Encoded Pilots for Iterative Receiver Improvement

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Abstract[1]—This paper proposes a novel iterative channel estimation and low density parity check (LDPC) (not turbo) decoding scheme where the pilot symbols are encoded and can be used for both channel estimation and decoding. To achieve this objective, this paper will employ *systematic* LDPC codes so that pilot symbols can be encoded as data. In this way, initial channel estimation can be made before decoding by using systematic coded pilot symbols. In addition, the known pilot symbol positions have higher reliability than data and can significantly improve the initial decoding. Moreover, the encoded pilot symbols are not necessary to be transmitted for decoding purpose. So, the encoded pilot symbols can be called artificial symbols. This paper has wide applications in wireless communications systems because many of them require channel estimation and coding.

I. INTRODUCTION

Turbo codes have shown near-capacity performance; however, their high complexity is still costly [1]. In [2], Gallager introduced a low-density parity-check (LDPC) code. Later work [3] reveals that LDPC codes can approach the Shannon limit as closely as do the turbo codes. Moreover, recent studies show that LDPC codes match or even outperform turbo codes while requiring lower complexity [4-5].

On the other hand, Orthogonal Frequency Division Multiplexing (OFDM) has been widely adopted by many wireless communication systems because it offers the possibility of high data rates with low decoding complexity [6]. The OFDM demodulator requires channel estimation. Also, other wireless communication systems such as multiple-input multiple-output (MIMO) systems with various forward error correction algorithms, e.g., turbo codes, convolutional codes, Reed Solomon (RS) codes, and LDPC codes were pursued to mitigate a channel fading effect.

Since channel estimation in coded systems is essential for coherent demodulation and detection, much of the literature has focused on obtaining the channel state information (CSI) in coded OFDM systems [7-8]. In order to provide better CSI, iterative channel estimation algorithms combined with a decoding algorithm have been suggested [9-11]. Most studies regarding the iterative channel estimation and decoding of coded systems have considered techniques that use the pilot symbols only for channel estimation [9, 11-13]. If the pilot symbols are encoded, then the initial channel estimation may be difficult to perform.

Departing from the iterative receivers using time-multiplexed pilots in [11-13], this paper employs *systematic* LDPC codes so that pilot symbols can be encoded as data. In this way, initial channel estimation can be made before decoding by using the systematic coded pilot symbols. In addition, the known pilot symbol positions have higher reliability than data and can significantly improve the initial and following decoding. In other words, for the proposed LDPC-coded OFDM systems, the encoded pilots are placed in the OFDM block at known positions as code symbols. This is possible because we use a systematic encoder, and these pilot code symbols can help the LDPC decoding process. The encoded pilot symbols in the proposed system consume the same bandwidth (BW) as the traditional time-multiplexed un-coded pilot symbols in [11-13].

II. SYSTEM MODEL

This system model describes only a single transmit and single receive antenna (SISO) LDPC-based OFDM system with encoded pilot symbols to demonstrate the main concept. We will study MIMO systems in the future and extend our results to other coding and modulation systems.

Notation: Upper and lower boldface letters denote matrices and column vectors, respectively. Superscripts $(\cdot)^H$ and $(\cdot)^T$ indicate Hermitian and transpose, respectively. The $diag[\boldsymbol{x}]$ stands for a diagonal matrix with $\boldsymbol{x}$ on its main diagonal. Matrix $\boldsymbol{D}_N(\boldsymbol{h})$ with a vector argument denotes

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an $N\times N$ diagonal matrix $\boldsymbol{D}_N(\boldsymbol{h}) = diag[\boldsymbol{h}]$. For a vector, $\|\cdot\|$ denotes the Euclidean norm. $[\boldsymbol{A}]_{k,m}$ stands for the $(k,m)^{\text{th}}$ entry of a matrix $\boldsymbol{A}$, $x(m)$ for the m^{th} entry of the column vector $\boldsymbol{x}$, $\boldsymbol{I}_N$ for the $N\times N$ identity matrix, and $[\boldsymbol{F}_N]_{m,n} = N^{(-1/2)}\exp(-j2\pi mn/N)$ for the $N\times N$ fast Fourier transform (FFT) matrix $\boldsymbol{F}_N$.

