J. W. Murdock
Associate Professor Emeritus,
Drexel University,
Philadelphia, PA 19104
Life Fellow ASME

D. R. Keyser
Senior Project Engineer,
Naval Air Development Center,
Warminster, PA 18974
Mem. ASME

Theoretical Basis for Extrapolation of Calibration Data of PTC 6 Throat Tap Nozzles[1]

Equations for the extrapolation of calibration data for ASME/PTC 6 throat tap nozzles are derived from boundary layer theory. The results match published coefficients with a maximum difference of +0.03 percent. It is also shown that the effects of transition in the boundary layer extend to throat Reynolds numbers in excess of 10,000,000, far beyond the capacity of any known calibration laboratory. The present PTC 6 requirement that calibration data must be in the fully turbulent range cannot be met with current facilities.

Introduction

This paper was developed because page and space limitations prevented us from including derivations in our original paper [1]. We have included all pertinent information from Hall's paper [2] so that each step may be followed without recourse to any other document.

We have also tried to include all pertinent information on flat plate boundary layer theory, which is the basis for our method of predicting the coefficient of discharge at higher Reynolds number using calibration data at lower Reynolds numbers. Theoretical numerical values needed for calculations were taken from Schlichting [3].

Flat Plate Boundary Layer Theory

Figure 1 shows fluid approaching a flat plate with a uniform velocity profile of U. As the fluid passes over the plate, the velocity at the plate surface is zero and increases to U at some distance from the surface. The region in which the velocity varies from 0 to U is called the boundary layer. The thickness of this layer is δ. For some distance along the plate, the flow within the boundary layer is laminar, with viscous forces predominating. As the inertia forces begin to exceed the viscous, in the transition zone a turbulent layer begins to form and increases as the laminar sublayer decreases.

Figure 2 shows the relation of a throat tap nozzle to Hall's [2] simplified nozzle. This model is a straight piece of pipe whose diameter equals the nozzle diameter, and whose length, x_d, is equal to the diameter. It is primarily the difference between the Hall's model and the real nozzle, with throat taps installed, which gave rise to the individual calibration number, N, a characteristic for each nozzle [1]. From the continuity equation for an incompressible fluid, the ideal flow rate, Q_i, with no boundary layer is given by equation (A12), but the actual flow is less than this. The theoretical coefficient of discharge is essentially an area ratio formed by removing the annulus of the boundary layer from the area available to the flow. There was considerable discussion concerning the use of Hall's simplified model for the throat tap nozzle as compared with the more sophisticated models in the literature. We continue to support the use of this model because it produces the same results without requiring extensive computer modeling. Further, the flow phenomena are physically correct, and it is easier to understand.

1 It provides a rational basis for extrapolation.

2 It agrees with published PTC 6 coefficients in all regions of interest.

3 The present PTC 6 method is impractical and does not take advantage of most of the calibration data.

From Hall's analysis [2] the four flow regions shown in Fig. 3 were presented in our paper [1]. The equations for each region are derived herein.

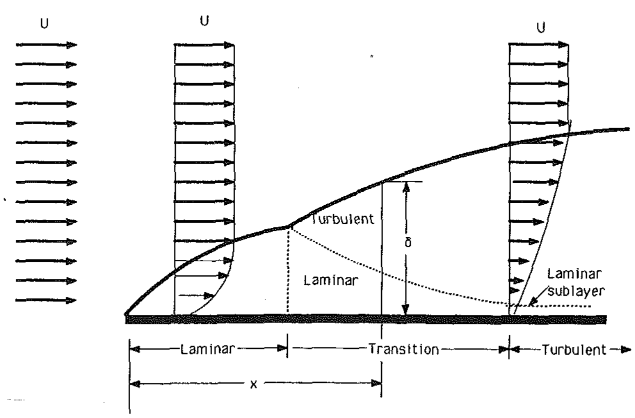
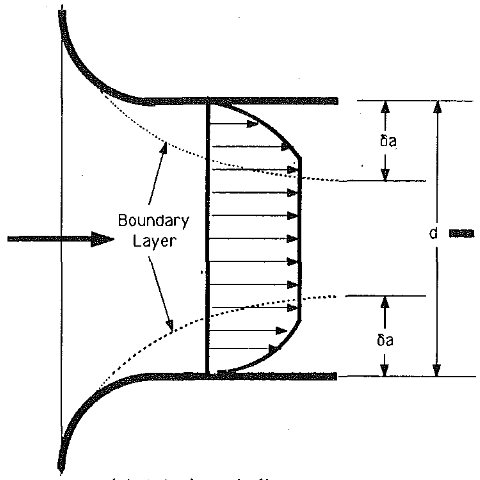


