Shock and Vibration 15 (2008) 599–617
IOS Press

Asymmetric vibration of polar orthotropic annular circular plates of quadratically varying thickness with same boundary conditions

N. Bhardwaj[a,*], A.P. Gupta[a] and K.K. Choong[b]
[a]*Department of Applied Sciences and Humanities, Huda- Sector 23-A, Institute of Technology and Management, Gurgaon- Haryana-122017, India*
[b]*School of Civil Enginnering, Universiti Sains Malaysia, Engineering Campus, 14300 Nobong Tebal, Seberang Perai Selatan, P. Pinang, Malaysia*

Received 2 July 2006
Revised 24 April 2007

Abstract. In the present paper, asymmetric vibration of polar orthotropic annular circular plates of quadratically varying thickness resting on Winkler elastic foundation is studied by using boundary characteristic orthonormal polynomials in Rayleigh-Ritz method. Convergence of the results is tested and comparison is made with results already available in the existing literature. Numerical results for the first ten frequencies for various values of parameters describing width of annular plate, thickness profile, material orthotropy and foundation constant for all three possible combinations of clamped, simply supported and free edge conditions are shown and discussed. It is found that (a) higher elastic property in circumferential direction leads to higher stiffness against lateral vibration; (b) Lateral vibration characteristics of F-F **plates** is more sensitive towards parametric changes in material orthotropy and foundation stiffness than C-C and S-S **plates**; (c) Effect of quadratical thickness variation on fundamental frequency is more significant in cases of C-C and **S-S plates** than that of F-F **plates**. Thickness profile which is convex relative to plate center-line tends to result in higher stiffness of annular plates against lateral vibration than the one which is concave and (d) Fundamental mode of vibration of C-C and S-S **plates** is axisymmetrical while that of F-F **plates** is asymmetrical.

Keywords: Annular plate, variable thickness, orthotropy, asymmetric vibration, Winkler elastic foundation, Jacobi method

1. Introduction

Annular circular plate is the simplest and widely used structural element in various engineering fields. The vibration of such plates has been the subject of various studies. Leissa [1–8] summarized the information in his well-known monograph and six comprehensive review articles. For the orthotropic plates, except for a few cases, no closed form solution exits. Researchers have used different approximation methods. Among them Vijaya Kumar and Ramaiah [9], Narita [10,11] and Gutierrez et al. [12] used Rayleigh-Ritz method, Greenberg and Stavsky [13] used finite difference method and Ginesu et al. [14] and Gorman [16] used finite element method.

A lot of information on annular circular plates having varying thickness is also available in the existing literature. The Chebyshev collocation method was used by Soni and Amba-Rao [17] to study the axisymmetric vibration of an

*Corresponding author: Dr. Neeraj Bhardwaj, Tel.: +91 124 2365811/2365812 (Ext. 223); Fax: +91 124 2367488; E-mail: bneerajdma@yahoo.co.in or bneerajdma@gmail.com.

ISSN 1070-9622/08/$17.00

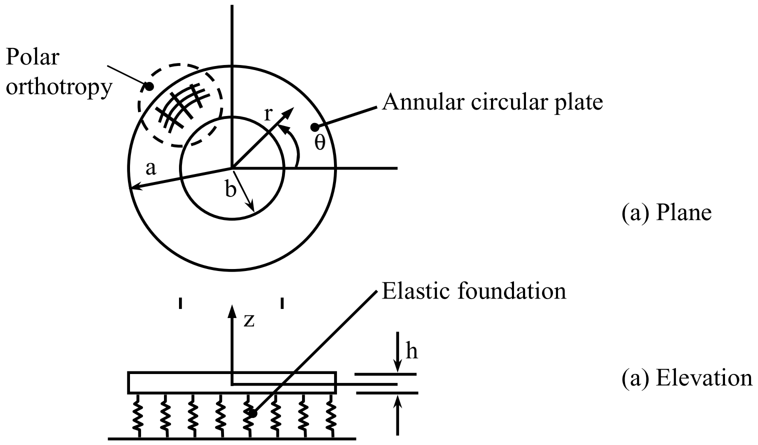


Fig. 1. Annular circular plate resting on elastic foundation.

annular plate with linearly varying thickness. Gupta and Lal [18] have extended the above paper to include the effect of in-plane forces. Lal and Gupta [19] solved the same problem for polar orthotropy. The axisymmetric vibration of such plates was further considered by Gupta et al. [20] using spline technique. Gorman [16] employed finite element method to compute natural frequencies of axisymmetric and asymmetric modes of polar orthotropic annular plates of linearly varying thickness, Raju et al. [21] used the same technique to analyze axisymmetric vibration of linearly tapered isotropic annular plates. Exact closed form solutions have been presented by Conway et al. [22] for linearly tapered isotropic annular plates and Lenox and Conway [23] for polar orthotropic plates of parabolic thickness variation. Kim and Dickinson [24] have analyzed composite circular plates as a particular case of annular plates by taking inner radius very small but only few results are given on circular plates and that too are for uniform thickness only. Wang et al. [25] studied free vibration analysis of annular plates by differential quadrature method. Laura et al. [26] have analysed annular circular plates having cylindrical anisotropy and non-uniform thickness using polynomial coordinates functions. Chen and Ren [27] studied lateral vibration of isotropic and orthotropic thin annular and circular plates of arbitrarily varying thickness along radius using finite element method and obtained natural frequencies and mode shapes of the axisymmetric and asymmetric modes. Gutierrez et al. [12] have analyzed annular plates of polar orthotropy using Rayleigh -Ritz method. Recently, Neeraj et al. [32] have studied effect of elastic foundation on the vibration of orthotropic elliptic plates with varying thickness.

In all of the above papers, variation of thickness depends only on one taper parameter and there is no mention of nodal lines and mode shapes.

In the present paper, asymmetric vibration of annular plates of polar orthotropic material having quadratically varying thickness along radial direction and resting on Winkler elastic foundation is analyzed by using boundary characteristic orthonormal polynomials in Rayleigh-Ritz method. Two taper parameters are used for the quadratic thickness variation, which give more flexibility to study thickness variation. Many thickness variations can be approximated by it by suitably choosing the values of taper constants. Frequencies for the first ten normal modes of vibrations are computed for various values of inner radius, taper, orthotropy and foundation parameters for all three possible combinations of clamped-clamped, simply supported-simply supported and free-free conditions for inner and outer edges respectively. Convergence of frequencies at least upto five significant figures is observed. Comparison of frequencies in particular cases are made with the results already available in the literature. Apart from close agreement, it is also found that the present results are better in almost all the cases. Figures are shown for nodal lines and their corresponding three dimensional mode shapes.

