Photonic Crystal at Millimeter Waves Applications

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Abstract— The photonic band gap (PBG) crystals have been used as a perfectly reflecting substrate for many millimeter wave applications. In his work the fin line directional coupler with PBG substrate was analyzed using the TTL — Transverse Transmission Line — method. Comparing with the other full wave methods the TTL is efficient, making possible a significant algebraic simplification of the equations involved in the process. In order to analyze the structure the coupling were determined. Numerical results obtained for this finline coupler are presented.

DOI: 10.2529/PIERS060901105337

1. INTRODUCTION

Photonic band gap crystals have emerged as a new class of periodic dielectric structures where propagation of electromagnetic waves is forbidden for all frequencies in the photonic band gap [1]. This material has a periodic arrangement of cylinders immersed in air with diameters and spacing of less than a wave length [2–4]. This substrate can improve the band width and eliminate the propagation of undesirable modes. Many integrated circuits for millimeter wave applications can be made using fin line techniques. This includes, beyond the fin line circuit, circuits inserted in metal and other standards circuits, mounted in the wave guides E-plane [5].

This letter demonstrates an application of the 2D layer-by-layer PBG crystal; an efficient unilateral fin line directional coupler. This type of coupler can be realized by the use of the natural coupling between the 2 slots symmetrically localized. The analysis is made using the TTL method and the coupling definitions. The Fig. 1 shows a project of the device.

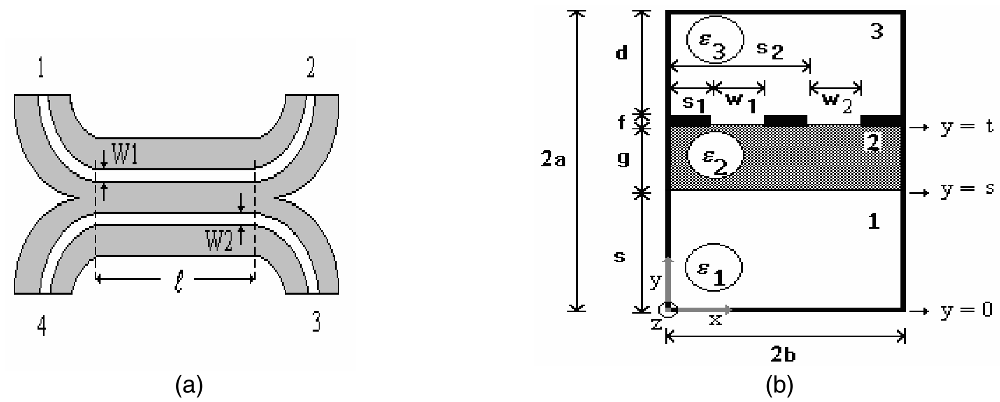


Figure 1: (a) Fin line coupler superior and intern view, and (b) transversal section of unilateral fin line coupler.

The coupled unilateral fin lines consist of a rectangular wave guide with three dielectric regions inside being the second region a substrate placed in the center of the wave guide and having three conductors fins on top of the substrate and the other two regions being air.

2. THEORY

Starting for the rotational Maxwell equations the electromagnetic fields are developed. The "x" and "z" components the final fields equations in the Fourier Transform Domain for the structures

ith regions are obtained:

$$\widetilde{E}_{xi} = \frac{1}{\gamma_i^2 + k_i^2}\left[-j\alpha_n\frac{\partial}{\partial y}\widetilde{E}_{yi} - j\omega\mu\Gamma\widetilde{H}_{yi}\right] \quad (1)$$

$$\widetilde{E}_{zi} = \frac{1}{\gamma_i^2 + k_i^2}\left[-\Gamma\frac{\partial}{\partial y}\widetilde{E}_{yi} - \omega\mu\alpha_n\widetilde{H}_{yi}\right] \quad (2)$$

$$\widetilde{H}_{xi} = \frac{1}{\gamma_i^2 + k_i^2}\left[-j\alpha_n\frac{\partial}{\partial y}\widetilde{H}_{yi} + j\omega\varepsilon\Gamma\widetilde{E}_{yi}\right] \quad (3)$$

$$\widetilde{H}_{zi} = \frac{1}{\gamma_i^2 + k_i^2}\left[-\Gamma\frac{\partial}{\partial y}\widetilde{H}_{yi} + \omega\varepsilon\alpha_n\widetilde{E}_{yi}\right] \quad (4)$$

were: , is the propagation constant in "y" direction; α_n is the spectral variable in "x" direction.
$k_i^2 = \omega^2\mu\varepsilon = k_0^2\varepsilon_{ri}^*$, is the wave number of ith term of dielectric region;
$\varepsilon_n^* = \varepsilon_{ri} - j\frac{\sigma_i}{\omega\varepsilon_0}$, is the relative dielectric constant of the material with losses;
$\varepsilon_i = \varepsilon_{ri}^* \cdot \varepsilon_0$, is the dielectric constant of the ith region;
$\Gamma = \alpha + j\beta$, is the complex propagation constant; and,
$\omega = \omega + j\omega_i$, is the complex angular frequency.