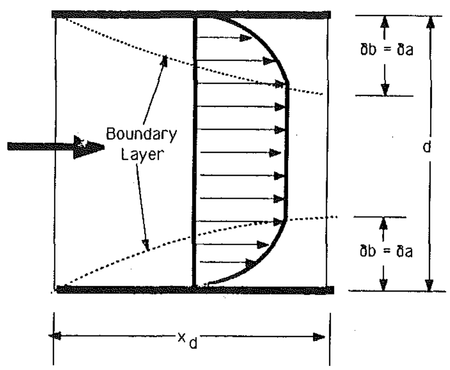
Fig. 1 Boundary layer along a smooth flat plate

[1] The opinions expressed in this paper are those of the authors, and do not necessarily reflect those of the Navy Department or the Naval Establishment at large.

Contributed by the Board on Performance Test Codes and presented at the Joint Power Generation Conference, Boston, Massachusetts, October 21-25, 1990. Manuscript received by the Board on Performance Test Codes July 1990. Paper No. 90-JPGC/PTC-1.



(a) Actual nozzle flow



(b) Hall's simplified flow model

Fig. 2 Transformation to Hall's simplified flow model

Region I: Boundary layer wholly laminar ($Re_d < 5 \times 10^5$) (equation (A16))

$$\delta^*_{max} = \delta^*_L$$

Region II: Boundary layer partly laminar and partly turbulent, with laminar displacement thickness greater than turbulent, ($5 \times 10^5 < Re_d < 7 \times 10^5$) (equation (A22))

$$\delta^*_{max} = \delta^*_{Lt}$$

Region III: Boundary layer partly laminar and partly turbulent, with turbulent displacement thickness greater than laminar ($7 \times 10^5 < Re_d < 10^7$) (equation (A32))

$$\delta^*_{max} = \delta^*_{Tt}$$

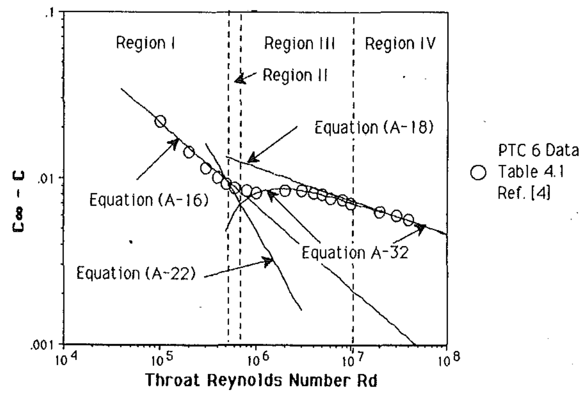


Fig. 3 Flow regions

Region IV: Boundary layer wholly turbulent ($Re_d > 10^7$)

$$\delta^*_{max} = \delta^*_T$$

Figure 3 is a Hall-type plot of the flow regions. Superimposed on this plot are the PTC 6 coefficients from Table 4.1 of [4]. Note that equation (A32) converges to equation (A18) at approximately $Re_d = 10^7$. This means that equation (A32) may be used for Region IV as well.

For extrapolation of PTC 6 calibration data, only Regions III and IV are of interest. From Appendix A, equation (A32), the boundary layer coefficient equation is:

$$C_b = C_{Tt} = 1 - \frac{T}{Re_d^{1/5}} \left(1 - \frac{Re_t}{Re_d} \left(1 - \frac{(H_T/H_L)^{5/4}(L/T)^{5/4}}{Re_t^{3/8}}\right)\right)^{4/5} \tag{1}$$

By using all the theoretical equations for C_b for Regions I–IV, the boundary layer in Hall's model nozzle can be computed at the location of the throat tap pressure measurement. When this curve of boundary layer versus Reynolds number is inverted to calculate the theoretical coefficient of discharge, the traditional, familiar, and historical shape of the calibration curve is obtained, shown in Fig. 4. This is a most satisfying result, and it should remove all lingering doubts as to the correctness of our theoretical method. It only remains to customize, or fit, this equation to the individual calibration data for each peculiar nozzle, which is described in our parallel paper [8].

