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Optimum maintenance policy with Markov processes

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Abstract

This paper presents a method to find the optimum maintenance policy for a component. Random failures and failures due to deterioration are considered. Using Markov processes, the state probabilities are calculated and the optimal value of the mean time to preventive maintenance is determined by maximizing the availability of single component with respect to mean time to minimal preventive maintenance. Using the state probabilities, the problem is set up as Markov decision processes and an optimum maintenance policy using the policy iteration algorithm is determined. An example is used to illustrate the method. Maple V and Matlab software have been used to solve the equations.

Keywords: Preventive maintenance; Availability; Deterioration; Markov processes; Markov decision processes; Random failure; Optimal policy

1. Introduction

In principle, improving system reliability and reducing operations and maintenance (O&M) costs are top priorities of electric utilities. In an increasingly competitive power delivery environment, electric utilities are forced to apply more proactive methods of utility asset management. One of the main elements of electric power delivery asset management is the capital budget and O&M of existing facilities. Since in many cases the cost of construction and equipment purchases are fixed, O&M expenditures are the primary candidate of potential savings. As system equipments continue to age and gradually deteriorate, the probability of service interruption due to component failure increases.

Electric utilities are confronted with many challenges in this new era of competition: rising O&M costs, growing demand on systems, maintaining high levels of reliability and power quality, and managing equipment aging. Therefore, the health of equipment is of utmost importance to the industry because revenues are affected by the condition of equipment when demand is high, and when equipment is in working order, substantial revenues can be realized. On the contrary, unhealthy equipment can result in service interruption, customer dissatisfaction, loss of good will, and eventual loss of customers. An effective maintenance strategy is essential in delivering safe and reliable electric power to customers economically.

In general, maintenance is either planned or unplanned. Corrective maintenance is a reactive strategy which is unplanned and is carried out after failure has occurred. The intention is to restore an item to a state that can perform its required function. Planned maintenance strategies are proactive in nature and can be divided into two groups: preventive and predictive. Preventive maintenance (PM) which is sometimes called scheduled, is a maintenance carried out at regular intervals. The purpose of PM is to eliminate the need for radical treatment sometime in the future (which is almost always much more expensive). PM, by its very nature, can be scheduled and controlled for a minimum cost. Clearly, too little maintenance may have very costly consequences but on the other hand, it may not be economical to perform it too frequently [1]. Predictive maintenance (PdM) is a maintenance carried out when it is deemed necessary, based on periodic inspections, diagnostic tests or other means of condition monitoring.

Quantitative analysis of a maintenance scheme is usually based on the deterministic assumption that the consequences of the maintenance actions are non-random. For example, after an overhaul, the future trend of operating cost is known and the random failures of the equipment have no bearing on the maintenance frequency. In reality, however, the equipment failure may require system replacement, and scheduled maintenance activity

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may be either postponed or canceled, thus the maintenance interval may become random in nature. A maintenance policy which is based on the probabilistic principles would not only better reflect the random nature of the equipment operating times, but could also lead to substantial savings in maintenance costs.

The problem of replacement or overhaul of equipment, which deteriorates with usage, is one of the standard applications of Markov processes [2]. Continuous operation, or daily start–stop operation, without failure requires a comprehensive plant preventive maintenance policy and diagnostic system for each equipment and component. In [3], multi-state system models for reliability evaluation have been proposed. Models and appropriate methods for parameter estimation of degradation data have been developed [4]. These models incorporate catastrophic failure as well as degradation failure. Literature related to optimal maintenance policies for repairable components are given in [5]. Such policies which require to make choices among actions (such as "repair", "overhaul", or "do nothing") can be formulated as a Markov decision process [6]. The goal is to find an optimal maintenance policy which maximizes the expected benefits.

2. Markov processes

To date, in the power systems context, continuous parameter Markov chains have been applied most extensively to model power system reliability and maintenance problems. Each equipment is assumed to be repairable. The time to repair depends on the type of failure. Periodically, the component is removed from operation for minimal preventive maintenance. Minimal preventive maintenance is a preventive maintenance activity of limited effort and effect. If deterioration is modeled as occurring in a limited number of discrete steps, then minimal preventive maintenance sets back the process by one step. This improves the component from stage i to stage $i-1$ of deterioration. If the component is in stage one of deterioration, it remains in that stage on completion of minimal preventative maintenance. It is assumed that the duration of each stage of deterioration as well as times for repairing a failed equipment are exponentially distributed.

