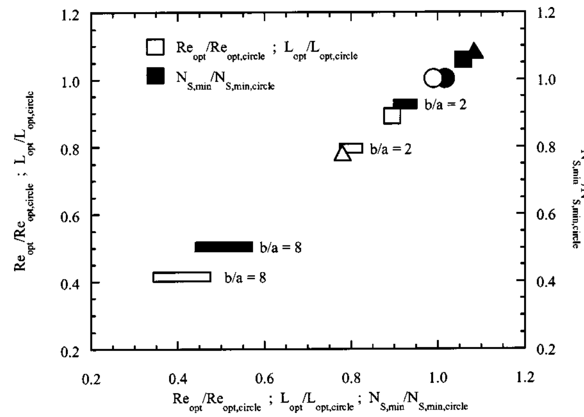


Fig. 4 Optimal solutions for laminar flow

For an incremental change in the Reynolds number from the optimal condition, more entropy is generated in the direction of viscous dissipation than heat transfer dissipation.

For turbulent flow, the Dittus-Boelter equation [14] constants for Eq. (10) are $C_h = 0.023$, $\alpha = 4/5$, $\beta = 0.4$ (heating) and 0.3 (cooling). The constants [14] for Eq. (11) are $C_f = 0.316$ and $\gamma = 1/4$ for $\mathrm{Re}_D < 2\times10^4$, and $C_f = 0.184$ and $\gamma = 1/5$ for $2\times10^4 < \mathrm{Re}_D < 3\times10^5$. Substituting the constants into Eq. (12) provides the optimal Reynolds number for turbulent flow and is presented in Fig. 2 (dashed lines). The optimum is plotted for two Prandtl numbers. As the Prandtl number increases, the optimal Reynolds number decreases. Equation (14) for turbulent flow is plotted in Fig. 3. Note that the turbulent solution is more symmetric around the optimum in comparison to the laminar solution. With an incremental change in Reynolds number from the optimal condition, the increase in heat transfer loss is closer in value to the increase in viscous loss. Both the heat dissipation and viscous dissipation losses are larger than for the laminar case. In addition the solution by Bejan [6] is also plotted for comparison.

Noncircular Cross-Section Tubes

The model is applicable to noncircular cross-section tubes. Table 1 presents the different cross-sections to be considered. The table provides the geometric parameters: perimeter, cross-sectional area, and hydraulic diameter. The noncircular solutions were compared to the circular solution. The optimal Reynolds number for the noncircular cross-sections with respect to the optimal Reynolds number for the circular cross-section is

$$\frac{\mathrm{Re}_{\mathrm{opt}}}{\mathrm{Re}_{\mathrm{opt,circle}}} = \left[\frac{(\chi^4 C_h C_f)_{\mathrm{circle}}}{(\chi^4 C_h C_f)}\right]^{1/(7-\gamma+\alpha)} \tag{15}$$

The minimum entropy generation for the noncircular cross-section with respect to the minimum entropy generation for the circular cross-section is

$$\frac{N_{S,\mathrm{min}}}{N_{S,\mathrm{min,circle}}} = \frac{(\chi C_h)_{\mathrm{circle}}}{(\chi C_h)}\left[\frac{\mathrm{Re}_{\mathrm{opt,circle}}}{\mathrm{Re}_{\mathrm{opt}}}\right]^{\alpha+1} \tag{16}$$

The solution for Eq. (15) is shown in Fig. 4 for laminar flow. For the optimal Reynolds number, the shape order from highest to lowest is the circle, square (90 percent), rectangle $(b/a=2)$ (80 percent), triangle (78 percent), and rectangle $(b/a=8)$ (41 percent). The optimal Reynolds number is a strong function of the shape ratio, χ. The larger its value, the smaller the Reynolds number. Shapes with a larger value of χ can reduce the hydraulic diameter to increase heat transfer without excessive viscous dissipation losses.

The optimal Reynolds number fixes the tube diameter and it also fixes the tube length. It can be shown that the ratio of the noncircular tube length to the circular tube length is equal to the ratio of the Reynolds number of the noncircular tube to the Reynolds number of the circular tube. The optimal length ratio is plotted in Fig. 4. The rectangle $(b/a=8)$ is the shortest and the circle is the longest.

For the minimum entropy generation, the shape order from most irreversible to least irreversible is the triangle (109 percent), square (105 percent), circle, rectangle $(b/a=2)$ (93 percent), and rectangle $(b/a=8)$ (51 percent). The triangle and square generate more entropy than the circle. Equation (16) is strongly dependent on the product of the heat transfer correlation coefficient and the shape ratio. All of the cross-sections accept for the rectangle $(b/a=2)$ are ordered with respect to the value of the correlation coefficient. For higher values of the coefficient, less entropy is generated. The rectangle $(b/a=2)$ has a high enough perimeter-to-diameter ratio to reduce the entropy generation below the circle.

The solution for Eq. (15) is shown in Fig. 5 for turbulent flow. For the optimal Reynolds number, the shape order from highest to lowest is the circle, square (88 percent), rectangle $(b/a=2)$ (83 percent), triangle (77 percent), and rectangle $(b/a=8)$ (54 percent). The optimal Reynolds number is solely a function of the shape ratio. The heat transfer and friction factor coefficients are the same for all cross-sections. The optimal length ratio is plotted in Fig. 5. The rectangle $(b/a=8)$ is the shortest and the circle is the longest.