The pressure measured by a static pressure tap differs from the true static pressure, by an amount that depends upon the hole size and shape. Since static pressure measurement is not included in Hall's simplified nozzle, the complete equation requires a correction for this effect. Based on Rayle's analysis for a finite tap diameter [6], Benedict shows this value to be

Nomenclature

C = coefficient of discharge
H = shape parameter = δ^*/Θ
K = momentum coefficient
L = laminar slope
Re = Reynolds number
T = turbulent slope
u = local velocity within boundary layer
U = velocity outside the boundary layer
x = distance from leading edge, flat plate
β = d/D = ratio of nozzle throat diameter to pipe internal diameter
δ^* = boundary layer displacement thickness
Θ = boundary layer momentum thickness
ν = fluid kinematic viscosity
ρ = fluid density

Subscripts

a = actual
avg = average
b = based on boundary layer theory
d = based on throat diameter
i = ideal
L = laminar
m = measured
max = maximum
o = beginning of turbulent boundary layer
t = transition
T = turbulent
x = flat plate
∞ = at $Re_d = \infty$
6 = for a PTC 6 throat tap nozzle

Table 1 Values from Schlichting [3]

L	Laminar slope	6.88
T	Turbulent slope	0.185
H_L	Laminar shape parameter	2.59
H_T	Turbulent shape parameter	1.28
Re_t	Transition Reynolds number	5×10^5

Table 2 Difference between published and calculated coefficients of discharge

Throat Reynolds Number Re_d	Discharge coefficients PTC 6 1976 C_6	Equation (2) $C_{(2)}$	Difference ΔC
1,000,000	0.9972	0.9972	0.0000
2,000,000	0.9970	0.9967	0.0003
3,000,000	0.9970	0.9969	0.0001
4,000,000	0.9972	0.9972	0.0000
5,000,000	0.9974	0.9974	0.0000
6,000,000	0.9978	0.9976	0.0002
8,000,000	0.9980	0.9980	0.0000
10,000,000	0.9983	0.9982	0.0001
20,000,000	0.9991	0.9991	0.0000
30,000,000	0.9994	0.9995	−0.0001
40,000,000	0.9998	0.9999	−0.0001

+0.0054 for a plenum inlet to the nozzle and +0.0056 for a beta ratio of 0.43 [7]. It is noted that Benedict, during an extended debate of this topic within the PTC 19.5 flow measurement committee, contended that this value was not a constant over the full range of flow. However, it was the collective judgment that the "tap correction" most likely is a constant coefficient of the dynamic pressure, similar to the drag coefficient data of a small hole in the skin of an aircraft. The answer to the question of whether or not this value is constant over regions III and IV cannot be known until data are obtained well beyond our present-day laboratory capacities.

In any case, the question of the absolute value of the "tap correction" becomes moot, since the individual nozzle calibration includes these effects as well as any small effects of beta ratio. Consequently we decided to use the plenum inlet correction in keeping with the philosophy of our simplified nozzle—making the theory no more complicated than necessary to match the data. As a result, we chose the plenum inlet value of 1.0054 for the theoretical coefficient of discharge (as the Reynolds number approaches infinity).

$$C=C_\infty-\frac{T}{\mathrm{Re}_d^{1/5}}\left(1-\frac{\mathrm{Re}_t}{\mathrm{Re}_d}\left(1-\frac{(H_T/H)_L^{5/4}(L/T)^{5/4}}{\mathrm{Re}_t^{3/8}}\right)\right)^{4/5}=1.0054$$

$$-\frac{0.185}{\mathrm{Re}_d^{1/5}}\left(1-\frac{5\times10^5}{\mathrm{Re}_d}\left(1-\frac{(1.28/2.59)^{5/4}(6.88/0.185)^{5/4}}{(5\times10^5)^{3/8}}\right)\right)^{4/5}=$$

$$C=1.0054-\frac{0.185}{\mathrm{Re}_d^{1/5}}\left[1-\frac{361{,}239}{\mathrm{Re}_d}\right]^{4/5} \quad (2)$$

Table 2 shows that the maximum difference between the present PTC 6 coefficient and that calculated using equation (2) is 0.03 percent at a throat Reynolds number of 2,000,000.

