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# FDTD ANALYSIS OF ULTRA SHORT PULSE LASER PROPAGATION IN TRANSIENT LENS EFFECT

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## ABSTRSACT

*Recent advances in measurement techniques using the photo-thermal effect have drawn attention to the application in various fields. We focus on the transient lens effect, which is a well-known photo-thermal phenomenon. As this effect is caused by the change of thermal properties or other physical properties, the measurements using this effect have the possibility to measure various materials information. This phenomenon has the optical property of a concave lens since the refractive index distribution on the optical axis is formed when the sample is irradiated. One reason for the refractive index distribution in sample is the effect of the nonlinear effect when irradiated by ultra-short pulse laser. In this research, we investigate two-dimensional propagation of ultra short pulse laser in the sample with nonlinear effect by using FDTD method. Effect of the third-order susceptibility on the two-dimensional pulse propagation has been demonstrated.*

## NOMENCLATURE

| | |
|---|---|
| E | Electric field, V/m |
| D | Electric displacement field, C/m$^2$ |
| B | Magnetic field, T |
| H | Magnetizing filed, A/m |
| t | Time, s |
| x,y,z | Coordinate, m |
| $\varepsilon_0$ | Electric permittivity in vacuum, F/m |
| $\varepsilon_s$ | Electric permittivity in electrostatic field, F/m |
| $\mu_0$ | Magnetic permittivity in vacuum, H/m |
| n | Time step number in FDTD method |
| i,j | Spatial step number in FDTD method |
| $P_L$ | Linear polarization density, C/m$^2$ |
| $P_{NL}$ | Nonlinear polarization density, C/m$^2$ |
| $\chi^{(1)}$ | First-order electric susceptibility |
| $\chi^{(3)}$ | Third-order electric susceptibility, m$^2$/V$^2$ |
| $\omega_i$ | Resonance angular frequency, rad/s |
| $\delta_i$ | Attenuation coefficient |
| $G_i$ | Normalization coefficient |
| $\delta_{(t-\tau)}$ | Delta function |
| $\alpha$ | Ratio of Kerr effect to Raman effect |
| $g_R$ | Term of transient Raman scattering |
| $\tau_1$ | Inverse of optical phonon frequency, s |
| $\tau_2$ | Phonon lifetime, s |
| $\lambda$ | Wavelength, m |
| c | Speed of light |
| $n_0$ | Refractive index |
| $A_0$ | Peak amplitude of pulse, A/m |
| $b_t$ | Parameter of linear chirp |
| $\psi_0$ | Phase at time at zero, s |

## INTRODUCTION

Gordon et al. [1] has reported that the beam shape of incident laser light is expanded after passing through a liquid medium. It is called “the thermal lens effect”. Nowadays, the thermal lens effect is a well-known photo-thermal phenomenon. Phenomenological, optical, and spectroscopic studies of the thermal lens effect have been carried out and its application to ultra-sensitive spectrometry has been reported [2]. However, the recent progress of laser technology makes the research on thermal lens effect more improved [3]. Based on these efforts, people are more conscious about the mechanism of the lens effect, that it is not only due to the temperature distribution but also some other elements: e.g. density of atomic number, molecular volume, electronic population, molecular orientation et al. Recent studies consider this phenomena and call it "the transient lens effect" [4, 5]. The main advantage of using the transient lens effect in Photo-Thermal-Spectroscopy is that its sensitivity is 100 or 1000 times more than a traditional absorptiometry [6].

However, almost all of conventional research about the transient lens effect investigated only the spatial intensity distribution in the near-field axis of laser beam [7, 8] and it lost the general information of spatial intensity distribution of the probe beam. Therefore, taking a full view of the relationship between light, material and temperature field was difficult. It is necessary to investigate the relationship between the profile of spatial density distribution of the probe beam and the refractive index distribution in the medium [9], and use it as a new application in optical fluidic devices and non-invasive, non-destructive measurement.

On laser technology development, ultra-short pulse laser is expected for application in various fields such as in ultra-fast measurement technique, non-invasive sensor, and laser processing techniques. [4, 5, 10] We focus on the pulse width of the laser as the parameter of the laser properties. As pulse-width become shorter, nonlinear phenomena (optical Kerr effect, self-focusing effect) will occur in the sample [11]. In our previous experiment which is investigation of the profile of spatial density distribution of the probe beam between continuous laser and ultra-short pulse laser in transient lens effect, the profile of the laser passed though the sample has a difference between continuous laser and ultra-short pulse laser [12].

