

# Josephson Voltage Standard as a Current Reference and Practical Method for Stabilization of High Current

Kyu-Tae Kim, *Member, IEEE*, Mun-Seog Kim, Myungsoo Kim, and Jürgen Niemeyer

**Abstract**—A stable reference source with a high current capability is useful for many metrological applications. Unlike the conventional Josephson voltage standard, one can couple the programmable Josephson voltage standard (PJVS) to a current source below milliamp range to stabilize the current. For higher current ranges, we tested a feedback method which allowed us to stabilize the 0.1-A output current of a commercial current source to within a standard deviation of  $2 \times 10^{-8}$  A/A.

**Index Terms**—Current reference source, current stabilization, programmable Josephson voltage standard (PJVS), reversible thermal electromotive force (EMF), voltage standard.

## I. INTRODUCTION

A programmable Josephson voltage standard (PJVS) can generate a large and well-defined Shapiro step [1]. Unlike the conventional Josephson voltage standard, the output voltage ( $V_J$ ) of a PJVS can definitely be controlled by the selection of bias lines to the binary divided sections of the PJVS Josephson junction series array.

$$V_J = \frac{\sum_{k=1}^n b_k 2^{k-1} f}{K_J} \quad (1)$$

where  $b_k$  is the  $k$ th digit's binary number 1 or 0, depending on whether the bias current is flowing or not through the corresponding section containing  $2^{k-1}$  Josephson junctions,  $f$  is the driving microwave frequency, and  $K_J = 2e/h$  is the Josephson constant. Fig. 1 shows the typical current–voltage characteristic of a PJVS array of  $2^{13}$  superconductor–insulator–normal metal–insulator–superconductor (SINIS) junctions fabricated at Physikalisch-Technische Bundesanstalt (PTB). The current width of the Shapiro step is about 1 mA with 65-GHz irradiation. When adjusting the bias current to the optimum, i.e., the middle of the Shapiro step corresponding in this case to about 3.6 mA, one can obtain maximum stability of the output voltage  $V_J$  against external noise. This stability and easy controllability of any desired voltage allow various dc and low-frequency (LF) measurements, such as the precise determination of the resistance ratio [2] and the LF and fast reversed dc waveform synthesis for the evaluation of the ac-dc transfer standards

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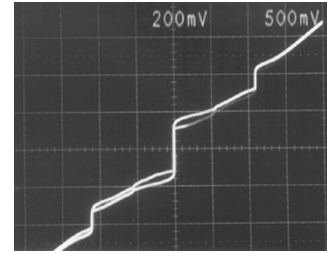


Fig. 1. Typical  $I$ - $V$  characteristic of the PJVS array with  $2^{13}$  SINIS junctions irradiated with microwave of 65 GHz. X: 500 mV/div, Y: 2 mA/div.

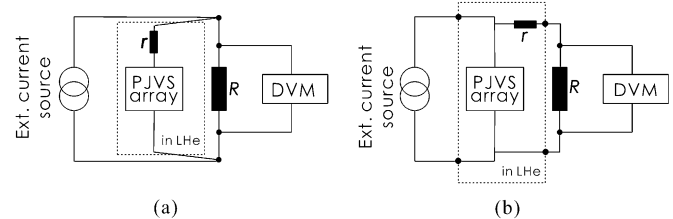


Fig. 2. Two possible circuit configurations for direct parallel connection of the PJVS to a standard resistor of four-terminal type.

[3]–[6]. However, so far the PJVS has only been used for zero or small current applications. Here, we would suggest that it could also be used for the stabilization of current sources. We will describe the principle of the proposed stabilization method and the result of an experimental demonstration. This will be followed by a discussion and finally by the conclusion.

