

# Modeling of Soft Materials: Integrating Bond Graphs with Finite Element Analysis

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## Abstract

In this paper, modeling of system dynamics during the soft contact interaction between a rigid body and a soft material is carried out using the Bond Graph technique and the Finite Element method. The Finite Element method is employed to discretize the elastic continuum and obtain material characteristics of inertia, stiffness and damping. An effort has been made to integrate the Bond Graph formulation for dynamic modeling and analysis with the advantages of the Finite Element method. The force-deformation relationship is represented using *C-fields* in Bond graphs, and its parameters are obtained from the Finite Element formulation. The model has been tested through simulation. Simulation code has been developed using MATLAB directly from the Bond graph.

**Keywords:** Bond Graph, Modeling soft materials, Soft contact, Finite element method.

## 1 Introduction

Soft contact interaction plays an important role in many applications. During the soft contact interaction, the contact force is distributed over an area of contact due to the effect of material compliance between interaction bodies. The area itself can continuously change. In case of hard bodies the contact interaction can be considered to occur at a point. The estimation of contact force and its distribution requires knowledge of contact characteristics, including relationship between forces and contact area and pressure distribution profile at the contact interface. It is therefore quite interesting to analyze the contact mechanics of rigid bodies interacting with soft materials, especially when viewed from the perspective of robotics and control.

The soft material in between the object and robotic fingers affects the kinematics and dynamics of the sys-

tem during grasping and manipulation. The deformation effect of soft material of robotic fingertips is a useful feature in general robotic application. Moreover, soft material contact mechanics plays an important role in grasping stability as well as safe object prehension and handling during manipulation. However, the deformation effect of soft material during the contact tasks is not well understood and hence is drawing a lot of research attention in robotics, especially due to its applications in biomechanics and prosthetics. The development of forces and moments at the contact interface due to the change in the contact area during manipulation also needs analysis.

The conventional approach so far has been to consider soft contact based on point contact. Hertz first studied contact mechanics in 1882, based on contact between two linear elastic materials [1]. Hertz's model can be used to predict the elasticity at each contact as well as the pressure distribution across each contact patch. Later, the study of contact mechanics became a branch in mechanics [1]. Kwi-Ho Park, Byoung-Ho-Kim, and Shiniichi Hirai [2] developed a hemisphere-shaped soft fingertip for soft fingers and presented its modeling based on force distribution. Assuming that the materials of soft fingers, e.g. Rubber, Silicon and human tissue are non-linear elastic, they proposed a relationship between the normal force and area of contact. Nicholas Xydas and Imin Kao proposed a soft-contact model for soft fingers [3]. M. R. Cutkosky [4], Nicholas Xydas and Imin Kao [3] continued with the conventional approach in their contact models. The conventional approach, however, is not able to provide answers to questions of area generated during contact, generation of forces and their distribution at the area of contact interface, and so on.

In this paper, modeling of the system dynamics during the soft contact interaction between a rigid body and a soft material is carried out using the Bond Graph technique and Finite Element method. The Finite Element method is convenient for static analysis, and can be employed to discretize the elastic continuum [5]. One can obtain characteristics of inertia, stiffness and damping, based on such discretization. The models are of special significance for (1) design and analysis for robotics hand

development for dexterous manipulation, soft fingertips, etc., and (2) the control of robotic manipulation.

### 1.1 Bond graph modeling

The Bond graph technique is especially convenient and computationally advantageous for dynamic analysis [6-8]. Bond graph is the pictorial representation of the dynamics of the system and are convenient for modeling of physical system dynamics in multiple energy domains. A Bond graph model is based on the interaction of power between the elements. The method of Bond graphs is an attractive and powerful technique as it offers a unified framework for modeling the mechanism, and, the actuation and control systems due to its capability of handling multi-energy domains and depiction of cause and effect. Due to these advantages the Bond graph technique is used for modeling of soft contact. The finite element method has become a powerful tool for the numerical solution of a wide range of engineering problems. In this method, a continuum is discretized into simple geometric shapes called finite elements. The main emphasis in this paper is to analyze the effect of soft material, specially the deformation of soft material under the action of contact forces.