The solutions of the fields equations for the three regions in study are given by example: For region 2:

$$\widetilde{E}_{y2} = A_{2e} \cdot \mathrm{senh}\gamma 2y + B_{2e} \cdot \cosh\gamma 2y \quad (5)$$

$$\widetilde{H}_{y2} = A_{2h} \cdot \mathrm{senh}\gamma 2y + B_{2h} \cdot \cosh\gamma 2y \quad (6)$$

For the determination of the unknown constants described above the boundary conditions are applied.

For the propagation constant determination, the $\widetilde{E}_{xt}$ and $\widetilde{E}_{zt}$ (still unknown) components must be isolated in the magnetic field equations in the slots regions. Theses equations are shows in sequence:

$$\widetilde{H}_{x2} - \widetilde{H}_{x3} = \widetilde{J}_{zt} \quad (7)$$

$$\widetilde{H}_{z2} - \widetilde{H}_{z3} = -\widetilde{J}_{xt} \quad (8)$$

were $\widetilde{J}_{xt}$ and $\widetilde{J}_{zt}$ are the electric current densities in the fins.

Substituting the above equations in the magnetic fields equations (presented in the last section) and isolating the electric fields terms the admittance functions are obtained.

$$Y_{xx}\widetilde{E}_{xt} + Y_{xz}\widetilde{E}_{zt} = \widetilde{J}_{xt} \quad (9)$$

$$Y_{zx}\widetilde{E}_{xt} + Y_{zz}\widetilde{E}_{zt} = \widetilde{J}_{zt} \quad (10)$$

The $\widetilde{E}_{xt}$ and $\widetilde{E}_{zt}$ fields are expanded in terms of base functions and have a contribution of the two slots,

$$\widetilde{E}_{xt} = \sum_{i=1}^{n} a_{xi} \cdot \widetilde{f}_{xi} \quad (11)$$

$$\widetilde{E}_{zt} = \sum_{j=1}^{m} a_{zj} \cdot \widetilde{f}_{zj} \quad (12)$$

The base functions utilized in the Fourier Transform Domain are obtained; for the odd mode:

$$\widetilde{f}_{ixm}(\alpha_n) = \mathrm{Re}\left\{\frac{\pi wi}{4}e^{j\alpha_n(xi+wi/2)}\left[e^{jm\pi/2}J_0\left[\frac{1}{2}(\alpha_n wi + m\pi)\right] + e^{-jm\pi/2}J_0\left[\frac{1}{2}(\alpha_n wi - m\pi)\right]\right]\right\} \quad (13)$$

$$\widetilde{f}_{izm}(\alpha_n) = \mathrm{Im}\left\{-\frac{j\pi wi}{4}e^{j\alpha_n(xi+wi/2)}\left[e^{jm\pi/2}J_0\left[\frac{1}{2}(\alpha_n wi + m\pi)\right] - e^{-jm\pi/2}J_0\left[\frac{1}{2}(\alpha_n wi - m\pi)\right]\right]\right\} \quad (14)$$

And for the even mode:

$$\widetilde{f}_{ixm}(\alpha_n)=\text{Im}\left\{\frac{\pi wi}{4}e^{j\alpha_n(xi+wi/2)}\left[e^{jm\pi/2}J_0\left[\frac{1}{2}(\alpha_n wi+m\pi)\right]+e^{-jm\pi/2}J_0\left[\frac{1}{2}(\alpha_n wi-m\pi)\right]\right]\right\} \quad (15)$$

$$\widetilde{f}_{izm}(\alpha_n)=\text{Rm}\left\{-\frac{j\pi wi}{4}e^{j\alpha_n(xi+wi/2)}\left[e^{jm\pi/2}J_0\left[\frac{1}{2}(\alpha_n wi+m\pi)\right]-e^{-jm\pi/2}J_0\left[\frac{1}{2}(\alpha_n wi-m\pi)\right]\right]\right\} \quad (16)$$

were J_0 is the first species and zero order Bessel function; x_i e w_i are the dimensional terms presented in Fig. 1(b) with $I = 1, 2$ for first and second slots.