## II. PRINCIPLE OF THE CURRENT STABILIZATION

### A. Direct Use of the PJVS Output

Fig. 2 shows two possible circuit configurations for the direct parallel connection of the PJVS array to a standard resistor of the four-terminal type. In Fig. 2(a), both the external current source output and the array output are connected to the standard resistor input, while in Fig. 2(b), the external current is first directed to the array input and then, through the array output leads, to the standard resistor input. In Fig. 2(a), if the total series resistance of the cryoprobe leads for the array bias is  $r$ , the voltage across the standard resistor  $R$  will be  $V_J + I_0 r$  for the optimum bias  $I_0$ . The  $I_0 r$  term makes an undesired additional voltage drop, therefore,  $r$  should be as small as possible to achieve the desired stability. Now, because the Shapiro step of the PJVS array has a zero differential resistance, any fluctuation  $\Delta I_b$  in the external current will be divided by the ratio of  $r$  to  $R$ , leading to a voltage drop,  $(\Delta I_b) R r / (r + R)$ . Therefore, the relative change in the

current  $I_R$  through the standard resistor due to the change in the external current  $I_b$  will be given by the following equation:

$$\frac{\Delta I_R}{I_R} = \frac{\Delta V_R}{V_R} = \frac{\frac{\Delta I_b R r}{(R+r)}}{V_J + I_0 r} = \frac{\frac{\Delta I_b r}{(R+r)}}{(I_b - I_0)}. \quad (2)$$

The current stabilization depends on the relative values of  $r$ ,  $R$  and  $V_J/I_0$ . Typical values of  $r$  and  $V_J/I_0$  for our experiment were estimated to be  $\sim 1 \Omega$  and  $1 \text{ V}/3.6 \text{ mA} \sim 0.3 \text{ k}\Omega$ , respectively. In case  $R$  is sufficiently smaller than  $r + V_J/I_0$  and larger than  $r$ , most of the external current will be directed into  $R$  and (2) will then be  $\Delta I_R/I_R \sim (\Delta I_b/I_b)/(\eta + 1)$ , which means that the current stability is improved with increasing  $\eta \equiv R/r$ . In case  $R$  is sufficiently larger than  $r + V_J/I_0$ , most of the external current will be directed into  $r$  and  $I_b$  will, thus, approach  $I_0$ . Equation (2) will then be  $\Delta I_R/I_R \sim \Delta I_b r/V_J \sim (\Delta I_b/I_b)(I_0 r/V_J)$  if  $V_J \gg I_0 r$ , which means that a significant improvement is expected.

In Fig. 2(b), the fluctuation on the external current has no influence on the output voltage of the array if the fluctuation does not exceed the margin  $\pm \Delta I_0$ , so that the original Shapiro step can be maintained. However, one should take care of the fact that the voltage applied to the standard resistor contains the thermal EMF of the cryoprobe output leads and that, if the cryoprobe leads carry a large current, the thermal EMF may contain a reversible component due to the Thomson effect [7], which the polarity reverse technique does not help to compensate. Nevertheless, if the load current through the cryoprobe leads is much smaller than  $I_0$ , the thermal EMF effect in the case of Fig. 2(b) can be much smaller than in the case of Fig. 2(a), because in the latter the current through the cryoprobe leads will always carry a bias current around  $I_0$ .

### B. Feedback Stabilization With the PJVS

For the application to the higher current ranges, we would suggest a feedback stabilization method as shown in Fig. 3, where the current to the load resistor is mostly supplied by a main current source and where an additional small fine-tuning current is supplied by a voltage-to-current converter in such a way that the change of the voltage drop across the current sensing resistor is compensated in accordance with the detector reading. The detector compares the voltage drop with a reference voltage and sends its analog output to the voltage-to-current converter. The use of several ranges of current-sensing resistors in the circuit facilitates the proper combination of the current-sensing resistance with the output voltage of the reference, enabling multirange operation. By using the PJVS as the voltage reference and a precalibrated standard resistor as the current sensing resistor, one can accurately control any high current through the arbitrary load resistor. The compliance voltage to the load depends only on the compliance voltage of the main current source and the voltage-to-current converter, regardless of the output voltage level of the PJVS.