A. *Transmitter and Channel*

We consider the discrete-time equivalent baseband model of an LDPC-coded OFDM system communicating over frequency-selective channels, as shown in Fig. 1, where the pilots and information data are also encoded. The information data $\boldsymbol{m}_b$ of length N_b and pilot symbols $\boldsymbol{m}_p$ of length N_p are mixed by using the mutually orthogonal permutation matrices $\boldsymbol{P}_A$ and $\boldsymbol{P}_B$ to form $\boldsymbol{s}$ with N_s elements as,

$$\boldsymbol{s} := \boldsymbol{P}_A\boldsymbol{m}_b + \boldsymbol{P}_B\boldsymbol{m}_p \tag{1}$$

where $\boldsymbol{P}_A$ and $\boldsymbol{P}_B$ satisfy $\boldsymbol{P}_A^T\boldsymbol{P}_B = \boldsymbol{O}_{N_b\times N_p}$. Note that $N_b+N_p=N_s$. One example of such matrices is to form $\boldsymbol{P}_A$ with the first N_b columns of $\boldsymbol{I}_{N_s}$ and $\boldsymbol{P}_B$ with the last N_p columns of $\boldsymbol{I}_{N_s}$.

Then $\boldsymbol{s}$ is encoded into $\bar{\boldsymbol{c}}$ of length $N\cdot\log_2 M$ by the systematic LDPC encoder G_{sys}, where the code rate is $R:=N_s/(N\cdot\log_2 M)$ for M-ary modulation. We note that $\bar{\boldsymbol{c}}$ consists of the original data (information and pilot symbols) and the corresponding parity data, since G_{sys} is a systematic generator matrix. In the QPSK modulator, pairs of LDPC-encoded bits are used to form a sequence of complex symbols. The QPSK-modulated block $\breve{\boldsymbol{c}}$ of length N is permutated by the $N\times N$ permutator $\boldsymbol{P}$ to form $\boldsymbol{c}$, i.e., $\boldsymbol{c} = \boldsymbol{P}\breve{\boldsymbol{c}}$, where $\boldsymbol{P}$ is used to place the pilot symbols in the systematic part at the desired positions $\{i_0,\cdots,i_{N_p/\log_2 M-1}\}$ based on the optimal pilot design for channel estimation in OFDM systems.

Following the permutation, we perform the OFDM operation. Specifically, we implement N-point inverse FFT (IFFT) on $\boldsymbol{c}$ and insert the cyclic prefix (CP) to form $\boldsymbol{x}$ of length P. After the parallel to serial (P/S) conversion, each data segment is transmitted through the multi-path channel.

The frequency-selective channel in discrete-time baseband equivalent form is denoted by $\boldsymbol{h}:=[h(0),\ldots,h(L)]^T$ with order L. This channel incorporates transmitter-filter $g_{tx}(t)$, receiver-filter $g_{rx}(t)$, and frequency-selective multipath $g(t)$: i.e., $h(l)=(g_{tx}(t)*g(t)*g_{rx}(t))|_{t=lT}$, where * denotes convolution, and T is the sampling period. Then the samples at the receive-antenna filter output can be written as

$$\bar{y}(n) = \sum_{l=0}^{L} h(l)x(n-l) + \bar{w}(n) \tag{2}$$

where $\bar{w}(n)$ is zero-mean, white Gaussian noise with variance $\sigma_{\bar{w}}^2 := N_0/2$. The sequence $\bar{y}(n)$ is then serial to parallel (S/P) converted into $\bar{\boldsymbol{y}}$ of length P.

B. *Receiver*

After removing the CP and taking the FFT operation, we can obtain the input-output relationship

$$\boldsymbol{y} = \boldsymbol{D}_N(\tilde{\boldsymbol{h}})\boldsymbol{c} + \boldsymbol{w} \tag{3}$$

where $\boldsymbol{D}_N(\tilde{\boldsymbol{h}})$ is a diagonal matrix, and its element is the channel frequency response values on the FFT grid, i.e., $\tilde{\boldsymbol{h}} := [\tilde{h}(0),\ldots,\tilde{h}(2\pi(N-1)/N)]^T$. We note that (3) renders a set of flat-fading sub-channels equivalent to the frequency-selective channel [14].