In the sequence the Gallerkin method, particular case of Moment method, is used and, a new matrix homogeneous matrix with two variables is obtained.

$$\begin{bmatrix} K_{xx}^{11} & K_{xx}^{12} & K_{xz}^{11} & K_{xz}^{12} \\ K_{xx}^{21} & K_{xx}^{22} & K_{xz}^{21} & K_{xz}^{22} \\ K_{zx}^{11} & K_{zx}^{12} & K_{zz}^{11} & K_{zz}^{12} \\ K_{zx}^{21} & K_{zx}^{22} & K_{zz}^{21} & K_{zz}^{22} \end{bmatrix} \begin{bmatrix} a_{1x} \\ a_{2x} \\ a_{1z} \\ a_{2z} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (17)$$

were for example:

$$K_{xx}^{1j} = \sum_{-\infty}^{\infty} Y_{xx}\widetilde{f}_{jx} \cdot \widetilde{f}_{1x}^{*} \quad (18)$$

$$K_{xz}^{1j} = \sum_{-\infty}^{\infty} Y_{xz}\widetilde{f}_{jz} \cdot \widetilde{f}_{1x}^{*} \quad (19)$$

The determinant of (17) is represented by a transcendent equation witch the roots are the attenuation constant "α" and the phase constant "β", and the complex propagation constant $\Gamma = \alpha + j\beta$, is obtained.

Finally effective dielectric constant is determined:

$$\varepsilon_{ef} = \left(\frac{\beta}{k_0}\right)^2 \quad (20)$$

3. DIRECTIONAL COUPLER

In the directional coupler when the signal arrives in port 1, port 3 will be coupled and port 4 isolated. The even and odd modes propagates with different velocities and coupling is periodical along the fin length.

The length required for total power transference from port 1 to port 3 is [6].

$$L = \pi/(\beta_{\text{even}} - \beta_{\text{odd}}) \quad (21)$$

were β_{even} and β_{odd} are the phase constants for the even and odd modes respectively, and are calculated with the TTL method. The coupling amplitude coefficient between ports 1 and 3 is,

$$|S_{13}| = \text{sen}(\pi/2 \cdot l/L) \quad (22)$$

and the amplitude coefficient between ports 1 and 2 is,

$$|S_{12}| = \cos(\pi/2 \cdot l/L) \quad (23)$$

The length l is calculated with Eq. (48). The expression for the coupling is defined as [6]:

$$C_3 = 20.\log\left(\frac{1}{|s_{13}|}\right) \quad (24)$$

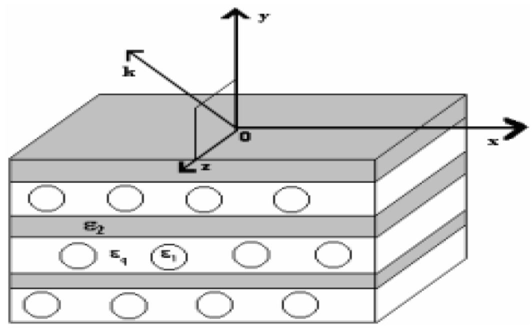


Figure 2: Homogenized bidimensional crystal.

4. PBG STRUCTURE

For a non-homogeneous structure submitted, the incident sign goes at the process of multiple spread. A solution can be obtained through a numerical process called homogenization [5]. The process is based in the theory related to the diffraction of an incident electromagnetic plane wave imposed by the presence of a air immerged cylinders in a homogeneous material.

In the Cartesian coordinates system of axes (O, x, y, z), are shown in the Fig. 2. A cylinder is considered with relative permittivity ε_1, with a traverse section in the plane xy, embedded in a medium of permittivity ε_2. For this process the two-dimensional structure is sliced in layers whose thickness is equal at the cylinder diameter. In each slice is realized the homogenization process.

According to homogenization theory the effective permittivity depends on the polarization. For the s and p polarization, respectively, we have:

$$\varepsilon_{eq} = \beta(\varepsilon_1 - \varepsilon_2) + \varepsilon_2 \tag{25}$$

$$\frac{1}{\varepsilon_{eq}} = \frac{1}{\varepsilon_1}\left\{1 - \frac{3\beta}{A_1 + \beta - A_2\beta^{10/3} + O(\beta^{14/3})}\right\}, \tag{26}$$

where:

$$A_1 = \frac{2/\varepsilon_1 + 1/\varepsilon_2}{1/\varepsilon_1 - 1/\varepsilon_2} \tag{27}$$

$$A_2 = \frac{\alpha(1/\varepsilon_1 - 1/\varepsilon_2)}{4/3\varepsilon_1 + 1\varepsilon_2} \tag{28}$$

And β is defined as the ratio between the area of the cylinders and the area of the cells, α is an independent parameter whose value s equal to 0.523. The A1 and A2 variables in (27) and (28) were included only for simplify (26) equation.