## III. MEASUREMENT

### A. Direct Use of the PJVS Output

We experimentally tested the direct use of the PJVS output as shown in Fig. 2. We used a calibrator for the external current source, a PJVS array with 8191 junctions, and Gunn diode oscillators of 65- and 70-GHz microwave frequency which were

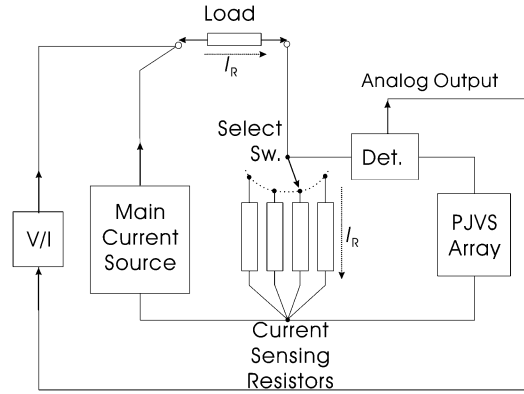


Fig. 3. Circuit diagram of the feedback stabilization method.

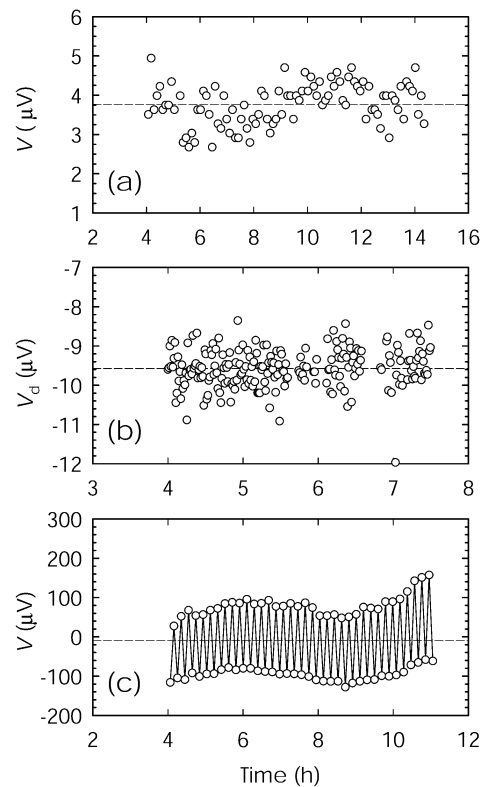


Fig. 4. Measurement results for the method of Fig. 2(a). (a) The voltage across a load resistance of  $1 \Omega$  for a current of  $1 \text{ A}$ . (b) The voltage difference ( $V_d$ ) with respect to a reference when the PJVS is not connected. (c) The load of  $10 \Omega$  and a current of  $0.1 \text{ A}$  to which an artificial square noise of  $\pm 0.1 \text{ mA}$  has been added. In this figure, data for (a) and (c) are shifted by  $-1.11828 \text{ V}$  and  $-0.99 \text{ V}$ , respectively.

locked to the external frequency reference of an atomic clock by a source lock counter. At first, we tested the stability of a  $1\text{-A}$  current by means of the circuit configuration shown in Fig. 2(a). After warming up for more than  $4 \text{ h}$ , the voltage across a  $1\text{-}\Omega$  resistor was measured by an  $8\frac{1}{2}$  digit digital voltmeter (DVM) the input polarity of which was alternately reversed. Fig. 4(a) shows the measurement result. The typical standard deviation for a period of  $2 \text{ h}$  was  $0.40 \mu\text{V}$ . Fig. 4(b) is similar, but this time the PJVS was not connected to the resistor. The voltage difference measured against a reference PJVS showed almost the same stability (typical standard deviation of  $0.44 \mu\text{V}$ ) as in