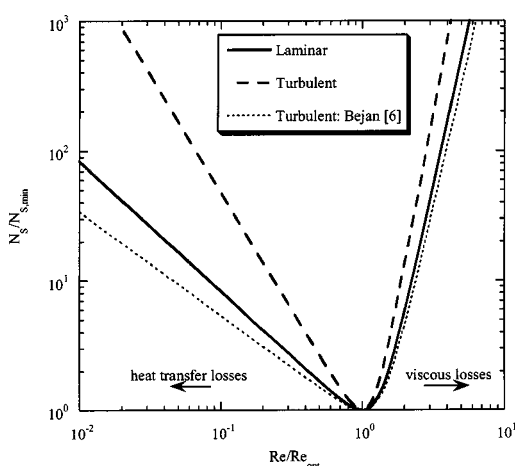


Fig. 3 Entropy generation for circular cross-section

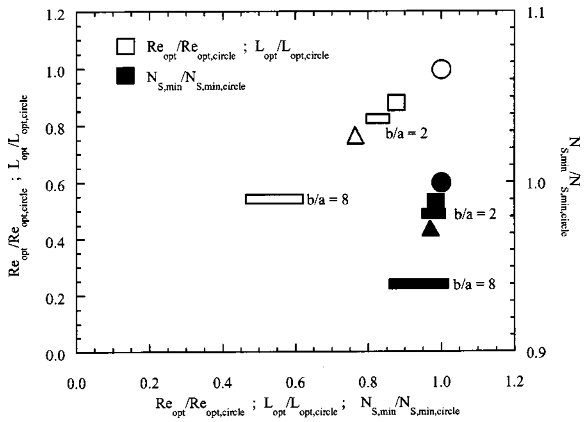


Fig. 5 Optimal solutions for turbulent flow

For the minimum entropy generation, the shape order from most irreversible to least irreversible is the circle, square (98.8 percent), rectangle $(b/a=2)$ (98.3 percent), triangle (97.5 percent), and rectangle $(b/a=8)$ (94.1 percent). The entropy ratio is only dependent on the shape ratio, but not as strong with respect to the optimal Reynolds number ratio.

Conclusions

The paper presented the optimal configuration for laminar and turbulent flow in a tube with a constant and uniform heat flux for a given total heat transfer rate and mass flow rate using the EGM method. Considering the heat transfer and viscous momentum transfer entropy generation, the total entropy generation was minimized providing the optimal Reynolds number, hydraulic diameter, and tube length. By the EGM method, the following conclusions were made:

- For fixed heat transfer rate and mass flow rate with a constant heat flux, there is an optimal Reynolds number for laminar and turbulent flow.
- For the same deviation from optimal Reynolds number in laminar flow, the increase in entropy generation is smaller for heat dissipation than for viscous dissipation. The same is true for turbulent flow, but not as pronounced.
- In turbulent flow, the optimal Reynolds number is larger for smaller Prandtl numbers.
- The optimal Reynolds number in laminar and turbulent flow scales inversely with the shape ratio. The shape order from highest to lowest is the circle, square, rectangle $(b/a=2)$, triangle, and rectangle $(b/a=8)$.
- The minimum entropy generation in laminar flow is strongly dependent on the inverse of the product of heat transfer correlation coefficient and shape ratio. The shape order from most irreversible to least irreversible is the triangle, square, circle, rectangle $(b/a=2)$, and rectangle $(b/a=8)$.
- The minimum entropy generation in turbulent flow is dependent solely on the inverse of the shape ratio. The shape order from most irreversible to least irreversible is the circle, square, rectangle $(b/a=2)$, triangle, and rectangle $(b/a=8)$.
- The optimal tube length scales with the optimal Reynolds number. The longest to shortest length is the same order as the optimal Reynolds number. For the cross-sections considered, the rectangle $(b/a=8)$ is the shortest length and the circle is the longest length.

Nomenclature

A_c = cross-section area, m^2
D = diameter, m
D_h = hydraulic diameter, m
C_h = correlation coefficient
C_f = correlation coefficient
f = friction factor
h = enthalpy, $J\,kg^{-1}$
k = thermal conductivity, $W\,m^{-1}\,K^{-1}$
L = tube length, m
N_S = entropy generation number
$N_{S,\min}$ = entropy generation number
$N_{S,\Delta T}$ = entropy generation number
Nu = Nusselt number
$\dot{m}$ = mass flow rate, $kg\,s^{-1}$
p = perimeter, m
P = Pressure, $N\,m^{-2}$
Pr = Prandtl number
Re = Reynolds number
Re_{opt} = Reynolds number, optimum
$\dot{q}''$ = heat transfer flux, $W\,m^{-2}$
s = entropy, $J\,kg^{-1}\,K^{-1}$
$\dot{S}'_{gen}$ = entropy generation gradient, $W\,K^{-1}\,m^{-1}$
$\dot{S}_{gen}$ = entropy generation rate, $W\,K^{-1}$
T = temperature, K
T_w = wall temperature, K
$\dot{Q}$ = heat transfer rate, W
x = coordinate, m
α = correlation exponent
β = correlation exponent
γ = correlation exponent
μ = dynamic viscosity, $N\,s\,m^{-2}$
ρ = density, $kg\,m^{-3}$
τ_w = wall shear stress, $N\,m^{-2}$
ϕ = Eq. (7)
χ = shape ratio: ratio of perimeter to hydraulic diameter

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