2. Equation of motion

A thin annular plate of outer radius a, inner radius b, variable thickness $h(r)$, is made up of orthotropic material and resting on Winkler elastic foundation is considered. Figure 1 shows a sketch of the plate problem treated in this paper. The plate is referred to cylindrical coordinate by taking the axis of the plate along the z-axis and the middle plane of the plate in the $r-\theta$ plane. The displacement components u, v and w in the directions of r, θ and z axes respectively, are taken as

$$u(r,\theta,\ z,t) = -z\, w_{,r}\ ,$$
$$v(r,\theta,\ z,t) = -\frac{z}{r} w_{,\theta} \tag{1}$$

and $w(r,\ \theta, z, t) = \ w(r,\theta,t)$.

2.1. Energy considerations

The strain energy due bending of the plate is given by

$$S_e = \frac{1}{2}\int_b^a\int_0^{2\pi}\left[\frac{E_r\, h^3}{12\,(1-\nu_r\nu_\theta)}\left\{w_{,rr}\left(w_{,rr}+\frac{\nu_\theta\, w_{,\theta\theta}}{r^2}+\frac{\nu_\theta\, w_{,r}}{r}\right)+\frac{E_\theta}{E_r}\left(\nu_r w_{,rr}+\frac{w_{,\theta\theta}}{r^2}+\frac{w_{,r}}{r}\right)\right.\right.$$
$$\left.\left.\left(\frac{w_{,\theta\theta}}{r^2}+\frac{w_{,r}}{r}\right)\right\}+G\left(\frac{2}{r}\,w_{,r\theta}-\frac{2}{r^2}\,w_{,\theta}\right)^2\right]\, r\,d\theta\ dr. \tag{2}$$

The kinetic energy of the plate is given by

$$K_e = \frac{\rho}{2}\int_b^a\int_0^{2\pi}\left[\frac{h^3}{12}\left(w_{,rt}^2+\frac{w_{,\theta t}^2}{r^2}\right)+h\ w_{,t}^2\right]\ r\,d\theta\ dr. \tag{3}$$

where ρ is the density of the plate.

The potential energy due to Winkler elastic foundation [31] can be taken as

$$P_e = \frac{1}{2}\int_b^a\int_0^{2\pi} k_f\, w^2\ \ r\ d\theta\ dr, \tag{4}$$

where k_f is vertical stiffness of the foundation per unit area.

Introducing the following non-dimensional variables:

$H = h/a, R_0 = (b/a), R = r/a, e_r = E_\theta/E_r = \nu_\theta/\nu_r, g_r = G/E_r\ \ s_r = \left(1-e_r\nu_r^2\right)$,
$K_f = 12\ s_r\, a\, k_f/\left(E_r, H_a^3\right)$, and $T = (t/a)\,\sqrt{(E_r/\rho)}$ and for harmonic vibration taking

$$w(r,z,t) = \ \ a\ W(R,\theta)\ \cos\,\omega T \tag{5}$$

and

$$W\,(R\,,\theta) = \ W_m\,(R)\ \cos\ \ m\theta \tag{6}$$

where m is the number of nodal diameters, Eqs (2), (3) and (4) could be reduced to

$$S_e = \frac{\pi\,E_r}{24\,s_r}\int_{R_0}^1\int_0^{2\pi} H^3\left[\left\{W_{m,RR}^2+2e_r\nu_r W_{m,RR}\left(\frac{-m^2}{R^2}W_m+\frac{1}{R}W_{m,R}\right)+e_r\left(\frac{-m^2}{R^2}W_m+\frac{1}{R}W_{m,R}\right)^2\right.\right.$$
$$\left.\left.+4\,s_r g_r\left(\frac{m}{R^2}W_m-\frac{m}{R}\,W_m\right)^2\right\}\ \right]\ \cos^{\,2}\omega\, T\ \ R\,d\theta\,dR, \tag{7}$$

$$K_e = \frac{\pi}{2}\int_{R_0}^1 E_r\,H\,\omega^{\,2}\,W_m^2\ \sin^2\ \omega\, T\ \ R\,dR. \tag{8}$$

$$P_e = \frac{\pi}{2}\int_{R_0}^1 k_f\ W_m^2\ R\,dR. \tag{9}$$

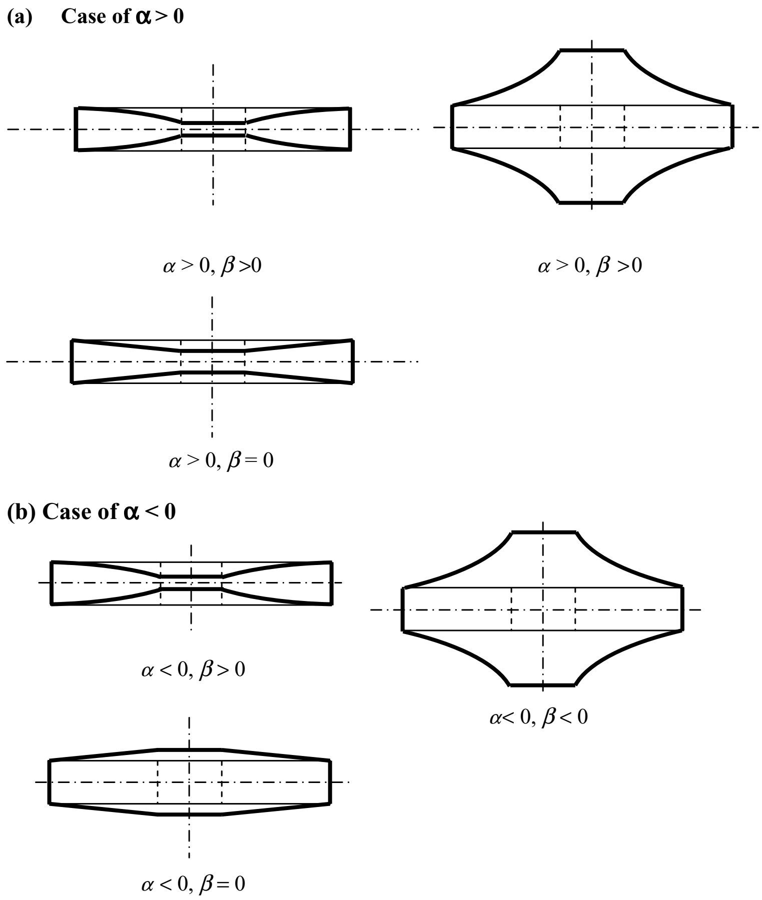


Fig. 2. Thickness variation under different combination of taper parameters α and β.