In this model [7], both random failure and failure due to deterioration can occur. Deterioration of the equipment is modeled as occurring in k discrete steps. The time spent in each stage of deterioration are exponentially distributed with an identical mean of $1/k\lambda_1$. Therefore, the time to deterioration failure is represented by an Erlangian distribution. An application of device-of-stages technique to electric power distribution systems have been proposed [8]. Two types of stage configurations are described and models parameters are estimated. Maintenance is assumed to improve the component's condition, but not to as-good-as-new state. Maintenance is modeled as a Poisson process with a parameter λ_m. Maintenance times are exponentially distributed with a mean of $1/\mu_m$. Repair is an activity that returns a failed component to working condition (e.g. overhaul). Assumption is made that repairs after failure due to deterioration will always produce as-good-as-new conditions with a mean of $1/\mu_1$, whereas repairs of random failures may or may not achieve this. Note that this assumption could be easily relaxed. State F_0 represents the random failures that can occur at any time but not while maintenance is performed on the component. Repair following random failure is completed at a rate of μ_0, and then return to state D_1. The state space diagram of the maintenance model is shown in Fig. 1.

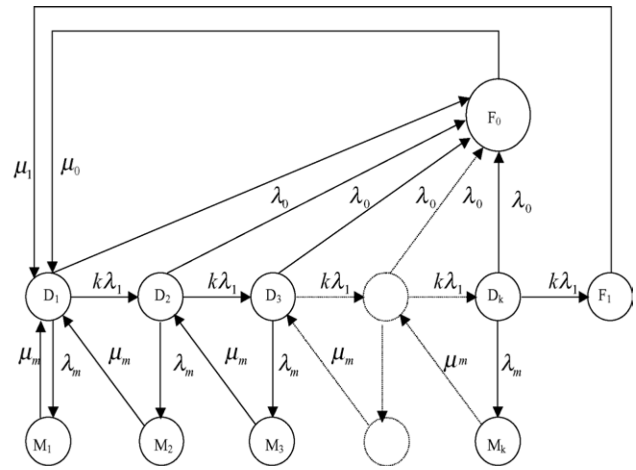

Fig. 1. Maintenance model with deterioration and full repair after random failure.

The state transition matrix for the model has the form

$$A=\begin{bmatrix}-\Sigma_1 & k\lambda_1 & 0 & \lambda_m & 0 & 0 & \lambda_0 & 0\\ 0 & -\Sigma_2 & k\lambda_1 & 0 & \lambda_m & 0 & \lambda_0 & 0\\ 0 & 0 & -\Sigma_3 & 0 & 0 & \lambda_m & \lambda_0 & k\lambda_m\\ \mu_m & 0 & 0 & -\mu_m & 0 & 0 & 0 & 0\\ \mu_m & 0 & 0 & 0 & -\mu_m & 0 & 0 & 0\\ 0 & \mu_m & 0 & 0 & 0 & -\mu_m & 0 & 0\\ \mu_0 & 0 & 0 & 0 & 0 & 0 & -\mu_0 & 0\\ \mu_1 & 0 & 0 & 0 & 0 & 0 & 0 & -\mu_1\end{bmatrix} \quad (1)$$

where Σ_i = the negative of the sum of all the remaining elements in row i. The set of steady state equations can be solved by [1] as:

$$PA^T = 0 \quad (2)$$

$$\sum_{i=1}^{8} P_i = 1.0 \quad (3)$$

where $P = [P1\ P2\ P3\ P4\ P5\ P6\ P7\ P8]$. P_i = steady state probability that the component is in state i and not undergoing minimal maintenance, $i = 1, 2, \ldots, k, F_0, F_1$.

The solution to these equations is obtained by using a recursive approach [9] with Maple V and Matlab software. The probability $A(\lambda_m)$ (availability) that the component is in service is given by

$$A(\lambda_m) = P_1 + P_2 + P_3 \quad (4)$$

where P_1, P_2, and P_3 are the probabilities that the component is in service.