Summary

Equation (2), finally developed in this paper, was derived from textbook boundary layer theory and well-developed static tap analysis. The combination of the two is an equation that predicts the published PTC 6 data within +0.0003 and −0.0001—a maximum of 3 parts in 10,000. This theory is valid throughout the range of usual nozzle calibrations and beyond without limit. It is therefore justified to use this equation for the extrapolation of nozzle calibration data from the laboratory to the test measurement installation. The following paper [8] describes the application, method, and extrapolation of calibration data to higher Reynolds numbers for individual nozzles.

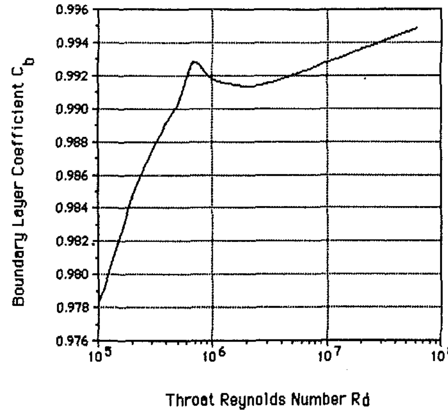


Fig. 4 Theoretical form for the coefficient of discharge of an ASME/PTC 6 throat tap nozzle

References

1 Murdock, J. W., and Keyser, D. R., "A Method of Extrapolating Calibration Data for Flow Nozzles," presented at the Joint ASME/IEEE Power Generation Conference, Milwaukee, WI, Oct. 20-24, 1985; ASME Paper No. 85-JPGC-PTC-2.

2 Hall, G. W., "Application of Boundary Layer Theory to Explain Some Nozzle and Venturi Peculiarities," *Trans. IME, London*, Vol. 173, No. 36, 1959, pp. 837-870.

3 Schlichting, H., *Boundary Layer Theory* (English translation by J. Kestin), McGraw-Hill, 7th ed., 1979.

4 ASME/ANSI Performance Test Code for Stream Turbines—PTC 6 1976 (reaffirmed 1982).

5 Cotton, K. C., and Westcott, J. C., "Throat Tap Nozzles Used for Accurate Flow Measurements," ASME Journal of Engineering for Power, Vol. 82, Oct. 1960, pp. 247-263.

6 Rayle, R. E., "Influence of Orifice Geometry on Static Pressure Measurements," presented at the ASME Annual Meeting, Atlantic City, NJ, Nov. 22-Dec. 4, 1959; ASME Paper No. 59-A-234.

7 Benedict, R. P., "The Plenum Inlet Discharge Coefficient of an ASME Nozzle," ISA Flow Symposium, Saint Louis, MO, 1981, Paper No. C1.81-533.

8 Murdock, J. W., and Keyser, D. R., "A Method for the Extrapolation of Calibration Data of PTC 6 Throat Tap Nozzles," presented at the Joint ASME/IEEE Power Generation Conference, Boston, MA, Oct. 22-24, 1990; ASME Paper No. 90-JPGC/PTC-2, ASME Journal of Engineering for Gas Turbines and Power, Vol. 113, 1991, this issue.

APPENDIX A

Derivation of Equations

Boundary Layer Thickness. The boundary layer thickness is defined as that point where the local velocity u is equal to the undisturbed velocity U. Figure 1 shows the boundary layer thickness δ at distance x from the leading edge of the plate. Because of the difficulty in determining the exact point where $u=U$, this thickness is defined as that distance where $u=0.99$ U. With this definition, the following equations are obtained from Schlichting [3] and referenced.