In the above expressions, E_r, E_θ and ν_r, ν_θ are the Young's modulii and Poission's ratio's in r and θ directions respectively, G is the shear modulus, K_f is the foundation constant and ω is the natural frequency of harmonic vibration.

The above formulation is well known and can be found for instance in [32].

2.2. Thickness variation

The non-dimensional thickness of the plate is taken as

$$H = H_a\, F(R,\theta)\,,$$

where $H_a = h_a/a$, h_a is the thickness of the plate at the outer periphery and

$$F\,(R,\theta) = 1 - \alpha(1 - R) - \beta\,(1 - R)^2 > 0\ , \quad \alpha(1 - R_0) + \beta\,(1 - R_0)^2 < 1, \tag{10}$$

where α and β are the taper constants. Figure 2 shows a sketch of the plate variation considered in this paper.

The quadratically varying thickness has two taper parameters, which give more flexibility in thickness variation. By suitably adjusting the taper parameters, many thickness variations can be approximated. As can be seen from Fig. 2, value of β will produce thickness variation which is convex ($\beta < 0$) and concave ($\beta > 0$) with respect to plate centerline. The special case of linear thickness variation when $\beta = 0$ is also shown.

The functional $J\,(W)$obtained by subtracting the maximum kinetic energy from the sum of the maximum strain energy and the maximum potential energy due to foundation is

$$J(W) = \frac{E_r H_a^3}{24\, s_r} \int_{R_0}^{1} \int_{0}^{2\pi} \left[F^3(R,\theta) \left\{ W_{m,RR}^2 + 2 e_r \nu_r W_{m,RR} \left(-\frac{m^2}{R^2} W_m + \frac{1}{R} W_{m,R} \right) + e_r \left(-\frac{m^2}{R^2} W_m + \frac{1}{R} W_{m,R} \right)^2 + 4\, s_r g_r \left(\frac{m}{R^2} W_m - \frac{m}{R} W_{m,R} \right)^2 \right\} + K_f\, W_m^2 - \Omega^2 F\; W_m^2 \right] R\; d\theta\, dR, \tag{11}$$

where

$\Omega = \sqrt{s_r}\,\Omega^*$. and $\Omega^* = \sqrt{12}\;\omega/h_a$

For applying the Rayleigh-Ritz method the functional $J(W)$ is to be minimized.

2.3. Generation of boundary characteristic orthonormal polynomials

The N-term approximation of the deflection function is taken as

$$W_m(R) = \sum_{j=1}^{N} c_{m\,j}\, \Phi_j(R), \tag{12}$$

where Φ_j are the boundary characteristic orthonormal polynomials satisfying at least the geometric edge conditions of the plate. Using three terms recurrence relation given by Chihara [30], Φ_j are generated as

$$\Phi_j = \phi_j / \sqrt{<\phi_j, \phi_j>}, \phi_1 = (R - R_0)^p \left(1 - R^2\right)^p, \quad j = 1, 2, ..., N, \tag{13}$$

$$\phi_{j+1} = \left(R - \frac{<R\,\phi_j, \phi_j>}{<\phi_j, \phi_j>} \right) \phi_j - \frac{<\phi_j, \phi_j>}{<\phi_{j-1}, \phi_{j-1}>} \phi_{j-1}\,,\ \phi_0 = 0\,, \quad j = 1, 2, ..., N-1, \tag{14}$$

$$<f, g> = \int_{R_0}^{1} F(R) f(R)\, g(R)\;\; R\, dR \tag{15}$$

The value of p is equal to 0,1 or 2 for free ($\boldsymbol{F}$), simply supported ($\boldsymbol{S}$) or clamped ($\boldsymbol{C}$), condition of the inner and outer edges of the plate, respectively. Substitution of $W_m(R)$ from Eq. (12) into energy Eq. (11) and then minimization of $J(W)$as a function of the coefficients $c_{m\,j}$ leads to the following standard eigenvalue problem:

$$\sum_{j=1}^{N} \left(a_{i\,j} - \Omega^2\,\delta_{i\,j}\right)\; c_{m\,j} = 0, \quad i = 1(1)N, \tag{16}$$

where

$$\delta_{ij} = \begin{cases} 1 & \text{when} \quad i = j \\ 0 & \text{when} \quad i \neq j \end{cases}$$

and

$$a_{i\,j} = \int_{R_0}^{1} \left[F^3 \left\{ \Phi_{i,RR}\Phi_{j,RR} + e_r\nu_r \Phi_{i,RR} \left(-\frac{m^2}{R^2}\Phi_j + \frac{1}{R}\Phi_{j,R} \right) + e_r\nu_r \Phi_{j,RR} \left(-\frac{m^2}{R^2}\Phi_i + \frac{1}{R}\Phi_{i,R} \right) + e_r \left(-\frac{m^2}{R^2}\Phi_i + \frac{1}{R}\Phi_{i,R} \right)\left(-\frac{m^2}{R^2}\Phi_j + \frac{1}{R}\Phi_{j,R} \right) + 4 s_r g_r \left(\frac{m}{R^2}\Phi_i - \frac{m}{R}\Phi_{i,R} \right)\left(\frac{m}{R^2}\Phi_j - \frac{m}{R}\Phi_{j,R} \right) \right\} + K_f \Phi_i \Phi_j \right] dR. \tag{17}$$

The integrals involved in Eqs (15) and (17) are evaluated by using the formula

Table 1
Convergence of $\Omega_{m,n}$ when $\alpha = \beta = 0.4$, $e_r = g_r = 5.0$, $R_0 = 0.5$, $K_f = 500$