However, most of all existent researches about interaction between ultra-short pulse laser focus on one-dimensional propagation of laser [13] and a few researches focus on nonlinear effect [14]. Therefore in order to establish theoretical framework as well as to develop application, in this research, we investigate properties of two-dimensional propagation of ultra-short pulse laser with numerical simulation.

### Basics of finite difference time domain method

In this research, finite difference time domain (FDTD) method is used for calculating the propagation of ultra-short pulse laser in the sample domain [15, 16]. In FDTD method, Maxwell's equation is differentiated in time domain and spatial domain, and update the value of electromagnetic field sequentially on time step. Therefore, one can obtain transient solution directly in this method. FDTD method has more simple algorithm and has higher resolution compare to other method and can apply to the sample which has a complex refractive index. Despite high calculation cost, FDTD method is attracted an attention for solving the electromagnetic field as progressing of computer performance.

In this research, $TE_z$ wave is used for calculation to consider two-dimensional propagation. The sample is assumed as isotropic and nonmagnetic. The Maxwell's equations are expressed as follows:

$$\frac{\partial D_x}{\partial t} = \frac{\partial H_z}{\partial y} \tag{1}$$

$$\frac{\partial D_y}{\partial t} = -\frac{\partial H_z}{\partial x} \tag{2}$$

$$\mu_0 \frac{\partial H_z}{\partial t} = \frac{\partial E_x}{\partial y} - \frac{\partial E_y}{\partial x} \tag{3}$$

By means of Yee's central difference scheme Eqs. (1), (2), (3) are differentiated in which the time and spatial steps are shifted by 1/2 step as follows:

$$D_x\Big|_{i+1/2,j}^{n} = D_x\Big|_{i+1/2,j}^{n-1} + \frac{\Delta t}{\Delta y}\left(H_z\Big|_{i+1/2,j+1/2}^{n-1/2} - H_z\Big|_{i+1/2,j-1/2}^{n-1/2}\right) \tag{4}$$

$$D_y\Big|_{i,j+1/2}^{n} = D_y\Big|_{i,j+1/2}^{n-1} - \frac{\Delta t}{\Delta x}\left(H_z\Big|_{i+1/2,j+1/2}^{n-1/2} - H_z\Big|_{i-1/2,j+1/2}^{n-1/2}\right) \tag{5}$$

$$\begin{aligned} H_z\Big|_{i+1/2,j+1/2}^{n+1/2} &= H_z\Big|_{i+1/2,i+1/2}^{n-1/2} \\ &+ \frac{\Delta t}{\mu_0 \Delta y}\left(E_x\Big|_{i+1/2,j+1}^{n} - E_x\Big|_{i+1/2,j}^{n}\right) \\ &- \frac{\Delta t}{\mu_0 \Delta x}\left(E_y\Big|_{i+1,j+1/2}^{n} - E_y\Big|_{i,j+1/2}^{n}\right) \end{aligned} \tag{6}$$

where n is the time step number and i, j is the spatial step number for x-direction and y-direction respectively. They are $t = n \cdot \Delta t$ and $x = i \cdot \Delta x$, $y = j \cdot \Delta y$.

In a dielectric material, the electric displacement field is expressed with electric field and polarization density which consist of linear polarization and nonlinear polarization as follows:

$$D = \varepsilon_0 E + P_{\mathrm{L}} + P_{\mathrm{NL}} \tag{7}$$

where $\varepsilon_0$ is electric permittivity in vacuum, $P_L$ is induced polarization depends linearly on the electric field strength, $P_{NL}$ is induced polarization depends on the three powered of electric field.

Next, linear polarization term $P_L$ is corresponding to Sellmeier's fitting equation as expressed as follows:

$$P_{\mathrm{L}} = \sum_k P_k \tag{8}$$

$$P_k = \varepsilon_0 \int_0^t \chi_k^{(1)}{}_{(t-\tau)} E_{(\tau)} d\tau \tag{9}$$

where $\chi^{(1)}$ has a frequency dispersion and is specified by Lorentz model in frequency domain and described as follows:

$$\chi_{k(\omega)}^{(1)} = \frac{G_k \omega_k^2 (\varepsilon_s - 1)}{\omega_k^2 - j\omega\delta_k - \omega^2} \tag{10}$$

where, j is the imaginary unit, $\varepsilon_s$ is electric permittivity in electrostatic field, $\omega_k$ is resonance angular frequency, $\delta_k$ is attenuation coefficient and $G_k$ is normalization coefficient and $\Sigma G_k = 1$.