For laminar layer flow:

$$\delta_L\approx5\sqrt{\frac{x\nu}{U}}\approx\frac{5x}{\sqrt{\frac{xU}{\nu}}}\approx\frac{5x}{\mathrm{Re}_x^{1/2}} \quad \text{(A1)}$$

[3, 7.35]

For turbulent layer flow:

$$\delta_T=0.37x\left(\frac{xU}{n}\right)^{-1/5}=\frac{0.37x}{\mathrm{Re}_x^{1/5}} \quad \text{(A2)}$$

[3, 21.8]

Displacement Boundary Layer Thickness. The boundary layer displacement thickness δ^* is defined as that thickness the layer would have if the flow in the layer were at the external velocity U. The displacement thickness is defined by equation (A-3) and illustrated by Fig. A1.

$$U\delta^*=\int_0^\delta(U-u)dy \text{ or } \delta^*=\int_0^\delta\left(1-\frac{u}{U}\right)dy \quad \text{(A3)}$$

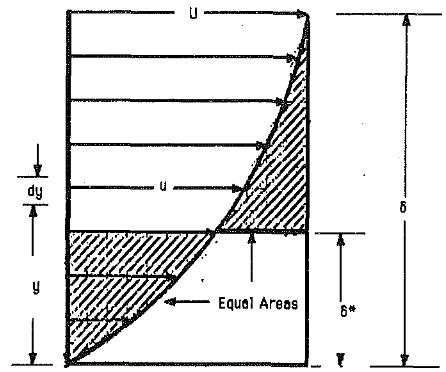


Fig. A1 Boundary layer displacement thickness

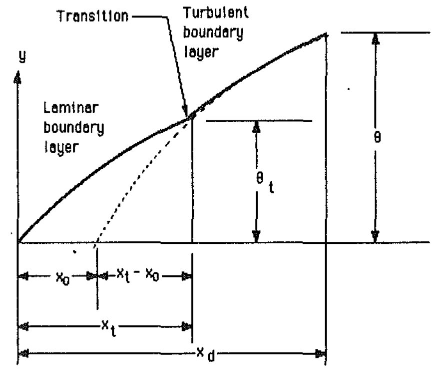


Fig. A2 Momentum boundary layer in transition

With this definition of displacement thickness, the following equations are obtained:

For laminar layer flow:

$$\delta_L^* = 1.7208\sqrt{\frac{\nu x}{U}} = \frac{1.7208x}{\sqrt{\frac{xU}{\nu}}} = \frac{1.7208x}{\mathrm{Re}_x^{1/2}} \qquad \text{(A4)}$$

[3, 7.37]

For turbulent layer flow:

$\delta_T^* = \frac{\delta_T}{8}$ from equation (A2)

$$\delta_T^* = \frac{\delta_T}{8} = \frac{0.37x}{8\mathrm{Re}_x^{1/5}} = \frac{0.04625x}{\mathrm{Re}_x^{1/5}} \qquad \text{(A5)}$$

[3, 21.6]

Momentum Boundary Layer Thickness. The loss of momentum in the boundary layer compared to the undisturbed flow gives rise to the momentum thickness, Θ, defined by equation (A6):

$$\theta = \int_0^\delta \frac{u}{U}\left(1 - \frac{u}{U}\right) dy \qquad \text{(A6)}$$

[3, 7.38]

For laminar layer flow:

$$\theta_L = 0.664\sqrt{\frac{\nu x}{U}} = \frac{0.664x}{\sqrt{\frac{Ux}{\nu}}} = \frac{0.664x}{\mathrm{Re}_x^{1/2}} \qquad \text{(A7)}$$

[3, 7.39]

For turbulent layer flow:

$$\theta_T = 0.036x\left(\frac{xU}{\nu}\right)^{-1/5} = \frac{0.036x}{\mathrm{Re}_x^{1/5}} \qquad \text{(A8)}$$

[3, 21.9]

Shape Parameters. The shape parameter, H, is a ratio of the displacement thickness δ^* to the momentum thickness, Θ:

$$H = \frac{\delta^*}{\theta} \qquad \text{(A9)}$$

For laminar layer flow:

$$H_L = \frac{\delta_L^*}{\theta_L} = \frac{1.7208x/\mathrm{Re}_x^{1/2}}{0.664x/\mathrm{Re}_x^{1/2}} \approx 2.59 \qquad \text{(A10)}$$