Edge conditions (inner-outer)	N	$\Omega_{0,0}$	$\Omega_{1,0}$	$\Omega_{2,0}$	$\Omega_{3,0}$	$\Omega_{4,0}$	$\Omega_{5,0}$	$\Omega_{0,1}$	$\Omega_{1,1}$	$\Omega_{6,0}$	$\Omega_{2,1}$
C-C	4	81.084	86.850	102.74	126.36	156.32	192.16	214.48	222.76	234.09	246.18
	5	81.083	86.846	102.72	126.30	156.20	191.94	214.19	222.55	233.82	246.11
	6	81.083	86.846	102.72	126.30	156.17	191.89	214.19	222.54	233.43	246.08
	7	81.083	86.846	102.72	126.30	156.17	191.89	214.19	222.54	233.43	246.08
		$\Omega_{0,0}$	$\Omega_{1,0}$	$\Omega_{2,0}$	$\Omega_{3,0}$	$\Omega_{4,0}$	$\Omega_{0,1}$	$\Omega_{1,1}$	$\Omega_{5,0}$	$\Omega_{2,1}$	$\Omega_{6,0}$
S-S	4	43.316	51.602	71.706	98.589	130.81	140.23	151.22	168.24	180.50	211.04
	5	43.316	51.602	71.706	98.589	130.81	140.21	151.17	168.24	180.37	211.04
	6	43.316	51.602	71.706	98.589	130.81	140.18	151.14	168.24	180.35	211.04
	7	43.316	51.602	71.706	98.589	130.81	140.18	151.14	168.24	180.35	211.04
F-F	4	24.579	28.792	31.928	43.647	47.750	70.448	70.933	86.854	107.66	152.76
	5	24.578	28.791	31.914	43.628	47.695	70.406	70.797	86.600	106.88	150.34
	6	24.578	28.791	31.912	43.625	47.689	70.394	70.788	86.544	106.78	150.18
	7	24.578	28.791	31.912	43.625	47.689	70.394	70.788	86.544	106.88	150.18

Table 2
Comparison of $\Omega_{m,n}$ for polar isotropic annular plates of uniform thickness when $e_r = 1.0$, $g_r = 0.384$, $\nu_r = 0.3$, $K_f = 0$

Edge condition (inner-outer)	$\rightarrow \Omega_{m,n}$ $\downarrow$ Ref.	R_0	$\Omega_{0,0}$	$\Omega_{0,1}$	$\Omega_{1,0}$	$\Omega_{1,1}$	$\Omega_{2,0}$	$\Omega_{2,1}$	$\Omega_{3,0}$	$\Omega_{3,1}$
C-C	**Present**	0.1	27.28056	75.36626	28.91363	78.63135	36.11467	90.43646	51.21047	112.08970
	[27]		27.28056	75.36631	28.91583	78.63537	36.61744	90.44888	51.21888	112.10877
	[28]		27.3	75.3	28.4	78.2	36.7	90.5	51.2	–
	[24]		27.281	75.369	28.918	78.642	36.622	90.463	51.221	
	Present	0.5	89.25075	246.34284	90.229157	247.73809	93.33169	251.96656	98.91900	259.14956
	[27]		89.25066	246.34249	90.23018	247.73932	93.32111	251.97236	98.92790	259.16263
	[28]		89.2	–	90.2	–	93.4	–	99.0	–
	[24]		89.251	–	90.230	–	93.321	–	98.928	–

$$\int_{R_0}^{1} (R - R_0)^k (1-R)^l R^s \, dR = \int_{R_0}^{1} R^{s+1} \left(\sum_{i=0}^{k} {}^kC_i \, (-R_0)^i \, R^{k-i} \right) \left(\sum_{j=0}^{l} {}^lC_j \, (-1)^j \, R^j \right) \, dR$$
$$= \sum_{i=0}^{k} {}^kC_i \, (-R_0)^i \left(\sum_{j=0}^{l} {}^lC_j (-1)^j \frac{(1 - R_0^{k+s+j-i+2})}{k+s+j-i+2} \right), k, l, s \geqslant 0. \tag{18}$$

The eigenvalues (Ω) and the eigenvectors $(c_{m\,j})$ are computed using Jacobi method. The mode shapes are computed from Eq. (12) and the nodal lines are computed from the same equation by putting $W_m(R) = 0$.

As can be seen from the description above, Rayleigh-Ritz method with orthonormally generated boundary characteristic polynomials requires a set of deflection shapes that satisfy at least the geometrical boundary conditions of the vibrating structures. It reduces the problem into eigenvalue problem. Also, the method in combination with the expression for thickness variation used, can be applied to practically any thickness variation provided the integrals can be evaluated accurately. For a polynomial variation, it is possible to evaluate them in closed form, so there is no loss of accuracy on that account but for other type of thickness variation some numerical methods have to be used.

3. Results and discussion

The following sets of computations have been carried out:

i) Convergent characteristics of the formulation used in this current study
ii) Comparison with available results by other researchers

Table 3
Comparison of $\Omega_{m,n}$ for isotropic annular plates of linearly varying thickness when $\alpha = 1.0$, $\beta = 0.0$, $e_r = 1.0, g_r = 0.384, K_f = 0$

Edge condition (inner-outer)	Source of results	ν	R_0	$\Omega_{0,0}$	$\Omega_{0,1}$	$\Omega_{1,0}$	$\Omega_{1,1}$	$\Omega_{2,0}$	$\Omega_{2,1}$	$\Omega_{3,0}$	$\Omega_{3,1}$
C-C	**Present**	1/3	0.1	13.345452	35.05977	15.481653	37.471833	21.956461	44.993661	32.219335	57.838787
	[27]			13.29217	34.87202	15.40388	37.25180	21.85060	44.72102	32.10169	57.55299
	[24]			13.406	35.274	15.501	37.633	21.904	45.023	32.119	–
	[22]			13.28	34.84	–	–	–	–	–	–
	Present		0.5	65.916233	180.70514	66.740116	181.778206	69.314345	185.02204	73.909881	190.50664
	[27]			65.91615	180.70326	66.72713	181.75935	69.26474	184.95349	73.80586	190.35920
	[24]			65.916	180.70	66.727	–	69.265	–	73.806	–
	[22]			66.0	180.8	–	–	–	–	–	–
	Present	0.3	0.1	13.410906	35.15651	15.544613	37.546778	22.010689	45.02049	32.255201	57.817126
	[27]			13.35744	34.96846	15.49996	37.36599	21.98872	44.87363	32.25351	57.73541
	[24]			13.472	35.370	15.597	37.746	22.042	45.173	32.270	–
	[29]			13.42	35.21	–	–	–	–	–	–
	Present		0.5	65.954113	180.75700	66.766592	181.812818	69.308426	185.00604	73.855777	190.41088
	[27]			65.954	180.75511	66.76739	181.81198	69.31175	185.00839	73.86281	190.41769
	[24]			65.954	180.76	66.767	–	69.312	–	73.863	–
	[29]			–	180.7	–	–	–	–	–	–

Table 4
Comparison of $\Omega_{m,n}$ for polar orthotropic annular plates of uniform thickness when $e_r = 5.0$, $g_r = 0.356$, $\nu_r = 0.06, R_0 = 0.5, K_f = 0$

Edge conditions (inner-outer)	Source of results	$\Omega_{0,0}$	$\Omega_{1,0}$	$\Omega_{2,0}$	$\Omega_{3,0}$	$\Omega_{4,0}$	$\Omega_{5,0}$	$\Omega_{6,0}$	$\Omega_{0,1}$	$\Omega_{1,1}$	$\Omega_{2,1}$
S-S	[15]	43.720	45.165	51.320	65.546	–	–	–	162.56	164.38	170.48
	[10]	43.720	45.160	–	–	–	–	–	–	–	–
	[24]	43.720	45.164	51.320	65.547	89.068	120.79	159.10	162.55	164.37	170.47
	(**Present**) 6 terms	43.720	45.164	51.320	65.547	89.068	120.78	159.10	162.55	164.37	170.47

iii) Vibration characteristics of annular circular plate resting on elastic foundation considering different combinations of the following parameters: material polar orthotropy e_r, foundation stiffness K_f, geometry of annular circular plate R_0 and taper parameters for thickness variation (α, β).