In this work, the Finite Element method is used for determination of the force-deformation relationship during contact as the deformation occurs when the rigid body makes contact with soft material. The force-deformation relationship is represented using *C-fields* in Bond graphs, and its parameters are obtained from the Finite Element formulation. Damping can also be handled likewise. Inertial distribution is accounted using point masses at nodes. System equations are derived algorithmically from the Bond Graph model.

## 2 Theoretical Developments

Hertz first studied contact mechanics in 1882, based on contact between two linear elastic materials [1]. Hertz's model can be used to predict the elasticity at each contact as well as the pressure distribution across each contact patch. Later, the study of contact mechanics became a branch in mechanics. One of the important results, in *Hertz's* seminal paper postulated that

$$a = CN^{\frac{1}{3}} \quad (1)$$

Where  $N$  is the normal force,  $a$  is the radius of circular contact area, and  $C$  is a proportional constant. Equation (1), derived by Hertz [1], describes the growth of radius of circular contact area as proportional to the normal force raised to the power of  $\frac{1}{3}$ . It is important to note that Hertz had assumed that (i) the contact is linear elastic and, (ii) the deformation is small.

Hertzian contact theory is probably the most widely used analytical contact model. Analytical models for elastic contacts, including the solution by Hertz, are discussed by Johnson [1]. Later, Tatara derived and proved that the Hertzian model does not apply to non-linear elastic materials involving large deformation [3] with

$$\frac{a}{r_o} > 0.3 \quad (2)$$

Where  $a$  is the radius of contact and  $r_o$  is the radius of curvature of the spherical fingertips. Tatara suggested that the soft materials can not be tested the same way as the linear elastic contacts, because the soft material tends to be non-linear and display a large deformation [1]. Kwi-Ho Park, Byoung-Ho-Kim, and Shinichi Hirai [2] developed a hemisphere-shaped soft fingertip for soft fingers and presented its modeling based on force distribution. Assuming that the materials of soft fingers e.g. Rubber, Silicon and human tissue are non-linear elastic; they derived the relationship between the normal force and area of contact.

In 1999, Nicholas Xydas and Imin Kao [3] proposed a soft-contact model for soft fingers. The soft contact model is

$$a = CN^{\gamma} \quad (3)$$

Where  $\gamma$  is a constant, depending on the material of contact, which ranges from 0 to  $\frac{1}{3}$ . Equations (1) and (3) are very similar, except the exponent. In equation (3) as the fingertip material become softer,  $\gamma$  will become smaller. In robotics, soft finger contacts have been explored by Cutkosky [4]. Sinha and Abel proposed an elastic contact stress model for finger-object contacts in multi-fingered grasping [4]. M. R. Cutkosky, Nicholas Xydas and Imin Kao have adopted the conventional approach in their contact models. Conventional approach so far has been to consider soft contact as based on point contact with empirical relations used to describe the stiffness interaction. The conventional approach, however, is unable to provide answers to questions of area generated during contact, generation of forces and their distribution at the area of contact interface, and so on.

## 3 Using the Finite Element Method

The finite element method has become a powerful tool for the numerical solution of a wide range of engineering problems. In this method, a continuum is discretized into simple geometric shapes called *finite elements*. The main emphasis in this paper is to analyze the effect of soft material, specially the deformation of soft material under the action of contact forces. [5] The material properties and the governing relationships are considered over these elements and expressed in terms of unknown values at element corners. An assembly process, element connectivity, duly considering the loading and constraints, results in a set of equations. Solution of these equations gives us the approximate behavior of the continuum. After discretizing the region, the element stiffness matrix can be calculated [5] as

$$k_e = t_e A_e B^T D B \quad (4)$$

Where  $t_e$  is the element thickness,  $A_e$  is the area of the element,  $D$  is the material property matrix, and,  $B$  is the element strain-displacement matrix. Using the element connectivity information, the global stiffness matrix can be calculated by placing the element stiffness matrix in

the appropriate locations in the global matrix. By considering the minimum potential energy approach the Finite element equations are developed after a consistent treatment of the boundary conditions [5].

$$KQ = F \quad (5)$$

$K$  is global stiffness matrix,  $F$  is global load vector and  $Q$  is the global displacement vector. Details have been provided in [9].