Once the state probabilities of the model are calculated, the optimal value of the mean time to preventive maintenance (λ_m)

is calculated by taking the derivative of $A(\lambda_m)$ with respect to λ_m and then equating it to zero.

3. Markov decision processes

Using the state probabilities found in the previous section, the optimal stationary policy of the model can be determined by setting up the problem as a Markov decision process (MDP). In general an MDP is a 4-tuple (S, K, R, T) where S = a set of system states (assumed to be finite); K = a set of available actions; R = a set of state- and action-dependent immediate rewards or costs; T = a set of state- and action-dependent transition probabilities.

A decision rule, d_n, is a scheme for assigning actions in K to states in S. A policy is a sequence of decision rules for a specified time horizon which can be finite or infinite. A "stationary policy" is a policy over a specified time horizon where the decision rules for each period are identical. If $p(j|i, k)$ represents the transition probability of state i to state j as a result of action k, and $r(i, k, j)$ represent returns that result from a transition from state i to j as a result of action k, then expected return from taking action k while in state i is given by

$$r(i, k) = \sum_{j \in S} p(j|i, k) r(i, k, j) \tag{5}$$

In order to find the optimal stationary policy for the model, the unichain policy iteration algorithm is used. A system with a single set of recurrent states and possibly some transient states is said to be unichain. In order to be able to use the unichain policy iteration algorithm, the MDP model should be checked to determine whether it is unichain. The result of this verification indicates that indeed this model is unichain. The unichain policy iteration algorithm involves the following steps [6]:

Step 1. Set $n = 0$ and select an arbitrary decision rule $d_n \in D$ where n = iteration count; D = set of decision rules.

Step 2. (Policy evaluation) obtain a scalar g_n and an $h_n \in V$ by solving:

$$rd_n - ge + (P_{dn} - I)h = 0 \tag{6}$$

where $v_i(n) = ng_i + h_i$ = expected total earnings in the next n transitions if the system is now in state i; P_d = Markov reward process with transition matrix p_d; I = identity matrix.

Step 3. (Policy improvement) choose $d_{n+1} = d_n$ to satisfy:

$$d_{n+1} \in arg\ \max \left\{ rd + \sum_{j=1}^{N} P_d h_n \right\} \tag{7}$$

where N is the number of states in the system, setting $d_{n+1} = d_n$ if possible.

Step 4. If $d_{n+1} = d_n$ for all states, stop and set $d^* = d_n$. Otherwise increment n by 1 and return to step 2.

4. Example

The following example illustrates the implementation of the model. Consider that the average random failure of a unit occurs once in 500 days and that the repair lasts 7 days. If the unit is not maintained, it would fail from deterioration every 1000 days (on average). Also, the mean repair time is 14 days and the minimal maintenance would take an average of half a day. Assume that all times are exponentially distributed and the number of stages of deterioration of the unit is 3. In this case, $k = 3$, $\lambda_0^{-1} = 500$ days, $\lambda_1^{-1} = 1000$ days, $\mu_0^{-1} = 7$ days, $\mu_1^{-1} = 14$ days, and $\mu_m^{-1} = 0.5$ days. By maximizing $A(\lambda_m)$ with respect to λ_m, the optimal value of the mean time to minimal maintenance can be determined [9]. Fig. 2 illustrates the effect of the mean time to minimal preventive maintenance on the availability of the component. Table 1 provides the state probabilities of the component (in percent). The availability of the model is calculated as 0.9808.

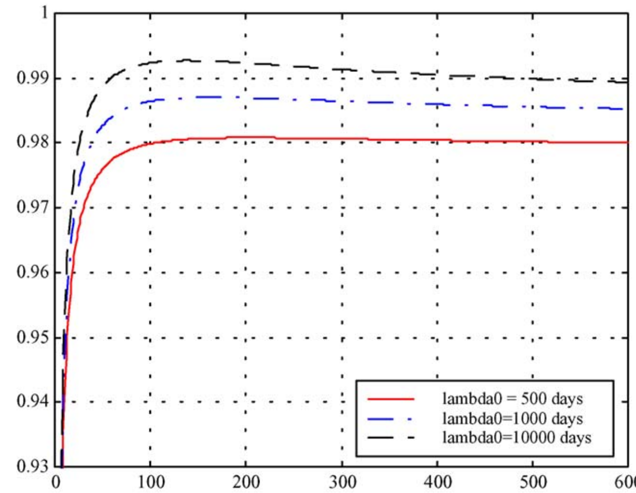

Fig. 2. The effect of mean time to minimal maintenance on availability.