If the sample is sufficiently short that attenuation term can be neglected and from $n_0 = 1 + \Sigma\chi^{(1)}$, then Eq. (10) can be described as follows:

$$\chi_{k(\omega)}^{(1)} = \frac{b_k \omega_k^2}{\omega_k^2 - \omega^2} \tag{11}$$

where $b_k = G_i(\varepsilon_s - 1)$.

While, nonlinear response has two dominant time scale. The shorter time scale gives the dominant contribution to the real part of $\chi^{(3)}$ and is modeled by instantaneous delta function response. The longer time scale of about 100 fs is largely given by the phonon lifetime and is modeled by a single Lorentzian line, centered on the optical phonon frequency. Hence, nonlinear polarization term $P_{NL}$ is assumed to be characterized by the nonlinear single time convolution and is expressed as follows:

$$P_{NL} = \varepsilon_0 \chi^{(3)} E \int_{-\infty}^{\infty} \left[\alpha\delta_{(t-\tau)} + (1-\alpha) g_{\mathrm{R}(t-\tau)}\right]\left[E_{(\tau)}\right]^2 d\tau \tag{12}$$

where $\delta_{(t-\tau)}$ is the delta function, $\alpha$ is ratio of intensities of Kerr effect to Raman effect, $g_R$, models transient Raman scattering, is expressed as follows:

$$g_R(t) = [(\tau_1^2 + \tau_2^2)/\tau_1\tau_2^2] e^{-t/\tau_2} \sin(t/\tau_1) \tag{13}$$

$g_R$ models a single Lorentzian line centered on the optical phonon frequency $1/\tau_1$ and having a band-width of $1/\tau_2$.

## NUMERICAL MODELING

As shown in Fig.1 ultra-short pulse laser is irradiated in the sample glass, and is simulated its propagation by using FDTD method. The simulation is performed with 1000 × 500 grid, where $\Delta x = \Delta y = 1.3842\times10^{-8}$m respectively. A Courant stability condition (the time step must be chosen for a two-dimensional problem with $\Delta x = \Delta y$ so that $((\Delta x/\sqrt{2}\cdot c) \geq \Delta t))$ is maintained, which means that the value of $\Delta t = 3.00\times10^{-17}$ s is chosen. The number of time steps is 800 to investigate the transient propagation of ultra-short pulse laser. A Mur's boundary condition is used at the domain boundary layer for absorbing the refraction of the beam propagation. In this calculation, laser wavelength $\lambda$ = 800 nm, pulse-width is 30fs are used, and profile of the laser is assumed as Gaussian profile in the time domain and spatial domain shown as follows:

$$H = A_0 \exp\left(-\frac{t^2}{2\Delta t_1^2}\right)\cos\left(\omega_0 t + \frac{1}{2} b_t t^2 + \psi_0\right) \tag{14}$$

where $A_0$ is the peak value of amplitude of the pulse, $b_t$ is a parameter of linear chirp, $\psi_0$ is phase at t = 0 [17].The radius of laser is 1μm. The parameters are set as $\psi_0=0$, $b_t=1.0$, $\omega_0=6.3\times10^{16}$.

In this research, sample is assumed as fused silica. The third-order susceptibility $\chi^{(3)}=1.85\times10^{-20}$ m$^2$/V$^2$ is given from $\chi^{(3)}=(4/3)\ \varepsilon_0\cdot c\cdot n_0(\omega_0)^2$, here, $\omega_0$ is the center angular frequency of the optical pulse. The ratio of the magnitude of Kerr effect and Raman effect is $\alpha$=0.8. The parameters in Eq. (11) are set as $b_1$=0.6961663, $b_2$=0.4079426, $b_3$=0.0684043 [18]. The electric permittivity in vacuum $\varepsilon_0 = 8.85418\times10^{-12}$ F/m, and the magnetic permittivity in vacuum $\mu_0=1.25663\times10^{-6}$ H/m are used. The parameters $\tau_1$, $\tau_2$ in Eq. (13) are set as $\tau_1$=12.2 fs, $\tau_2$=32 fs.