III. PROPOSED ITERATIVE RECEIVER

A. *Initial Channel Estimation*

For the first, i.e., initial, channel estimation, we use N_p pilot symbols, which are placed in the OFDM block at known positions by using the permutator $\boldsymbol{P}$ at the transmitter. For the results, we use $\boldsymbol{P}$ that places the pilot symbols at equally spaced positions in an OFDM block. From (3), we can extract observations corresponding to known pilot positions as

$$\boldsymbol{y}_p = \boldsymbol{B}\boldsymbol{h} + \boldsymbol{w}_p \tag{4}$$

where $\boldsymbol{y}_p = [y(i_0),\cdots,y(i_{N_p/\log_2 M-1})]^T$, and $\boldsymbol{B} := diag[c(i_0),\ldots,c(i_{N_p/\log_2 M-1})]\cdot\boldsymbol{F}$, with $\boldsymbol{F}$ denoting the first $L+1$ columns and pilot position-related $N_p/\log_2 M$ rows of $\boldsymbol{F}_N$. From (4), the linear minimum mean-square error (LMMSE) channel estimator for the first iteration is given by [14]

$$\hat{\boldsymbol{h}}_{LMMSE}^{(1)} := (\sigma_w^2\boldsymbol{R}_h^{-1} + \boldsymbol{B}^H\boldsymbol{B})^{-1}\boldsymbol{B}^H\boldsymbol{y}_p \tag{5}$$

where $\boldsymbol{R}_h:=E[\boldsymbol{h}\boldsymbol{h}^H]$ is the channel covariance matrix, and σ_w^2 denotes the noise variance.

If $\boldsymbol{R}_h$ is difficult to find in practice, the maximum likelihood (ML) channel estimator can be used. With $N_p/\log_2 M \geq L+1$ and the matrix $\boldsymbol{B}^H\boldsymbol{B}$ selected to have full rank, the ML channel estimator takes the following form [14]

$$\hat{\boldsymbol{h}}_{ML}^{(1)} = (\boldsymbol{B}^H\boldsymbol{B})^{-1}\boldsymbol{B}^H\boldsymbol{y}_p \tag{6}$$

In order to guarantee the ML estimation performance, we need a minimum number of $(L+1)\log_2 M$ pilot symbols. Note that the factor $\log_2 M$ is due to the M-ary modulation.

Use of the minimum number of pilot symbols may yield a high normalized mean square error (NMSE) in channel estimation due to deep fades in the frequency selective channel. Thus, more pilot symbols are preferable for reliable channel estimation at the expense of the bandwidth efficiency.

B. Iterative Channel Estimation

From the second iteration, the channel estimator has the same form as that of the first iteration, but it is slightly different. Instead of using N_p pilot symbols only for channel estimation as above, we can employ all the decoded symbols $\hat{\bar{\boldsymbol{c}}}^{(1)}$, which are initially obtained in the first iteration. Noticing that $\hat{\boldsymbol{c}}^{(1)}$ is the permutated block of N symbols after QPSK modulation on $\hat{\bar{\boldsymbol{c}}}^{(1)}$, we formulate the following

$$\boldsymbol{y} = \boldsymbol{B}^{(1)}\boldsymbol{h} + \boldsymbol{w} \tag{7}$$

where $\boldsymbol{B}^{(1)} := \sqrt{N} \cdot diag\left[\hat{c}^{(1)}\right] \cdot \boldsymbol{F'}$, with $\boldsymbol{F'}$ denoting the first $L+1$ columns of $\boldsymbol{F}_N$. Thus, the ML estimator for the second iteration can be expressed as

$$\hat{\boldsymbol{h}}_{ML}^{(2)} = (\boldsymbol{B}^{(1)^H}\boldsymbol{B}^{(1)})^{-1}\boldsymbol{B}^{(1)^H}\boldsymbol{y} \tag{8}$$

In this way, the μth CSI $\hat{\boldsymbol{h}}_{ML}^{(\mu)}$ also can be obtained by using the LDPC decoder output in the $(\mu\text{-}1)^{st}$ iteration, where $\mu=1,2,\ldots,\mu_{max}$ with μ_{max}, denoting the predefined maximum number of iterations in our iterative receiver. The performance of the estimator in (8) depends on the LDPC decoder output, which is also affected by the previous channel estimator. Therefore the iterative channel estimator can give a more accurate CSI, which in turn can improve the performance of the LDPC decoder.