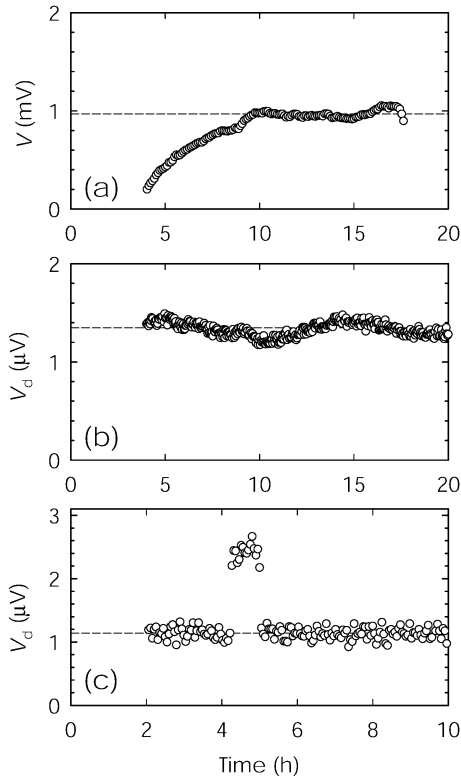


Fig. 5. Measurement results for the method of Fig. 2(b). (a) The voltage for a load of  $10\ \Omega$  and a current of  $0.1\ \text{A}$  to which an artificial square noise of  $\pm 0.1\ \text{mA}$  has been added. (b) The voltage difference ( $V_d$ ) with respect to a reference PJVS for a load of  $10\ \text{k}\Omega$  and a current of  $0.1\ \text{mA}$  (c) The same without resistors, where the peak was intentionally generated for calibration purpose by a change of the microwave frequency. In this figure, data for (a), (b), and (c) are shifted by  $-0.988\ \text{V}$ ,  $-139\ \mu\text{V}$ , and  $-82.185\ \text{mV}$ , respectively.

Fig. 4(a). This confirms the argument that the stability is not improved significantly for the low resistance load. Fig. 4(c) shows the stabilization effect of  $0.1\ \text{A}$  for the higher load resistance of  $10\ \Omega$ , where an artificial square noise of  $\pm 10^{-3}\ \text{A/A}$  was added. The observed voltage fluctuation was only  $\pm 0.09\ \text{mV}$  ( $\sim 10^{-4}$ ), thus ten times lower, which is well explained by (2). Second, we also tested the  $0.1\text{-A}$  stability by means of the circuit configuration of Fig. 2(b). Instead of the DVM input, the current polarity was reversed. The voltage measured across the  $10\text{-}\Omega$  standard resistor (with the artificial external noise of  $\pm 10^{-3}\ \text{A/A}$ ) is shown in Fig. 5(a), where a remarkable drift is seen. This drift is attributed to the large uncompensated thermal EMF caused by the Thomson effect in the cryoprobe leads due to the large current of  $0.1\ \text{A}$ . However, for the larger load resistance of  $10\ \text{k}\Omega$ , as shown in Fig. 5(b), the  $0.1\ \text{mA}$  current was found to be stabilized much better than shown in Fig. 5(a). The change in the voltage drop across the  $10\text{-k}\Omega$  standard resistor with respect to a reference PJVS was measured by a low-voltage DVM. The difference measured was observed to be stable within a standard deviation of  $40\ \text{nV}$ . The slow and large variation within  $300\ \text{nV}$  is attributed to the standard resistors themselves because, as manifested in Fig. 5(c), the difference between two PJVS measured directly showed no change over a period of  $10\ \text{h}$ . The rapid fluctuation of approximately  $\pm 90\ \text{nV}$  ( $1\sigma$ ) in Fig. 5(c) is attributed to the DVM on its  $100\text{-mV}$  range, and the peak was intentionally generated for calibration by a change in the microwave frequency. These experiments confirm the value of the stabilization method shown in Fig. 2(b) for sufficiently small currents.