For turbulent layer flow:

$$H_T = \frac{\delta_T^*}{\theta_T} = \frac{0.04625x/\mathrm{Re}_x^{1/5}}{0.036x/\mathrm{Re}_x^{1/5}} \approx 1.28 \qquad \text{(A11)}$$

Hall's Simplified Nozzle

Figure 2 shows the relation of a nozzle without throat taps to Hall's [2] simplified nozzle. This model is a straight piece of pipe whose diameter is the nozzle diameter and whose length is equal to the diameter. From the continuity equation for an incompressible fluid, the ideal flow with no boundary layer would be

$$Q_i = UA_i = U\frac{\pi d^2}{4} \qquad \text{(A12)}$$

but the actual flow, Q_a, is

$$Q_a = UA_a = U\frac{\pi(d - 2\delta_{\max}^*)^2}{4} = U\frac{\pi(d^2 - 4d\delta_{\max}^* + 4\delta_{\max}^{*2})}{4}$$

$$\approx U\frac{\pi(d^2 - 4d\delta_{\max}^*)}{4} \quad (\delta_{\max}^* \text{ small compared with } d) \qquad \text{(A13)}$$

$\delta_{\max}^*$ is the boundary layer affecting the downstream pressure measurement.

The theoretical coefficient of discharge is essentially an area ratio formed by removing the annulus of the boundary layer from the area available to the flow

$$C_b = \frac{Q_a}{Q_i} = \frac{U\pi(d^2 - 4d\delta_{\max}^*)/4}{U\pi d^2/4} = 1 - \frac{4\delta_{\max}^*}{d} \qquad \text{(A14)}$$

Region I—Boundary Layer Wholly Laminar ($\delta_{\max}^* = \delta^* L$). In this region the maximum displacement thickness is obtained from equation (A4) with the distance x equal to the diameter, or:

$$\delta_{L\max}^* = \frac{1.7208x}{\mathrm{Re}_x^{1/2}} = \frac{1.7208d}{\mathrm{Re}_d^{1/2}} \qquad \text{(A15)}$$

Substituting equation (A15) into equation (A14) yields

$$C_b = C_L = 1 - \frac{4\delta_{\max}^*}{d} = 1 - \left(\frac{4}{d}\right)\left(\frac{1.7208d}{\mathrm{Re}_d^{1/2}}\right)$$

$$= 1 - \frac{6.88}{\mathrm{Re}_d^{1/2}} = 1 - \frac{L}{\mathrm{Re}_d^{1/2}} \qquad \text{(A16)}$$

where L = the laminar slope.

Before we can derive the equations for Regions II and III, we must derive the equation for the fully turbulent boundary layer.

Region IV—Boundary Layer Wholly Turbulent ($\delta_{\max}^* = \delta_T^*$). In this region the maximum displacement thickness is obtained from equation (A5) with the distance x equal to the diameter, or

$$\delta_{T\max}^* = \frac{0.04625x}{\mathrm{Re}_x^{1/5}} = \frac{0.04625d}{\mathrm{Re}_d^{1/5}} \qquad \text{(A17)}$$

Substituting equation (A17) into equation (A14)

$$C_b = C_L = 1 - \frac{4\delta^*_{max}}{d} = 1 - \left(\frac{4}{d}\right)\left(\frac{0.04625d}{\mathrm{Re}_d^{1/5}}\right)$$

$$= 1 - \frac{0.185}{\mathrm{Re}_d^{1/5}} = 1 - \frac{T}{\mathrm{Re}_d^{1/5}} \quad \text{(A18)}$$

where T = the turbulent slope.