Because the channel compensator in Fig. 2 requires knowledge of all channel frequency responses on the FFT grid at the μ^{th} iteration, the estimate of $\boldsymbol{D}_N(\hat{\tilde{\boldsymbol{h}}}^{(\mu)})$ corresponding to $\hat{\boldsymbol{h}}_{ML}^{(\mu)}$ can be computed as

$$\boldsymbol{D}_N(\hat{\tilde{\boldsymbol{h}}}^{(\mu)}) = \sqrt{N}\boldsymbol{F'}\hat{\boldsymbol{h}}_{ML}^{(\mu)} \tag{9}$$

Based on (3) and (9), the channel compensator gives the following output

$$\hat{\boldsymbol{y}}^{(\mu)} := \boldsymbol{D}_N(\hat{\tilde{\boldsymbol{h}}}^{(\mu)})\boldsymbol{y} \tag{10}$$

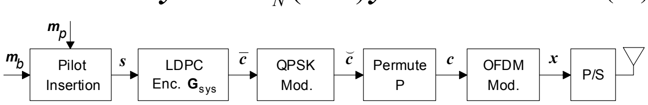

Fig: 1 Transmitter structure.

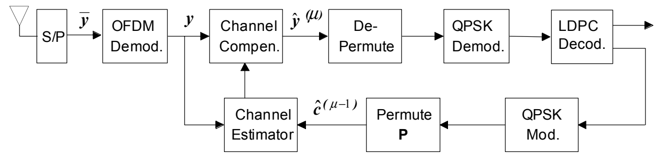

Fig: 2 Receiver structure.

C. LDPC Decoding

Once the CSI is obtained, the log-likelihood ratios (LLRs) for the LDPC decoder are computed. We will use the information of the encoded pilot symbol positions in order to increase the error correction capability at the LDPC decoder in addition to channel estimation. The received pilot symbol values will be used for the initial channel estimation.

Since the positions of the pilots are known at the receiver, we can employ a high *a priori* LLR, $LR(f_i) := \pm T$, where i belongs to the known pilot positions. Because of the high *a priori* LLR, these locations are highly reliable and thus they can affect the LLR of other information bit nodes $\{LR(p_i^{(v)})\}$, where $LR(p_i^{(v)})$ is the log-likelihood ratio of the i^{th} information bit node after v^{th} iteration. In this way, the encoded pilot symbols employed in the LDPC decoder can protect the information data.

In the simulation, T is set to be 50, which is relatively high compared to the normal input. Fig. 3 also indicates that $T=50$ is enough for the simulation set up because the number of message bit errors in the message field $\boldsymbol{m}_b$ of length *992* bits per frame does not decrease further after $T=10$.

To confirm the effect of data protection by the known pilot positions, we conduct a simulation as shown in Fig. 4, using the (*1024,2048*) parity check matrix of the column weight $j=3$ is used; 2.5% of known pilot positions are selected among the bit nodes ($N \cdot \log_2 M=2048$) are assumed with QPSK modulation and frequency selective channel is considered. It can be observed from Fig. 4 that the pilot-aided LDPC decoding algorithm has a fast convergence and a possibility of better BER performance. In other words, the pilots can help to correct a larger number of erroneous bit nodes in a few iterations. Fig. 4 shows that the proposed pilot-aided LDPC decoder can correct 11(=82-71) more message bit errors in the *992* message bit field $\boldsymbol{m}_b$ per frame than the uncoded pilot scheme at $E_b/N_0=1$ *dB*.

IV. SIMULATION RESULTS

For the simulations, the HyperLAN/2 channel model B [15] is used to generate the frequency selective channels of order L=15. For the purpose of comparison, we introduce another possible design (reference system) where pilots are uncoded and only used for channel estimation, i.e., they are inserted after the LDPC encoder and removed before the LDPC decoder, as was done in [11-13]. For both systems, we use QPSK modulation; hence the OFDM block length is 1024.