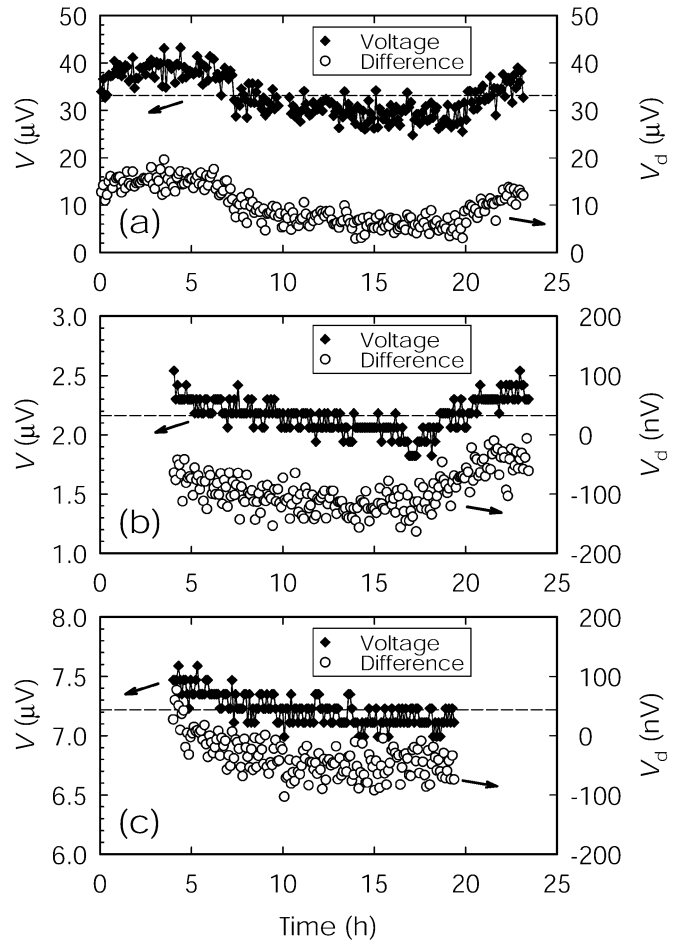


Fig. 6. Measurement result for the method of Fig. 3 (feedback circuit). (a) Without feedback; the open circles are the voltage difference ( $V_d$ ) against a PJVS and the solid diamonds are the total voltage ( $V$ ) across the resistor. (b) The same measurement, but with feedback stabilization. (c) The same as (b), but using a  $1.018\ \text{V}$  Zener voltage standard for reference instead of the PJVS. In this figure, the voltage data for (a), (b), and (c) are shifted by  $-1.189\ \text{V}$ ,  $-1.11458\ \text{V}$ , and  $-1.01814\ \text{V}$ , respectively.

### B. Feedback Stabilization With the PJVS

We set up a simple voltage-to-current converter with a single op-amp to test  $0.1\ \text{A}$  stabilization by means of the feedback circuit of Fig. 3. We chose a  $10\text{-}\Omega$  standard resistor as current sensing resistor, a shorting cable as load resistor, and a nanovoltmeter as the detector. We supplied  $0.1\ \text{A}$  from the calibrator which was intentionally put on the  $1\ \text{A}$  range to simulate a poor current source. At the same time, we measured the voltage difference between the current sensing resistor and the PJVS and the voltage across the resistor by means of the nanovoltmeter ( $10\ \mu\text{V}$  range and filter time constant of  $2.5\ \text{s}$ ) and the  $8\ 1/2$  digit DVM, respectively. Fig. 6(a) shows the voltage difference (open circles) and the voltage (solid diamonds) when the feedback loop is disconnected. Typical standard deviations of the voltage difference and the voltage are  $1.4$  and  $1.5\ \mu\text{V}$ , respectively. Fig. 6(b) shows the same measurements, this time, however, with feedback stabilization. The typical standard deviations of the voltage difference and the voltage were  $21$  and  $70\ \text{nV}$ , respectively. The direct reading of the PJVS output of  $1.1\ \text{V}$  with reversed output polarity was also recorded (not shown here), which showed a typical standard deviation of about  $50\ \text{nV}$ . Thus,