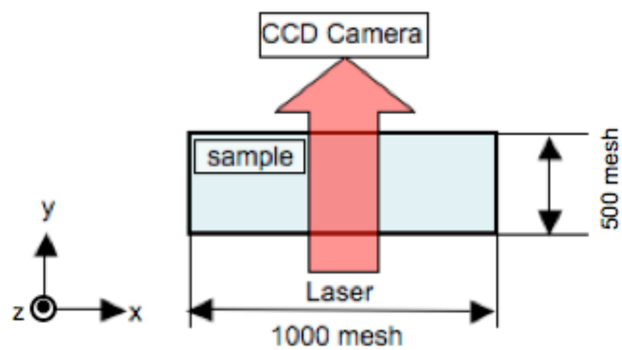
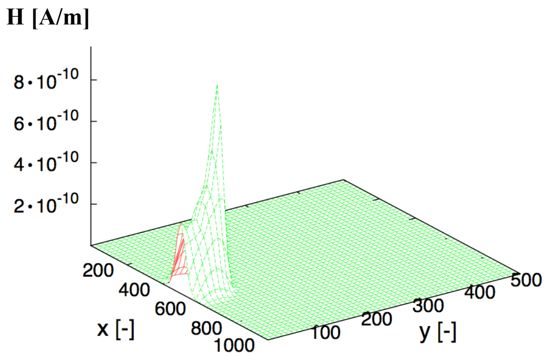



**FIGURE.1** MODEL OF NUMERICAL SIMULATION DOMAIN

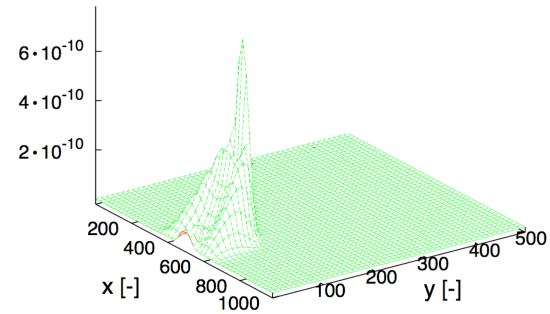
## RESULTS AND DISSCUSSION

### Time evolution

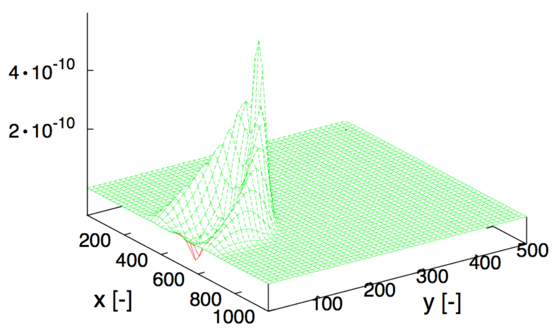
First, Fig.2 shows time evolution of magnitude of magnetizing field in sample at t = 3 fs, t = 6 fs, t = 9fs, t = 12 fs respectively. As shown in Fig.2, profile of magnetizing filed is different from Gaussian profile e.g. in the propagation front has higher intensity and the tail part has gradually decreasing profile. This trend is caused by the nonlinear parameter e.g. profile of third power of electric field which similar to Gaussian profile in spatial domain provide derivative of nonlinear polarization profile which has minus value in the propagation front and has plus value in the tail part. Hence, considering Eq.(6) and Eq.(7), the profile of magnetizing field is leaded to the asymmetric shape for the spatial domain and is similar with the results of one-direction propagation [19].