Region II—Boundary Layer Partly Laminar and Partly Turbulent, With the Laminar Displacement Thickness Greater Than the Turbulent. This is a very narrow region, which could be ignored, except that it provides the physics of the beginning of the "transition hump" often discussed in the literature in connection with PTC-6 nozzle calibrations. Previously it had been assumed that once the data were "over the hump" the flow was in the fully developed turbulent region from which the C_d could be extrapolated to Reynolds numbers beyond those obtained from the flow calibration laboratory. The maximum laminar displacement thickness will occur at the end of Region I, the transition point x_t. From equation (A4)

$$\delta^*_{Lt\max} = \frac{1.7208 x_t}{\mathrm{Re}_t^{1/2}} \quad \text{(A19)}$$

Substituting equation (A19) into equation (A14)

$$C_b = C_{Lt} = 1 - \frac{4\delta^*_{max}}{d} = 1 - \left(\frac{4}{d}\right)\left(\frac{1.7208 x_t}{\mathrm{Re}_t^{1/2}}\right) = 1 - \frac{6.88 x_t}{\mathrm{Re}_t^{1/2}} \quad \text{(A20)}$$

Noting that

$$\frac{x_t}{d} = \frac{\mathrm{Re}_t}{\mathrm{Re}_d} \text{ or } x_t = \frac{d\mathrm{Re}_t}{\mathrm{Re}_d} \quad \text{(A21)}$$

Substituting equation (A21) into equation (A20)

$$C_b = C_{Lt} = 1 - \frac{6.88 x_t}{d\mathrm{Re}_t^{1/2}} = 1 - \frac{6.88(d\mathrm{Re}_t/\mathrm{Re}_d)}{d\mathrm{Re}_t^{1/2}} =$$

$$1 - \frac{6.88\mathrm{Re}_t^{1/2}}{\mathrm{Re}_d} = 1 - \frac{L\mathrm{Re}_t^{1/2}}{\mathrm{Re}_d} \quad \text{(A22)}$$

Region III—Boundary Layer Partly Laminar and Partly Turbulent, With the Turbulent Displacement Thickness Greater Than the Laminar. At the beginning point of Region III, the momentum boundary layer thickness, θ_t, must be the same for both laminar and turbulent layers. The distance x_0 in Fig. A2 is the distance from the leading edge that a hypothetical fully turbulent boundary layer would begin in order to equal the momentum boundary layer thickness of θ_t at the transition point distance, x_t.

For laminar flow, from equation (A7)

$$\theta_L = \theta_t = \frac{0.664 x_t}{\mathrm{Re}_t^{1/2}} = \frac{K_L x_t}{\mathrm{Re}_t^{1/2}} \quad \text{(A23)}$$

where K_L is the laminar momentum coefficient

For turbulent flow, from equation (A8)

$$\theta_T = \theta_t = \frac{0.036(x_t - x_0)}{\left(\frac{(x_t - x_0)U}{\nu}\right)^{1/5}} = \frac{K_T x_t^{1/5}(x_t - x_0)^{4/5}}{\left(\frac{x_t U}{\nu}\right)^{1/5}} = \frac{K_T x_t^{1/5}(x_t - x_0)^{4/5}}{\mathrm{Re}_t^{1/5}} \quad \text{(A24)}$$

where K_T is the turbulent momentum coefficient.

Equating (A23) and (A24)

$$\theta_t = \frac{K_T x_t^{1/5}(x_t - x_0)^{4/5}}{\mathrm{Re}_t^{1/5}} = \frac{K_L x_t}{\mathrm{Re}_t^{1/2}}$$

$$(x_t - x_0)^{4/5} = \left(\frac{K_L}{K_T}\right)\left(\frac{x_t}{x_t^{1/5}}\right)\left(\frac{\mathrm{Re}_t^{1/5}}{\mathrm{Re}_t^{1/2}}\right) = \frac{x_t^{4/5}}{\mathrm{Re}_t^{3/10}}\left(\frac{K_L}{K_T}\right)$$

$$x_t - x_0 = \frac{x_t}{\mathrm{Re}_t^{3/8}}\left(\frac{K_L}{K_T}\right)^{5/4} \quad \text{(A25)}$$

Solving equation (A25) for x_0

$$x_0 = x_t - \frac{x_t}{\mathrm{Re}_t^{3/8}}\left(\frac{K_L}{K_T}\right)^{5/4} = x_t\left(1 - \frac{(K_L/K_T)^{5/4}}{\mathrm{Re}_t^{3/8}}\right)$$

Note that $\frac{x_t}{x_d} = \frac{\mathrm{Re}_t}{\mathrm{Re}_d}$ so that

$$x_0 = \frac{x_d \mathrm{Re}_t}{\mathrm{Re}_d}\left(1 - \frac{(K_L/K_T)^{5/4}}{\mathrm{Re}_t^{3/8}}\right) \quad \text{(A26)}$$