Table 1 shows the convergence of the first ten frequencies $\Omega_{m,n}$ of at least upto five significant figures for all possible three combinations ***C-C***, ***S-S*** and ***F-F*** of edge conditions at inner and outer edges when $\alpha = \beta =$0.4, $e_r = g_r = 5.0, R_0 = 0.5$ and $K_f = 500$. The suffixes m and n in $\Omega_{m,n}$ denote number of nodal diameters and nodal circles, respectively. $m = 0$ corresponds to axisymmetric modes whereas $m = 1, 2, 3, \ldots$ corresponds to asymmetric modes of vibration. It can be seen that 7 terms are required to get the accuracy of upto five significant figures in all the cases.

The values of ν_r and g_r are taken as 0.3 and 5.0, respectively, for all computations except for the results presented in Tables 2, 3, 4 and 5, where other values are used for the purpose of comparison to known results. The variations in parameters for all possible three combinations of edge conditions for inner and outer edges are taken as follows:

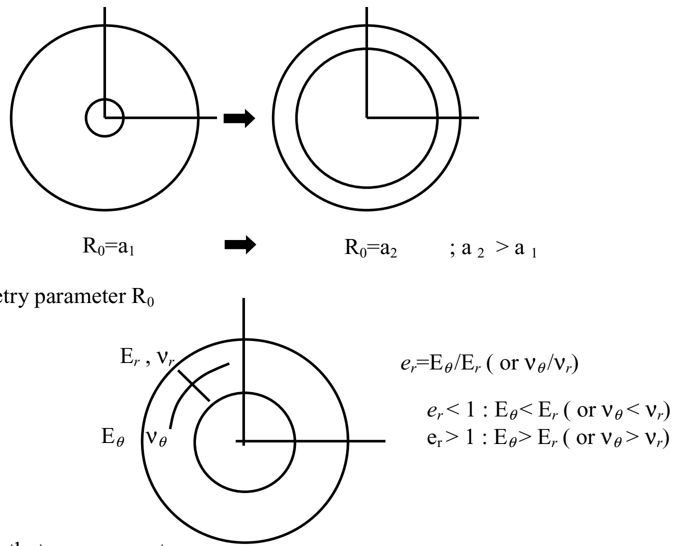
1. e_r from 0.25 to 8.0 by doubling the value at each steps;
2. K_f from 0 to 1000 in steps of 200;
3. R_0 from 0.1 to 0.6 in steps of 0.1;
4. α and β from -0.4 to 0.4 in steps of 0.1.

Figure 3 shows the physical meaning of variations in R_0 and e_r.

Comparison of $\Omega_{m,n}$ for ***C-C*** edge condition for isotropic annular plates of uniform thickness with Chen and Ren [27], Vogels [28] and Kim and Dickinson [24] is given in Table 2 when $\nu_r = 0.3$ and for isotropic annular plates of linearly varying thickness with Chen and Ren [27], Kim and Dickinson [24], Conway's exact solution [22] and Sankarnarayanan et al. [29] for clamped periphery is given in Table 3 when $\alpha = 1.0$, $\beta = 0.0$, $e_r = 1.0$, $g_r = 0.384$, $K_f = 0$and $\nu_r = 1/3$ and 0.3.

Table 5
Comparison of $\Omega_{m,n}$ for polar orthotropic annular plates of linearly varying thickness with clamped Peripheries when $\beta = 0, e_r = 50$, $g_r = 0.6652,\ \nu_r = 0.0052, K_f = 0$

$R_0,\ \alpha$	No. of terms	$\Omega_{0,0}$	$\Omega_{1,0}$	$\Omega_{0,1}$	$\Omega_{2,0}$	$\Omega_{1,1}$	$\Omega_{0,2}$	$\Omega_{2,1}$
0.1, 1.0	(**Present**) 6 terms	30.667	38.942	65.577	70.108	72.893	107.40	108.21
	[24] 8 terms	30.667	38.942	65.577	70.108	72.893	107.40	108.21
	[27]	30.42588	38.85691	64.98246	70.10823	72.52273	–	108.20416
		$\Omega_{0,0}$	$\Omega_{1,0}$	$\Omega_{2,0}$	$\Omega_{3,0}$	$\Omega_{4,0}$	$\Omega_{0,1}$	$\Omega_{1,1}$
0.5, 1.0	(**Present**) 6 terms	82.268	84.400	96.623	129.55	186.36	204.52	206.45
	[24] 8 terms	82.268	84.400	96.623	129.55	186.36	204.51	206.45
	[27]	82.26839	84.39951	96.62299	–	–	204.51273	206.45072
		$\Omega_{1,0}$	$\Omega_{0,0}$	$\Omega_{2,0}$	$\Omega_{2,0}$	$\Omega_{3,0}$	$\Omega_{4,0}$	$\Omega_{1,1}$
0.5, −2.0	(**Present**) 6 terms	160.84	163.42	166.09	208.91	295.71	405.66	406.38
	[24] 8 terms	160.84	163.42	166.09	208.91	295.71	405.66	406.38



(a) Annular geometry parameter R_0

(b) Material polar orthotropy parameter e_r

Fig. 3. Meaning of annular geometry parameter R_0 and material polar orthotropy e_r.

Table 4 shows the comparison of $\Omega_{m,n}$ with results of Gorman [15], Narita [10] and Kim and Dickinson [24] for polar orthotropic annular plates of uniform thickness with simply-supported peripheries when $e_r = 5.0$, $g_r = 0.356$, $\nu_r = 0.06$, $R_0 = 0.5, K_f = 0$. Comparison of $\Omega_{m,n}$ with Kim and Dickinson [24] and Chen and Ren [27] for polar orthotropic annular plates of linearly varying thickness with clamped peripheries is given in Table 5 when $\beta = 0.0, e_r = 50, g_r = 0.6652, v_r = 0.0052$ and $K_f = 0$.