In this paper, the Finite Element method is integrated with Bond graph for dynamic analysis of the soft material during contact interaction.

## 4 Integration with Bond Graphs

Modeling of the system is done with Bond graph Technique. Bond graph is the pictorial representation of the dynamics of the system. Bond graphs are used for modeling of physical system dynamics in multiple energy domains. A bond graph model is based on the interaction of power between the elements. The bond graph technique was developed in 1959 by Professor Henry Paynter [6, 7]. The method of Bond graphs is an attractive and powerful technique as it offers a unified framework for modeling the mechanism, and, the actuation and control systems due to its capability of handling multi-energy domains [6-8]. Cause-effect relationships are also depicted and help in deriving the system equations in an algorithmic manner. Bond Graphs can be easily extended to nonlinear systems.

Sometimes the graphical representation of large physical systems becomes very complex if made by scalar bond graph. It is convenient and advantageous to have more compact representation. This compact graphical representation is called multi-bond graphs. A multi-bond is represented by a harpoon arrow made of two parallel lines and its bond strength is indicated in between them. In this paper, we depict the scalar bond by a thin harpoon arrow and the multibond with three scalar bonds by a thicker harpoon arrow. The strength 3 is not explicitly depicted on the graph in order to avoid clutter. Wherever the bond strength exceeds three, it is explicitly depicted on the multibond.

$$\frac{e}{f} \quad \frac{\bar{e}}{\bar{f}}$$

Power =  $e \cdot f$       Power =  $\bar{e}^T \bar{f} = \bar{f}^T \bar{e}$

Scalar bond      Vector bond

Fig. 1: Notation for scalar and multibonds

## 5 Example

For checking the validity of the proposed model, an example of soft contact interaction is considered from a robotics perspective.

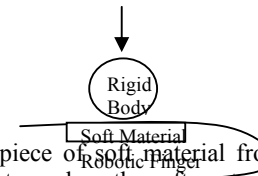


Fig. 2: A piece of soft material from robotic finger is considered to analyze the soft contact interaction.

A small part of soft material silicone rubber of robotic fingertip as shown in Fig. 2 dimensioning length 0.03 meters, thickness 0.005 meters and width 0.01 meters has been considered. In this work a planar case has been considered and it has also been considered that the deformation takes place in two dimensions. The thickness ( $t_c$ ) of the soft material assumed to remain constant. For Finite Element analysis, the whole material has been discretized in constant triangular elements (CST). As shown in Figure 1.3 the material is divided into small 12 equal parts of 0.005 meters length and 0.005 meters width, assuming the thickness as constant equal to 0.005 meters. Thus the material has two layers of 0.005 meters clubbed together. The effect of contact is analyzed on these two layers. Further each part is discretized into two triangular elements. Base of the material is taken as fixed as at the base of soft material of robotic finger, there is a rigid skeleton. The total material is discretized into 24 equal triangular elements. 1, 2 and 3 in triangular element shows the local nodal numbering while the numbering 1 to 24 with coordinates in brackets shows global coordinates in Fig. 3.