LDPC code parameters for the proposed system and reference system are detailed in Table 1. In the reference scheme, we delete those columns of the parity check matrix $\boldsymbol{H}$, where pilots are placed in coded-pilot case. After deleting 32 (number of pilots) columns from $\boldsymbol{H}$ we generate a new $\boldsymbol{G_{sys}}$ matrix (1024, 2016) to encode the information part of length 1024 bits (without pilots). The almost half-rate LDPC encoder outputs the code word of length 2016 bits. Then 32 pilots are added separately for channel estimation and uniformly spread out in the obtained code word by doing a permutation. In the same way after de-permutation, pilots are removed before decoding at the receiver. Thus, 1024 bits of information data is transmitted per OFDM block, but the comparison is made by taking the first 992 bits of both the proposed and reference system; hence, in the results we refer to the message block as 992 bits.

We now plot BER versus SNR in Figs. 5 and 6 with soft and hard decision feedback, respectively. The ideal case corresponding to a perfectly known CSI is also depicted as a benchmark. The results in both Figs. 5 and 6 show BER performance improvement with the increased iterations and substantiate our claim that the coded pilots can help to correct the erroneous symbols, which results in better BER performance. The proposed system of the parity check matrix $\boldsymbol{H}$ size = (1024, 2048) shows about 0.6 dB performance gain over the reference system with the uncoded pilots at BER=10^{-2} and $\mu=1$ iteration in Fig. 5. Soft and hard decision feedback is only 0.3 dB and 0.6 dB away from the perfect CSI at BER=10^{-2} and μ=2 iterations, respectively. We expect that the good LDPC codes of a large size $\boldsymbol{H}$ will enhance the refined channel estimation by the reliable estimates of data symbols from the LDPC decoder, which in turn improves the BER performance.

Table 1

Simulation parameters for results shown in Figs. 3-6.

Parameters	Proposed Model
Information	N_b= 992
H and G_{sys}	(1024,2048)
Row weight	6
Code rate	0.5
FFT Block	1024
Pilots	32

V. CONCLUSIONS

The proposed iterative receiver using both channel estimation and decoding in a SISO OFDM spread spectrum system meets the objectives of the future wireless communications systems because: of the following (1) the feedback from the decoder can reduce the number of pilot symbols, which will increase the bandwidth efficiency; (2) the feed-forward from the channel estimation to the decoder with the known pilot positions can significantly enhance the decoder performance, which will provide the low BER; and (3) the OFDM or spread spectrum systems can be effective against frequency selective fading channel and jamming.

Simulation results show that BER performance can be improved with the proposed iterative channel estimation and pilot-aided LDPC decoding for OFDM systems. Since the pilot-embedded LDPC coding technology is quite new, its theoretical analysis is far from complete.

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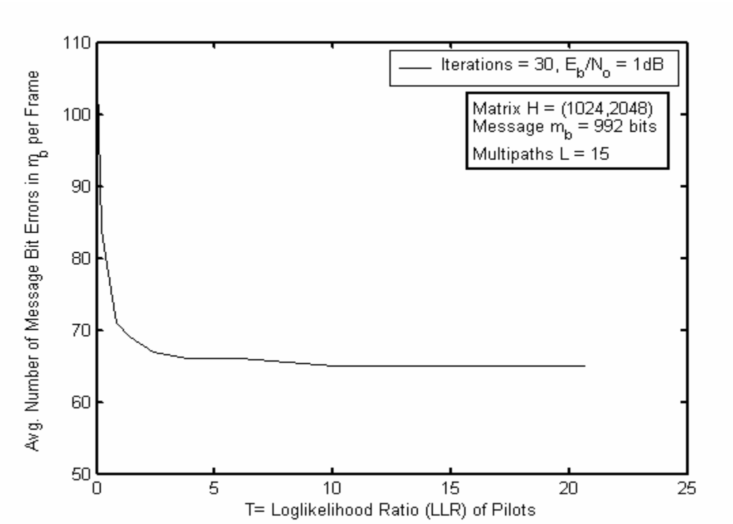

Fig: 3 Avg. number of message bit errors in $\mathbf{m}_b$=992 bits versus T= log likelihood ratio threshold.