From the computational results of convergence checking (Table 1), the main advantage of Rayleigh-Ritz method in monitoring rate of convergence through comparison of consecutive approximations is clearly seen. Increasing the order of approximation can increase accuracy of the result and the process can be terminated when the required number of frequencies has converged to the desired accuracy. Comparisons with available results (Tables 2 to 5) confirm that Rayleigh-Ritz method possesses faster rate of convergence than other methods such as Frobenious method, Chebyshev Collocation method, Spline method, differential quadrature method etc. Besides close agreement with the results compared, results obtained using the Rayleigh-Ritz formulation with the use of boundary characteristic orthonormal polynomials are found to be better even with lesser number of terms in almost all the cases. From the analysis, it is found that in the generation of boundary characteristic orthonormal polynomials, there is a loss of accuracy unless the precision is increased when proceeding to higher order polynomials. If precision is not increased, the results tend to show convergence up to a certain degree and subsequent divergence thereafter due to accumulation of rounding off errors.

Table 6
Variation in $\Omega^*_{m,n}$ with e_r for C-C, S-S and F-F plates when $\alpha = \beta = 0.4, g_r = 5.0, \mathrm{R}_0 = 0.5, K_f = 500$

Edge conditions (inner, outer)	e_r	$\Omega^*_{0,0}$	$\Omega^*_{1,0}$	$\Omega^*_{2,0}$	$\Omega^{*0}_{3,0}$	$\Omega^*_{4,0}$	$\Omega^*_{5,0}$	$\Omega^*_{0,1}$	$\Omega^*_{1,1}$	$\Omega^*_{6,0}$	$\Omega^*_{2,1}$
C-C	0.25	80.927	89.063	109.49	136.19	165.82	197.01	215.08	226.94	229.19	258.97
	0.5	81.932	90.012	110.48	137.35	167.36	199.17	217.68	229.50	232.27	261.48
	1.0	84.053	92.108	112.59	139.81	170.61	203.74	223.17	234.91	238.74	266.79
	2.0	88.796	96.751	117.34	145.38	177.96	213.98	235.46	247.04	253.15	278.76
	4.0	101.07	108.88	129.89	160.14	197.33	240.68	267.33	278.66	290.11	310.27
	8.0	154.50	162.66	187.14	227.97	285.18	358.64	406.60	418.36	448.01	452.72
		$\Omega^*_{0,0}$	$\Omega^*_{1,0}$	$\Omega^*_{2,0}$	$\Omega^*_{3,0}$	$\Omega^*_{4,0}$	$\Omega^*_{0,1}$	$\Omega^*_{1,1}$	$\Omega^*_{5,0}$	$\Omega^*_{2,1}$	$\Omega^*_{6,0}$
S-S	0.25	41.879	53.337	78.120	107.38	138.42	140.47	155.75	170.45	194.21	203.17
	0.5	42.517	53.935	78.767	108.25	139.73	142.21	157.45	172.44	195.90	206.13
	1.0	43.842	55.186	80.133	110.10	142.50	145.87	161.03	176.64	199.47	212.36
	2.0	46.734	57.948	83.200	114.28	148.73	154.03	169.03	186.03	207.52	226.16
	4.0	53.820	64.945	91.223	125.33	165.15	174.99	189.79	210.41	228.69	261.29
	8.0	82.458	94.693	127.46	176.20	239.52	265.44	317.22	281.13	324.99	409.16
		$\Omega^*_{2,0}$	$\Omega^*_{3,0}$	$\Omega^*_{0,1}$	$\Omega^*_{4,0}$	$\Omega^*_{5,0}$	$\Omega^*_{6,0}$	$\Omega^*_{7,0}$	$\Omega^*_{1,1}$	$\Omega^*_{2,1}$	$\Omega^*_{3,1}$
F-F	0.25	23.768	24.363	25.737	26.022	29.313	34.578	41.880	47.057	80.639	115.67
	0.5	24.138	25.307	26.280	28.458	34.338	43.130	54.667	47.511	81.138	116.47
	1.0	24.904	27.179	27.422	32.976	42.975	56.959	74.505	48.459	82.195	118.15
	2.0	26.561	30.915	29.875	41.191	57.519	79.129	105.39	50.526	84.571	121.92
	4.0	30.585	38.727	35.478	56.523	82.956	116.62	156.69	56.601	90.761	131.70
	8.0	46.127	60.249	54.416	91.498	137.44	195.78	265.47	74.434	116.53	173.42

Table 7
Variation in $\Omega_{m,n}$ with K_f for C-C, S-S and F-F plates when $\alpha = \beta = 0.4, e_r = g_r = 5.0, R_0 = 0.5$

Edge conditions (inner, outer)	K_f	$\Omega_{0,0}$	$\Omega_{1,0}$	$\Omega_{2,0}$	$\Omega_{3,0}$	$\Omega_{4,0}$	$\Omega_{5,0}$	$\Omega_{0,1}$	$\Omega_{1,1}$	$\Omega_{6,0}$	$\Omega_{2,1}$
C-C	0	77.434	83.452	99.869	124.00	154.32	190.40	212.82	221.23	232.20	244.90
	200	78.914	84.826	101.02	124.92	155.06	191.00	213.37	221.76	232.69	245.38
	400	80.367	86.178	102.15	125.84	155.80	191.59	213.92	222.28	233.18	245.85
	600	81.793	87.509	103.28	126.75	156.54	192.19	214.46	222.80	233.67	246.32
	800	83.195	88.820	104.39	127.66	157.27	192.79	215.00	223.33	234.16	246.79
	1000	84.574	90.112	105.49	128.56	158.00	193.38	215.54	223.85	234.65	247.26
		$\Omega_{0,0}$	$\Omega_{1,0}$	$\Omega_{2,0}$	$\Omega_{3,0}$	$\Omega_{4,0}$	$\Omega_{0,1}$	$\Omega_{1,1}$	$\Omega_{5,0}$	$\Omega_{2,1}$	$\Omega_{6,0}$
S-S	0	36.031	45.672	67.582	95.643	128.61	138.08	149.20	166.54	178.72	209.70
	200	39.108	48.132	69.261	96.832	129.49	138.92	149.98	167.22	179.38	210.24
	400	41.960	50.472	70.901	98.007	130.37	139.76	150.76	167.90	180.03	210.77
	600	44.630	52.708	72.503	99.168	131.24	140.60	151.53	168.58	180.67	211.31
	800	47.149	54.853	74.071	100.32	132.11	141.43	152.30	169.25	181.32	211.84
	1000	49.539	56.917	75.606	101.45	132.97	142.25	153.07	169.92	181.96	212.38
		$\Omega_{2,0}$	$\Omega_{0,1}$	$\Omega_{3,0}$	$\Omega_{1,1}$	$\Omega_{4,0}$	$\Omega_{2,1}$	$\Omega_{5,0}$	$\Omega_{0,2}$	$\Omega_{3,1}$	$\Omega_{1,2}$
F-F	0	7.8404	14.545	21.887	35.897	41.690	65.932	66.916	83.038	99.550	103.94
	200	16.692	21.352	26.359	39.168	44.188	67.752	68.491	84.458	100.76	105.08
	400	22.262	26.532	30.175	42.191	46.552	69.524	70.031	85.854	101.96	106.22
	600	26.692	30.893	33.559	45.015	48.801	71.253	71.537	87.229	103.14	107.34
	800	30.483	34.734	36.631	47.676	50.950	72.941	73.012	88.584	104.31	108.46
	1000	33.849	38.206	39.464	50.200	53.012	74.592	74.452	89.918	105.46	109.56