Solving equation (A26) for $x_d - x_0$ (the distance from the origin of the hypothetical turbulent boundary layer)

$$x_d - x_0 = x_d - x_d\left(\frac{\mathrm{Re}_t}{\mathrm{Re}_d}\right)\left(1 - \frac{(K_L/K_T)^{5/4}}{\mathrm{Re}_t^{3/8}}\right)$$

$$= x_d\left(1 - \frac{\mathrm{Re}_t}{\mathrm{Re}_d}\left(1 - \frac{(K_L/K_T)^{5/4}}{\mathrm{Re}_t^{3/8}}\right)\right) \quad \text{(A27)}$$

The maximum turbulent displacement from equation (A5)

$$\delta^*_{max} = \delta^*_{Tt} = \frac{0.04625(x_d - x_0)}{\left(\frac{(x_d - x_0)U}{\nu}\right)^{1/5}} = \frac{0.04625 x_d^{1/5}(x_d - x_0)^{4/5}}{\mathrm{Re}_d^{1/5}} \quad \text{(A28)}$$

Substituting equation (A28) into equation (A14) and noting that for Hall's model nozzle, $x_d = d$:

$$C_b = C_{Tt} = 1 - \frac{4\delta^*_{max}}{x_d} = \left(\frac{4}{x_d}\right)\left(\frac{0.04625 x_d^{1/5}(x_d - x_0)^{4/5}}{\mathrm{Re}_d^{1/5}}\right)$$

$$= 1 - \frac{0.185}{\mathrm{Re}_d^{1/5}}\left(\frac{x_d - x_0}{x_d}\right)^{4/5} = 1 - \frac{T}{\mathrm{Re}_d^{1/5}}\left(\frac{x_d - x_0}{x_d}\right)^{4/5} \quad \text{(A29)}$$

Substituting equation (A27) into equation (A29)

$$C_b = C_{Tt} = 1 - \frac{T}{\mathrm{Re}_d^{1/5}}\left(\frac{x_d\left(1 - \frac{\mathrm{Re}_t}{\mathrm{Re}_d}\left(1 - \frac{(K_L/K_T)^{5/4}}{\mathrm{Re}_t^{3/8}}\right)\right)}{x_d}\right)^{4/5}$$

$$= 1 - \frac{T}{\mathrm{Re}_d^{1/5}}\left(1 - \frac{\mathrm{Re}_t}{\mathrm{Re}_d}\left(1 - \frac{(K_L/K_T)^{5/4}}{\mathrm{Re}_t^{3/8}}\right)\right)^{4/5} \quad \text{(A30)}$$

By definition

$$\frac{K_L}{K_T} = \frac{\theta_L}{\theta_T} = \frac{\delta^*_L/H_L}{\delta^*_T/H_T} = \frac{H_T}{H_L}\frac{L}{T} \quad \text{(A31)}$$

Substituting equation (A31) into equation (A30) gives (finally) the correct theoretical equation for the coefficient of discharge for a throat tap nozzle throughout both Regions III and IV.

$$C_b = C_{Tt} = 1 - \frac{T}{\mathrm{Re}_d^{1/5}}\left(1 - \frac{\mathrm{Re}_t}{\mathrm{Re}_d}\left(1 - \frac{(H_T/H_L)^{5/4}(L/T)^{5/4}}{\mathrm{Re}_t^{3/8}}\right)\right)^{4/5} \quad \text{(A32)}$$

This equation can be fitted easily to individual calibration data to account for minor differences in installation or manufacture.

Coefficient of Discharge at $\mathrm{Re}_d = \infty$

The coefficient of discharge at Reynolds number of infinity is defined by equation (A33) as follows:

$$C_\infty = 1 + \text{"Tap Correction"} \quad \text{(A33)}$$

and consequently the equation of the actual coefficient of discharge of an ASME/PTC 6 throat tap nozzle is simply equation (A32) plus the throat tap correction. This equation is valid throughout regions III and IV. The average value for this correction is +0.0054; however [4] permits a variation of ±0.0025 about this value.