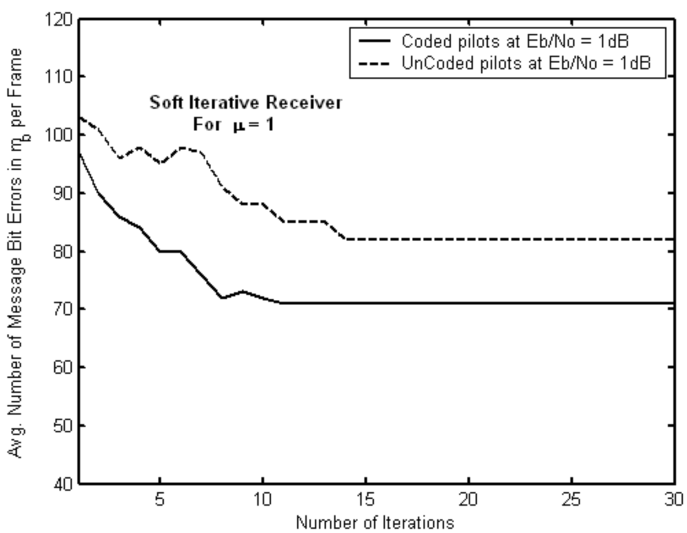

Fig: 4 Avg. number of message bit errors in $\mathbf{m}_b$=992 bits for uncoded and encoded pilots at E_b/N_0= 1 dB.

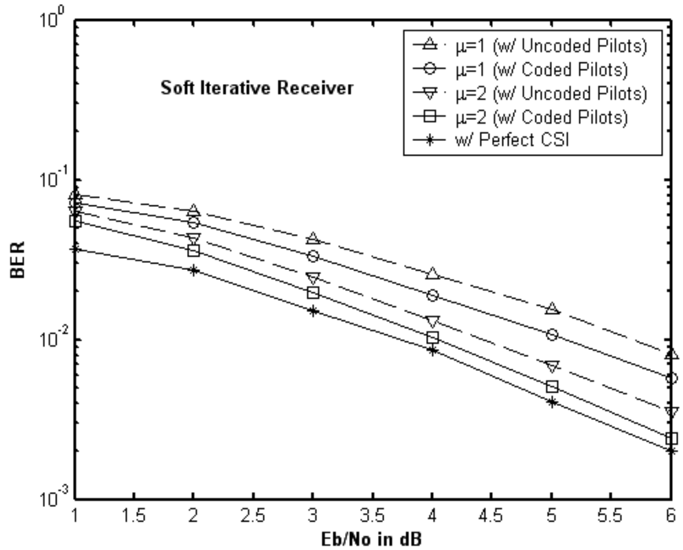

Fig: 5 BER vs E_b/N_0 for soft iterative receiver.

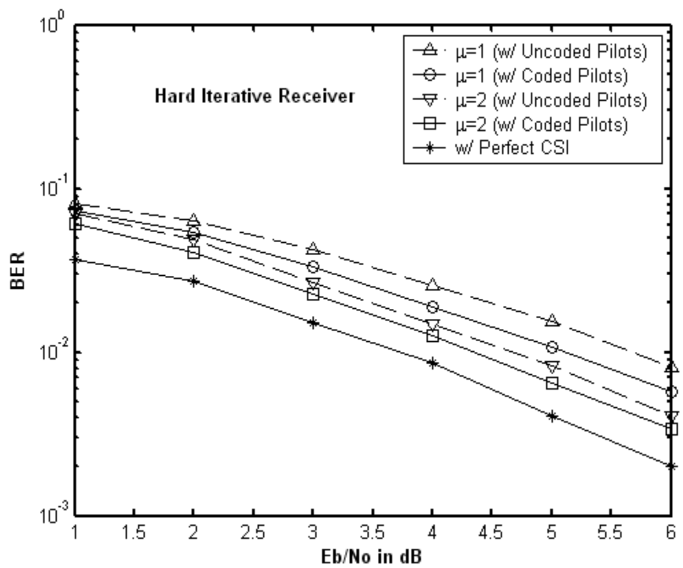

Fig: 6 BER vs E_b/N_0 for hard iterative receiver.