Variation in $\Omega^*_{m,n}$ with increasing e_r for ***C*-*C*, *S*-*S* and *F*-*F* plates** on elastic foundation is given in Table 6 when $\alpha = \beta = 0.4$, $g_r = 5.0$, $\mathrm{R}_0 = 0.5$ and $K_f = 500$. To show the full effect of e_r on frequencies, variation of $\Omega^*_{m,n}$ instead of $\Omega_{m,n}$ are taken in Table 6 because the parameter $\Omega_{m,n}$ contains e_r whereas $\Omega^*_{m,n}$ is free from e_r as can be seen from Eq. (3). It can be seen in Table 6 that all frequencies increase monotonically with increasing e_r. This is due to the fact that, stiffness of the plate increases with the increase in e_r. The rate of increase is however higher when $e_r > 1$. Since $e_r = E_\theta/E_r$, the results show that for the particular combination of parameters used, higher elastic property in circumferential direction tends to produce higher stiffness with respect to lateral vibration

Table 8
Variation in $\Omega_{m,n}$ with R_0 for C-C, S-S and F-F plates when $\alpha = \beta = 0.5, e_r = g_r = 5.0$ and $K_f = 500$

Edge conditions (inner, outer)	R_0	$\Omega_{0,0}$	$\Omega_{1,0}$	$\Omega_{0,1}$	$\Omega_{2,0}$	$\Omega_{1,1}$	$\Omega_{3,0}$	$\Omega_{2,1}$	$\Omega_{4,0}$	$\Omega_{3,1}$	$\Omega_{5,0}$
C-C	0.1	33.692	39.908	53.800	56.623	67.672	81.931	98.877	114.53	140.02	153.55
	0.2	35.991	42.205	68.718	58.608	80.362	83.196	108.76	115.16	147.18	153.81
	0.3	42.123	48.248	93.935	64.363	104.06	88.078	130.26	118.75	166.70	156.14
	0.4	54.425	60.349	138.85	76.187	143.83	99.348	168.20	128.86	203.40	164.60
	0.5	77.739	83.354	204.60	98.798	212.66	121.70	235.37	150.69	269.56	185.33
	0.6	123.72	128.97	335.38	143.84	342.68	166.55	363.80	195.59	396.78	230.12
		$\Omega_{0,0}$	$\Omega_{1,0}$	$\Omega_{0,1}$	$\Omega_{2,0}$	$\Omega_{1,1}$	$\Omega_{3,0}$	$\Omega_{2,1}$	$\Omega_{4,0}$	$\Omega_{3,1}$	$\Omega_{5,0}$
S-S	0.1	29.751	35.095	45.126	50.562	58.900	74.761	89.219	106.29	129.33	144.24
	0.2	29.875	35.644	51.885	51.381	65.001	75.359	94.632	106.59	133.42	144.37
	0.3	30.994	37.434	66.038	53.922	78.278	77.849	107.43	108.51	145.69	145.62
	0.4	34.357	41.558	90.840	59.283	102.18	83.796	130.82	114.23	169.48	150.54
	0.5	42.316	50.195	134.50	69.488	144.94	95.450	172.85	126.65	212.25	162.94
	0.6	60.348	68.620	217.50	89.518	227.12	117.83	254.00	151.26	293.96	189.20
		$\Omega_{2,0}$	$\Omega_{3,0}$	$\Omega_{0,1}$	$\Omega_{1,1}$	$\Omega_{0,2}$	$\Omega_{4,0}$	$\Omega_{2,1}$	$\Omega_{1,2}$	$\Omega_{5,0}$	$\Omega_{3,1}$
F-F	0.1	25.039	32.359	32.471	40.771	47.470	47.780	62.489	65.370	78.220	93.751
	0.2	25.030	32.345	32.138	40.697	46.002	47.771	62.395	66.153	70.213	93.522
	0.3	24.995	32.269	31.098	40.672	49.771	47.694	62.574	70.335	70.162	93.124
	0.4	24.903	32.040	29.912	41.115	61.022	47.388	64.065	74.127	69.878	94.121
	0.5	24.724	31.558	29.244	42.669	83.740	46.602	68.174	80.843	68.952	99.052
	0.6	24.446	30.756	29.561	46.342	129.276	45.104	76.683	146.82	66.868	110.99

of annular plate on elastic foundation.

Variation in $\Omega_{m,n}$ with K_f for ***C*-*C*, *S*-*S*** and ***F*-*F* plates** is given in Table 7 when $\alpha = \beta = 0.4$, $e_r = g_r = 5.0$ and $R_0 = 0.5$. It is observed that all frequencies increase monotonically with increasing K_f and the rate of increase falls in higher modes. This is due to the fact that, lateral stiffness of the annular plate-foundation combined increases with the increase in K_f. Comparison in terms of mode shapes shown in Tables 6 and 7 show that, there is no change in the first 10 mode shapes for cases of ***C*-*C*** and ***S*-*S* plates**. However for the case of ***F*-*F* plates**, other than the fundamental mode shape, all higher mode shapes are not coincident. This indicates that lateral vibration characteristics of ***F*-*F* plates** is more sensitive to parametric changes in material orthotropy and foundation stiffness.