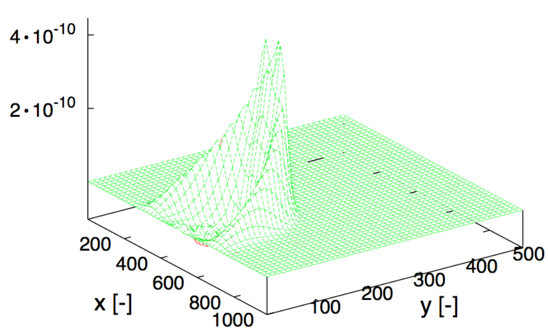

(a) t = 3 fs




(b) t = 6 fs

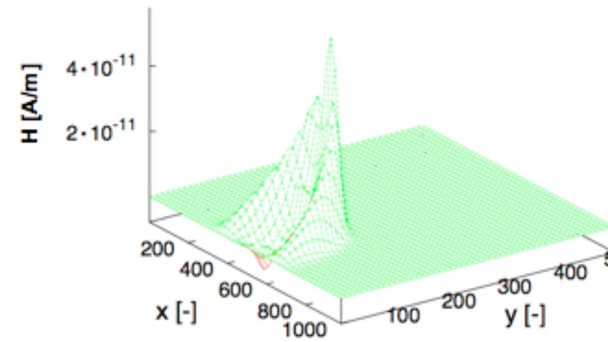



(c) t =9 fs

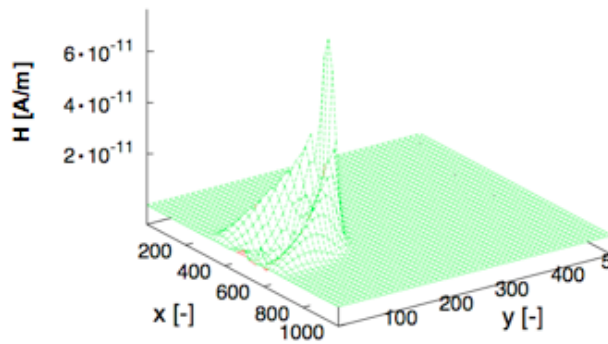



(d) t = 12 fs

**FIGURE 2** TIME EVOLUTION OF MAGNETIZING FIELD PROPAGATION IN THE SAMPLE;




(a) $\chi^{(3)}$=1.85×10$^{-22}$ m$^2$/V$^2$




$\chi^{(3)}$=1.85×10$^{-19}$ m$^2$/V$^2$

FIGURE 3. COMPARISON OF MAGNETIZING FIELD OF PROPAGATION WITH DIFFERENT THIRD-ORDER SUSCEPTIBILITY

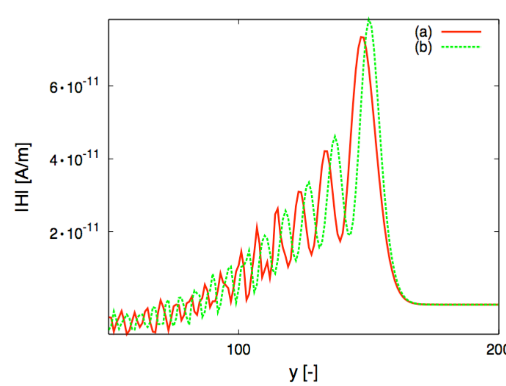



FIGURE 4. COMPARISON OF MAGNETIZING FIELD OF PROPAGATION IN NEAR FIELD OF LASER AXIS WITH DIFFERENT THIRD-ORDER SUSCEPTIBILITY;
(a) $\chi^{(3)}$=1.85×10$^{-22}$ m$^2$/V$^2$, (b) $\chi^{(3)}$=1.85×10$^{-19}$ m$^2$/V$^2$

### Effect of Kerr effect

Figure.3 shows the profile of the magnetizing field in samples with different electric permittivity. Fig.3 (a) shows one at $\chi^{(3)}$=1.85×10$^{-22}$ m$^2$/V$^2$ and Fig.3 (b) $\chi^{(3)}$=1.85×10$^{-19}$ m$^2$/V$^2$. It can be seen in Fig.3 that the two-dimensional pulse propagation characteristics change by changing the third-order susceptibility. Figure.4 shows change of the center profile of the laser propagation with the third-order susceptibility. Considering eq.(7) and eq.(12), as increasing $\chi^{(3)}$ , the effect of third power of electric field increase and the profile of derivative of $P_{NL}$ change. Hence, the peak magnitude of magnetizing field at $\chi^{(3)}$=1.85×10$^{-19}$ m$^2$/V$^2$ is higher than the peak magnitude of magnetizing field at $\chi^{(3)}$=1.85×10$^{-22}$ m$^2$/V$^2$. Oscillated profile in magnetizing field is derived from the effect of oscillation of Raman effect.

In the future, treatment of Raman effect and peak power of laser will be studied in detail for studying effect of nonlinear term.

## CONCLUSION AND FUTURE WORKS

In this paper, two-dimensional $TE_z$ wave propagation of ultra-short pulse laser in sample is simulated. In this research, nonlinear term affect the shape of electromagnetic wave with time evolution. The difference of magnitude of magnetizing field profile in the near field of laser axis appears with the change of the Kerr effect term. Therefore the propagation can change with the change of the Kerr term.

In the future, treatment of the effect of pulse width and peak power will be studied in detail and effect of the refractive index profile in the sample will be studied for transient propagation of laser in transient lens effect.

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