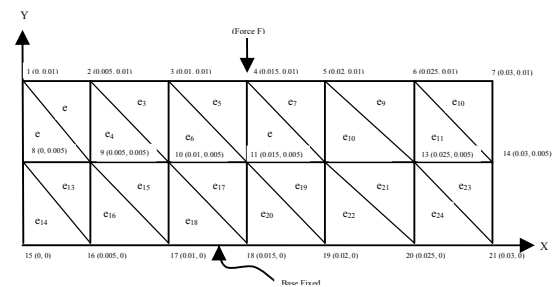
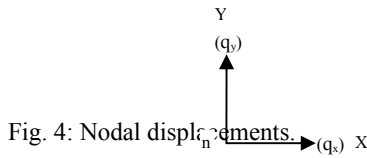


Fig. 3: Discretization of soft material for Finite Element Analysis.

The displacement of a node is shown in Fig. 4. The displacement coordinates of a local node  $j$  are represented as  $q_{2j-1}$  and  $q_{2j}$  in the  $x$  and  $y$  directions respectively.

Nodal Number (n)	Displacement in X-Direction $q_x$ Here $x = 2j-1$	Displacement in Y-Direction $q_y$ Here $y = 2j$
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Initially, the rigid body makes contact on the soft material at one point. This results in the application of a force at the corresponding node.



In this work, the Finite Element method is used for determination of the force-deformation relationship during contact as the deformation occurs when the rigid body makes contact with soft material. The force-deformation relationship is represented using *C-fields* in Bond graphs, and its parameters are obtained from the Finite Element formulation. The element stiffness matrix can be calculated using the potential energy approach [5]. This provides the requisite force-deformation relationship. Damping can also be handled likewise. Inertial distribution is accounted using point masses at nodes. The bond graph of the system is shown in Fig. 5.

System equations are derived algorithmically from the Bond graph model [7]. The MATLAB coding for simulations has been prepared directly from the Bond graph model.

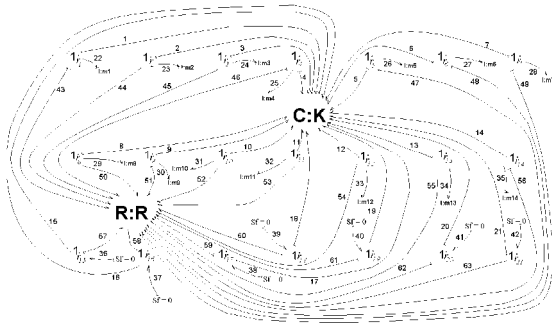


Fig. 5: Bond-Graph for the system of Fig. 3.

## 6 Simulation Results and Discussion

The total simulation time is taken as 10 seconds. Parameters used in simulation are given in Table 1.

Table-1: Material properties

Parameter	Value
Mass	0.01 kg
Length	0.03 m
Height	0.005 m
Thickness	0.005 m
Young's Modulus E	1000 N/m <sup>2</sup>
Poisson's Ratio $\nu$	0.5

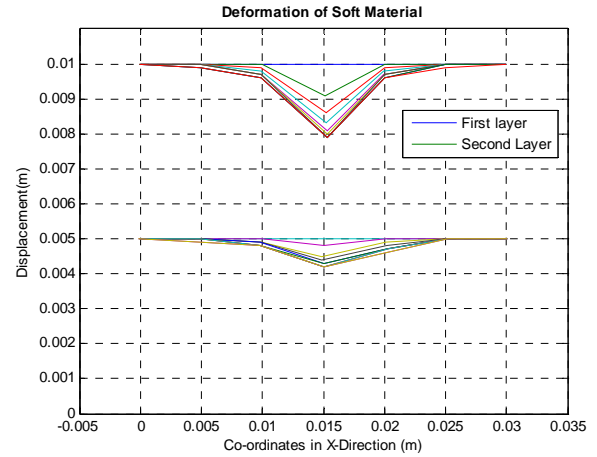


Fig. 6: Deformation of soft material at different intervals of time during simulation.

**Case 1:-** Here a force of 0.01N is applied vertically downwards at  $r_4$  (4<sup>th</sup>) point of the first layer. The damping is considered as  $R = 10 \text{ N}\cdot\text{s}/\text{m}$ . This deformation of both layers at different intervals of time is shown in Figures 2.1.