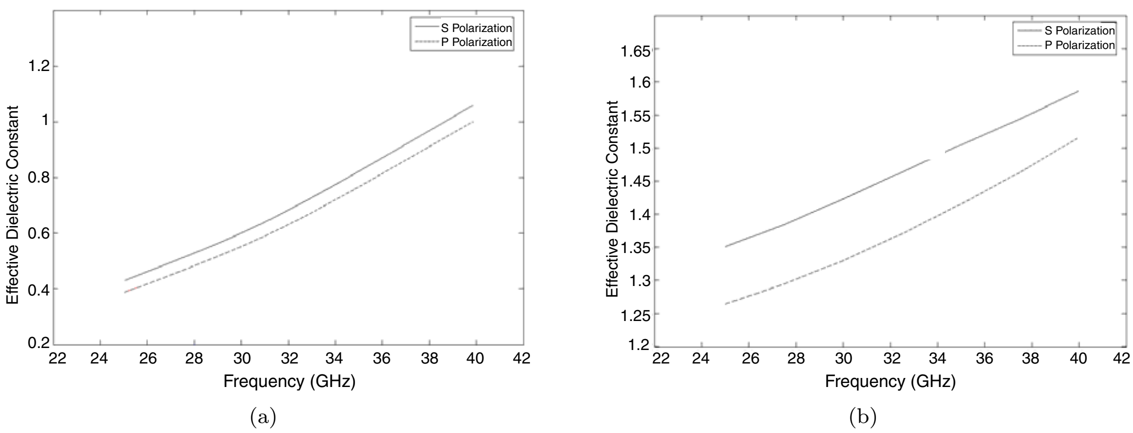


Figure 3: Effective dielectric constant as a function of the frequency for a unilateral directional coupler with 2D PBG substrate, in a WR-28millimeter wave guide, (a) for the even mode (b) for the odd mode.

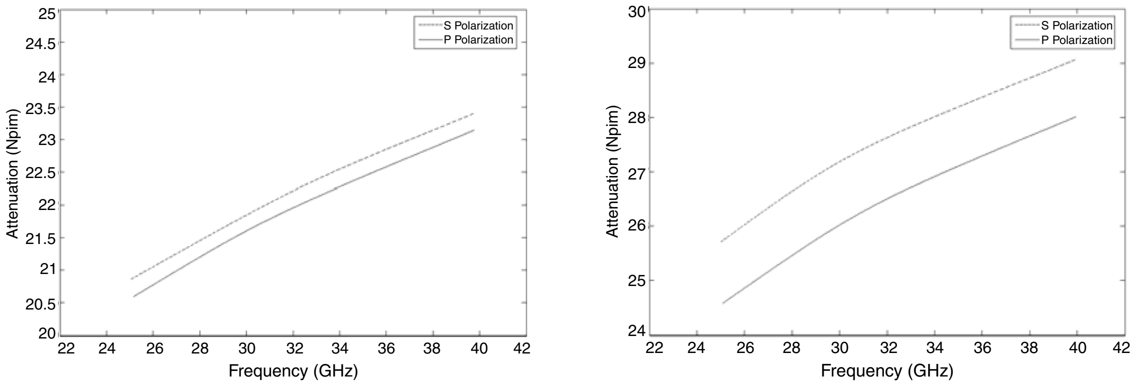
5. NUMERICAL RESULTS

For the numerical results determination a computational program in Fortran Power Station was developed according to the theory, using a Pentium IV, 3.4 GHz. The recourses present by the program include the determination of the attenuation, phase and effective dielectric constants for the even and odd modes.

The results were obtained for a unilateral directional coupler with 2D PBG substrate in WR-28 millimeter wave guide with dimensions $g = 0.254\,\text{mm}$, $s = 3.302\,\text{mm}$ (region 1 thickness1), $s_1 = 1.078\,\text{mm}$, $s_2 = 2.278\,\text{mm}$, $w_1 = w_2 = 0.2\,\text{mm}$, $\varepsilon_{r2} = 8.7209$ for p polarization and $\varepsilon_{r2} = 10.233$ for s polarization, $s' = 0.5\,\text{mm}$ (half the distance between the slots), $2a = 7.112\,\text{mm}$ and $2b = 3.556\,\text{mm}$.

Figure 3 shows the effective dielectric constant as a function of the frequency Fig. 3(a) for the even mode and Fig. 3(b) for the odd mode.

The Fig. 4 shows the attenuation constant as a function of the frequency (a) for the even mode and (b) for the odd mode.



(a) (b)

Figure 4: Attenuation constant as a function of the frequency for a unilateral fin line coupler with PBG substrate in a WR-28 millimeter wave guide; (a) for the even mode; (b) for the odd mode.

6. CONCLUSIONS

The full wave transverse transmission line (TTL) method was used to the characterization of the unilateral fin line directional coupler, considering a 2D Photonic Band Gap (PBG) substrate in the millimeter wave bands. The full wave TTL method was used to the electromagnetic fields determination. Numerical results for the attenuation, effective dielectric constant and coupling were presented.

ACKNOWLEDGMENT

This work was partially supported by CNPQ.

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