the voltage fluctuation of 70 nV is largely to be attributed to the DVM on its 2-V range. This experiment demonstrates that a current of 0.1 A from a calibrator having a stability of  $10^{-5}$  A/A to  $10^{-6}$  A/A can be stabilized within a standard deviation of  $2 \times 10^{-8}$  A/A. For comparison, Fig. 6(c) shows the measurement result with feedback stabilization using a Zener voltage standard (1.018 V) instead of the PJVS array. The typical standard deviation of the voltage difference and the voltage was 21 and 100 nV, respectively. The difference instability observed in Fig. 6(c) was similar to that of Fig. 6(b), but the voltage instability was larger due to the instability of the Zener voltage standard.

#### IV. DISCUSSION

##### A. Direct Use of the PJVS Output

As far as small current stabilization is of interest, both methods shown in Fig. 2 are possible if we take care of an undesired voltage drop at the lead resistance and the possibility of a reversible thermal effect. Especially when the load current is much smaller than  $I_0$ , the method of Fig. 2(b) should have smaller thermal effect and, thus, seems more suitable than the method of Fig. 2(a) where the thermal effect is generated by the array bias current  $I_0$  regardless of the magnitude of the load current. In the case of Fig. 2(b), as the PJVS output is sufficiently stable, excluding the DVM instability, the uncertainty of the current stabilization depends only on the stability of the thermal EMF in the cryoprobe leads and the stability of the cryoprobe lead resistance. As the heat generated by the current through the cryoprobe leads increases, so does the reversible thermal EMF and its variability. It would, therefore, be desirable to use thicker cryoprobe leads in order to achieve a better stability. Similar considerations are also valid for the application of PJVS arrays to the evaluation of thermal voltage converter ac-dc transfer standards, because the typical input resistance of the converters is in the range below 1 k $\Omega$ .

##### B. Feedback Stabilization With the PJVS

The feedback method can be used for many applications of the Josephson effect. One example is the calibration of dc current meters. Here, one can use a PJVS as a voltage reference and a precalibrated resistor as current sensing resistor. This configuration will give an accurate current value based on the Josephson and the quantum Hall effect for the current through any arbitrary load. Another example is the calibration of 10 V using the 1-V PJVS and a 10:1 resistive divider. The feedback method allows the external current to the 10:1 divider to be stabilized with respect to the 1-V PJVS by using any resistor out of the divider as current sensing resistor, making the total voltage of the divider nearly 10 V. By comparing the 10 V output of the 10:1 divider with an unknown 10-V source under test, and re-

peating the comparison for all ten current sensing resistors, one can determine the 10-V/1-V ratio and the unknown 10-V output under test simultaneously. Further details of the 10-V calibration method using the 1-V PJVS is outside the scope of this paper and will be published elsewhere. In addition, we would like to point out that the feedback method using the Zener voltage standard gives good result and would also be useful for many practical applications.

#### V. CONCLUSION

We have investigated three current stabilization methods that use a PJVS. By directly attaching the output of the PJVS to the load resistor, it was possible to stabilize a current of 0.1 mA within a standard deviation of  $4 \times 10^{-8}$  A/A. The effects of the resistance and of the thermal EMFs of the cryoprobe leads should be taken care of to further improve the stability. We have also suggested a feedback method allowing the stabilization of a high current source by using the PJVS voltage reference, and have demonstrated that a current of 0.1 A from a calibrator with an inherent stability of  $10^{-5}$  A/A to  $10^{-6}$  A/A can be stabilized within a standard deviation of  $2 \times 10^{-8}$  A/A. The advantage of the feedback method is that, in principle, any high current can be stabilized effectively if the appropriate current sensing resistors are available. Also, then the compliance voltage of the stabilized current to the load depends only on the compliance voltage of the main current source and of the voltage-to-current converter, regardless of the output level of the PJVS array. In conclusion, it is expected that the current stabilization method using the PJVS array will provide possibilities for a more extensive use of the Josephson effect in precise dc and LF measurements including for high current ranges.

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