**Case 2:-** In case of soft contact, the contact force is distributed over an area of contact. Due to the effect of compliance and compatibility, when we apply force at one point, the adjacent points will also deform. Here when a force acts at  $r_4$  (4<sup>th</sup>) point of first layer, it results in deformation of the whole first and second layers. This can be analyzed from the graph shown in Figure 2.1. Compatibility conditions ensure that the two layers do not intersect each other.

**Case 3:-** If the material damping is not considered, oscillations continue. Figures 2.2 and 2.3 show effects of no damping and damping respectively. The graphs show the change in co-ordinates of  $r_4$  (4<sup>th</sup>) point under the action of force of 0.001N and damping is considered to be 10 N·s/m. All other parameters in both the conditions remain the same.

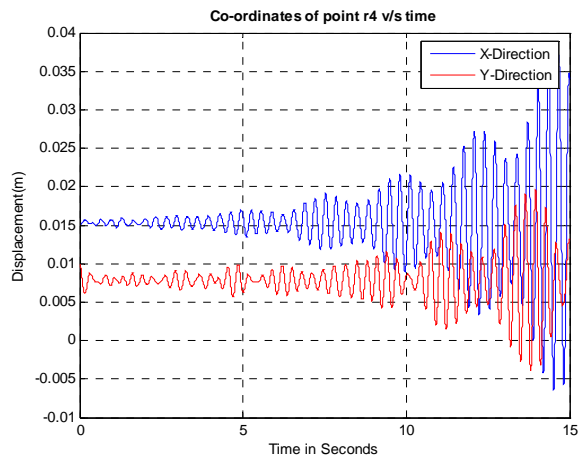


Fig. 7: Oscillations due to absence of damping.

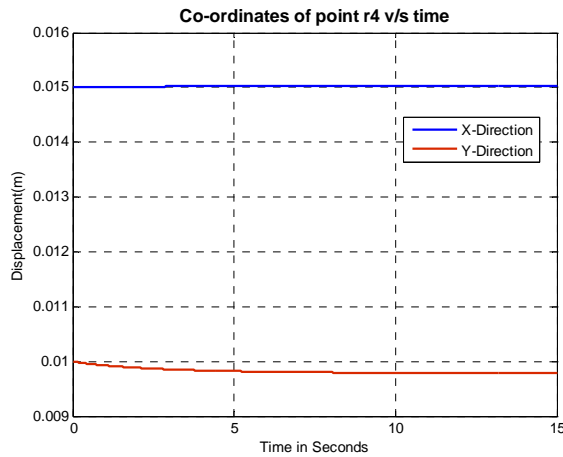


Fig. 8: Displacement coordinates for point  $r_4$  v/s time. Damping is considered.

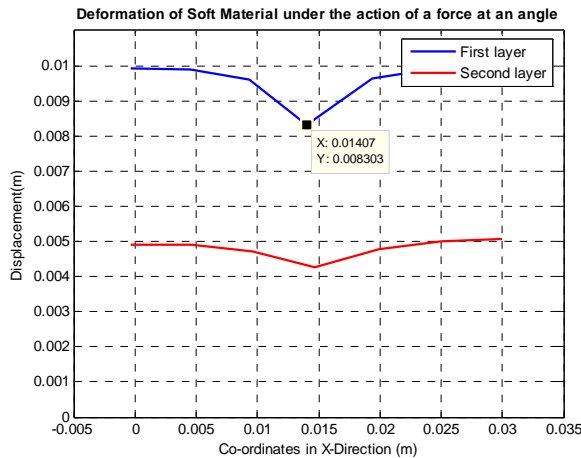


Fig. 9: Deformation of soft material under the action of a force at some angle to the surface.