The optimal value of the mean time to minimal preventive maintenance for the model is 203 days for a mean time to Poisson failure of 500 days. The availability decreases slowly as the mean time to minimal preventive maintenance exceeds its optimal value; the decrease is faster if the mean time to preventive maintenance is less than the optimal value. As the value of λ_0 increases from 500, to 1000, to 10,000, the optimal value decreases from 203 to 163 days and then to 136 days. The availability of the component increases as λ_o increases. This is because the value of λ_m depends on the values of λ_o.

In order to find the optimal policy, the best action in each state must be selected. There are two actions to be taken. Action I refers to "do nothing" while action II refers to "do maintenance". There are $2 \times 2 \times 2 \times 1 \times 1 \times 1 \times 1 \times 1 = 8$ stationary policies to choose from. Actions I and II can be chosen from states D_1,

Table 1
State probabilities

P_1	67.03
P_2	23.84
P_3	7.21
P_{M1}	0.165
P_{M2}	0.059
P_{M3}	0.018
P_{F1}	1.373
P_{F0}	0.303

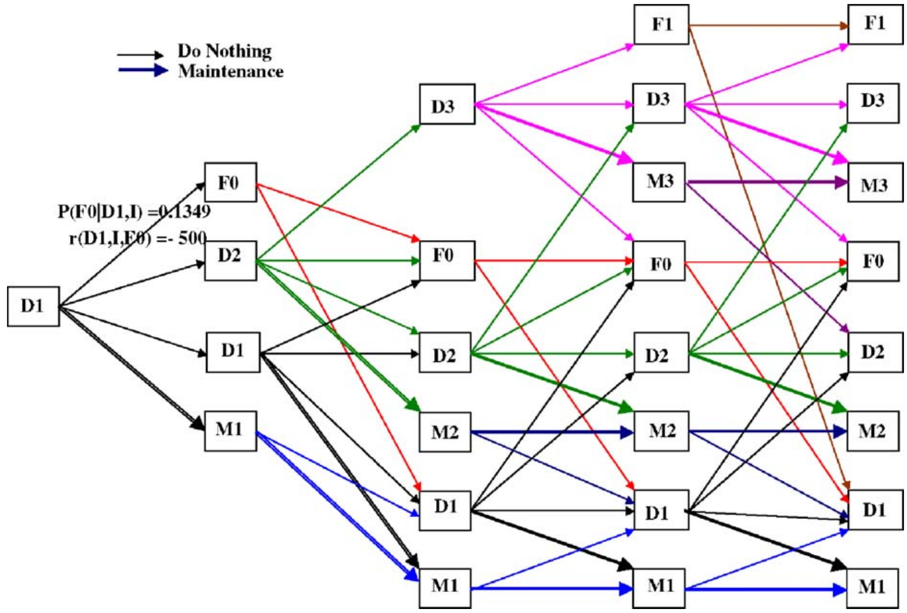

Fig. 3. State diagram.