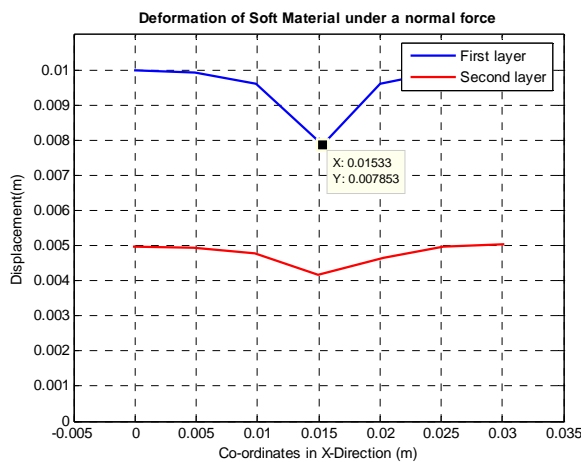


Fig. 10: Deformation of soft material under the action of a normal force.

**Case 4:-** Here the model is tested by applying a force  $F = 0.01 \cdot [-\sin(\pi/6) \ -\cos(\pi/6)]^T$  N inclined at an angle of  $30^\circ$  to the vertical. Fig. 9 and Fig. 10 show the comparison of effects on displacement due to inclined and normal forces respectively.

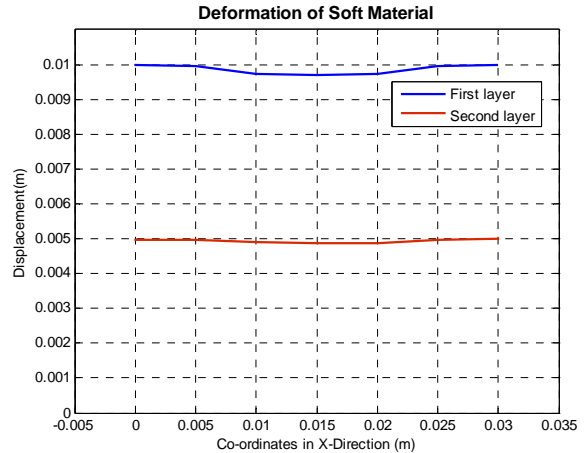


Fig. 11: Behavior of soft material under the action of a number of forces. The deformation takes place over a distributed area.

**Case 5:-** Here the behavior of soft material under the action of a number of forces (each force  $F = 0.001$  N) is being analyzed. The deformation takes place over a distributed area. The net force at the interface can be considered to be distributed over the points on the respective surface area. Corresponding deformation of soft material is shown in Fig. 11.

## Conclusions and Future Scope

Modeling of soft material during contact interaction is carried out in this paper using Bond graphs. An effort has been made to integrate the Bond graph formulation for dynamic modeling and analysis with the advantages of the finite element method. Finite element methods are used for calculating the stiffness matrix for the soft material or the force-deformation relationship during contact, as the deformation occurs when the rigid body interacts with soft material. The Bond graph models are further useful for control system analysis and design.

The models have been tested through simulation. Simulation code has been developed using MATLAB and results are plotted, analyzed and discussed. The graphical results show the behavior of soft material during contact interaction. During deformation of the material, the effect of compliance under compatibility conditions is observed.

Modeling of soft contact interaction is one of the important subjects from robotic perspective, especially in dexterous manipulation because grasping and manipulation are dictated by contact behavior. It has been established that the deformation effect of soft fingertips plays an

important role in the manipulation of an object by the hand. In case of soft contact, the contact force is distributed over an area of contact due to the effect of material compliance. Modeling of this aspect is indeed a challenging task.

The future scope of this work is to develop dynamic models for contact based manipulation. Manipulation, in general, includes both translational and rotational movements. Contact manipulation includes rolling with soft fingertips. The analysis of contact area as it continuously changes during manipulation is an interesting and open problem in robotics at present. The development of forces and moments at the contact interface due to the change in the contact area during manipulation also needs analysis. The models are of special significance for (1) design and analysis for robotics hand development for dexterous manipulation, soft fingertips, etc. (2) the control of robotic manipulation and prosthesis.

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