Table 2
Reward and transition probabilities

Reward	Transition probabilities	Reward	Transition probabilities
$r(D_1, I, F_0) = -500$	$P(F_0\|D_1, I) = 0.1349$	$r(M_1, I, D_1) = 1000$	$P(D_1\|M_1, I) = 0.9983$
$r(D_1, I, D_2) = 900$	$P(D_2\|D_1, I) = 0.1978$	$r(M_1, I, M_1) = -100$	$P(M_1\|M_1, I) = 0.0017$
$r(D_1, I, D_1) = 1000$	$P(D_1\|D_1, I) = 0.6703$	$r(M_2, I, D_1) = 1000$	$P(D_1\|M_2, I) = 0.9994$
$r(D_1, II, M_1) = -100$	$P(M_1\|D_1, II) = 1.0$	$r(M_2, I, M_2) = -100$	$P(M_2\|M_2, I) = 0.0006$
$r(D_2, I, D_3) = 800$	$P(D_3\|D_2, I) = 0.4570$	$r(M_3, I, D_2) = 900$	$P(D_2\|M_3, I) = 0.9998$
$r(D_2, I, F_0) = -500$	$P(F_0\|D_2, I) = 0.3046$	$r(M_3, I, M_3) = -100$	$P(M_3\|M_3, I) = 0.0002$
$r(D_2, I, D_2) = 900$	$P(D_2\|D_2, I) = 0.2384$	$r(F_0, I, F_0) = -500$	$P(F_0\|F_0, I) = 0.0030$
$r(D_2, II, M_2) = -100$	$P(M_2\|D_2, II) = 1.0$	$r(F_0, I, D_1) = 1000$	$P(D_1\|F_0, I) = 0.9970$
$r(D_3, I, F_1) = -1000$	$P(F_1\|D_3, I) = 0.5567$	$r(F_1, I, F_1) = -1000$	$P(F_1\|F_1, I) = 0.0137$
$r(D_3, I, D_3) = 800$	$P(D_3\|D_3, I) = 0.0721$	$r(F_1, I, D_1) = 1000$	$P(D_1\|F_1, I) = 0.9863$
$r(D_3, I, F_0) = -500$	$P(F_0\|D_3, I) = 0.3712$		
$r(D_3, II, M_3) = -100$	$P(M_3\|D_3, II) = 1.0$		

D_2, and D_3 whereas action I is available in states $M1$, $M2$, $M3$, $F0$, and $F1$.

The reward for the component to stay in states D_1, D_2, and D_3 is 1000, 900, and 800, respectively. The reward decreases from D_1 to D_3 because the component is deteriorating, and it may not work as good as new. The reward for the component to stay in states M_1, M_2, and M_3 is -100, but for states F_0 and F_1 the reward is -500 and -1000, respectively. The reward in these states is based on the duration of the unavailability; the longer time the component stays in such states, the greater the loss. The transition probabilities are based on the values obtained from Markov model.

Action sets are: $K_{D1} = \{\text{I, II}\}$, $K_{D2} = \{\text{I, II}\}$, $K_{D3} = \{\text{I, II}\}$ $K_{M1} = \{\text{I}\}$, $K_{M2} = \{\text{I}\}$, $K_{M3} = \{\text{I}\}$ $K_{F0} = \{\text{I}\}$, $K_{F1} = \{\text{I}\}$.

Fig. 3 shows the state diagram of the component for a case where the number of stages in the process is 4. The reward and transition probabilities for each transition with possible actions are given in Table 2.

Table 3 provides the summary of the results by using the policy iteration. The optimal policy of the model is: "do nothing" in states D_1, M_1, M_2, M_3, F_o, and F_1; "do maintenance" in states D_2 and D_3. The average reward or gain of this policy is 714. The optimal policy is expected, because the probabilities of the component transiting to failure state is getting higher in state D_2 and D_3, also the probability of deterioration failure which is in state D_3 is very high.

Table 3
Optimal policy

*D*1	*D*2	*D*3	Gain	*D*1	*D*2	*D*3
I	I	I	659	I	II	I
I	II	I	714	I	II	II
I	II	II	714	I	II	II

5. Conclusion

As the mean time to Poisson failure increases, the need for minimal preventive maintenance decreases. If minimal repair is carried out after random failures, the effect of random failure is eliminated and preventive maintenance is fully effective. As the mean time to deterioration failure increases, the availability also increases, and the need for minimal preventive maintenance will increase. Also, as the mean repair time increases, availability decreases. One of the best ways to reduce the mean time to minimal maintenance is to reduce the mean time of repairing a deterioration failure. In other words, for minimal PM to be effective, the deterioration failures must dominate the failure mix because the Poisson failure cannot be prevented through PM. The optimum maintenance policy will maximize the benefits. Assuming that the component is in state D_2, we may perform the maintenance more frequently than in state D_1, which may reduce the rate that the component will transit to state D_3.

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