Variation in frequencies with inner radius R_0 for ***C*-*C*, *S*-*S*** and ***F*-*F* plates** is given in Table 8 when $\alpha = \beta = 0.5$, $e_r = g_r = 5.0$ and $K_f = 500$. It is observed that as R_0 increases, all frequencies increase for ***C*-*C*** and ***S*-*S* plates**. For ***F*-*F* plate**, frequencies $\Omega_{1,1}$, $\Omega_{0,2}$, $\Omega_{2,1}$, $\Omega_{0,1}$ and $\Omega_{3,1}$ first decrease and then increase; $\Omega_{1,2}$ increases and $\Omega_{2,0}$, $\Omega_{3,0}$, $\Omega_{4,0}$ and $\Omega_{5,0}$ decrease. Closer examination of the results shown in Table 8 shows that change in width of annular plates has no significant influence on the fundamental frequencies of ***F*-*F* plates**. The effect of increasing R_0 on fundamental frequencies of ***C*-*C*** and ***S*-*S* plates** is more pronounce. This later fact is especially true for R_0 greater than 0.3. As R_0 increases, width of annular plate becomes narrower physically (refer Fig. 3(a)). Under such circumstance, boundary condition of plates tends to exert higher influence on lateral stiffness resulting in stiffer response to lateral vibration.

Variation in $\Omega_{m,n}$ with taper parameters α and β for ***C*-*C*, *S*-*S*** and ***F*-*F* plates** are given in Tables 9 to 11 when $e_r = g_r = 5.0$, $R_0 = 0.5$ and $K_f = 500$. It is observed that all $\Omega_{m,n}$ decrease continuously for ***C*-*C*, *S*-*S*** and ***F*-*F* plates**. However, close examination reveals that effect of thickness variation on fundamental frequencies of ***F*-*F* plates** is practically zero. Although the effect seems to be more pronounce for higher modes, nevertheless it is deemed to be not significant. Comparing to the case of ***F*-*F* plates**, effect of thickness variation on fundamental frequencies is more significant in the case of ***C*-*C* plates** than ***S*-*S* plates**. For both cases of ***C*-*C*** and ***S*-*S*** plates, thickness profile which is convex relative to plate center-line (both $\alpha < 0$ and $\beta < 0$, see Fig. 2(b)) tends to result in higher stiffness of annular plates against lateral vibration than the profile which is concave (both $\alpha > 0$ and $\beta > 0$, see Fig. 2(a)). The same observation applies to the case of ***F*-*F* plates** although the effect is very small relative to ***C*-*C*** and ***S*-*S* plates**.

Nodal lines and their corresponding three dimensional mode shapes for the first ten normal modes of vibration for all three combinations are shown in Fig. 4 to 9 when $\alpha = \beta = 0.4$, $e_r = g_r = 5.0$, $R_0 = 0.5$ and $K_f = 500$.

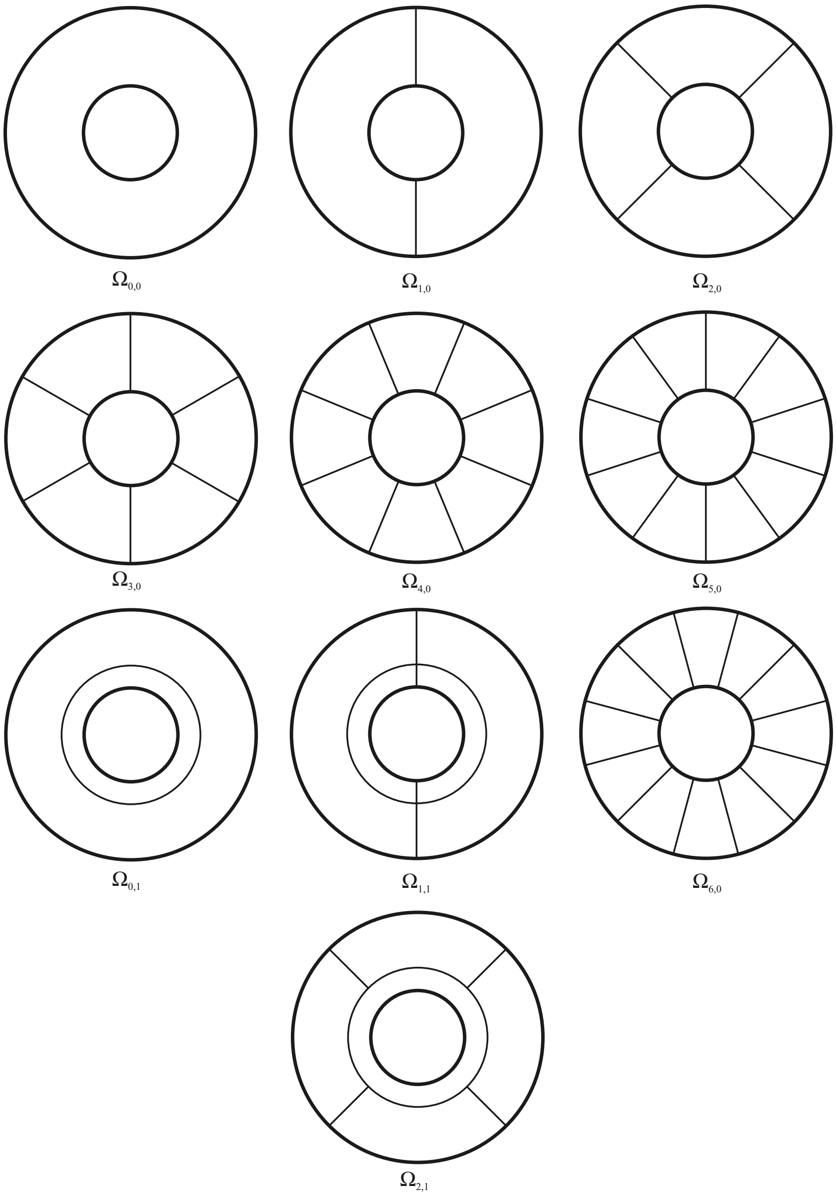


Fig. 4. First ten nodal lines of C-C plates when $\alpha = \beta = 0.4$, $e_r = g_r = 5.0$, $R = 0.5$, $K_f = 500$.

It can be seen from Figs 4, 5, 6 and 7 that the fundamental mode of vibration for both ***C*-*C*** and ***S*-*S* plates** are axisymmetrical mode, $\Omega_{0,0}$. Vibration modes for both cases are similar up until the 5^{th} mode. The case of ***F*-*F* plates** however exhibits fundamental mode of vibration which is asymmetry, $\Omega_{2,0}$.

4. Conclusion

Rayleigh- Ritz method with orthonormally generated boundary characteristic polynomials has been used to determine natural frequencies and mode shapes of annular circular plates resting on elastic foundation with ***C*-*C***, ***S*-*S*** and ***F*-*F*** edge conditions. Comparisons with available results have shown that the above formulation possesses faster rate of convergence. With regards vibration characteristics, it is found that:

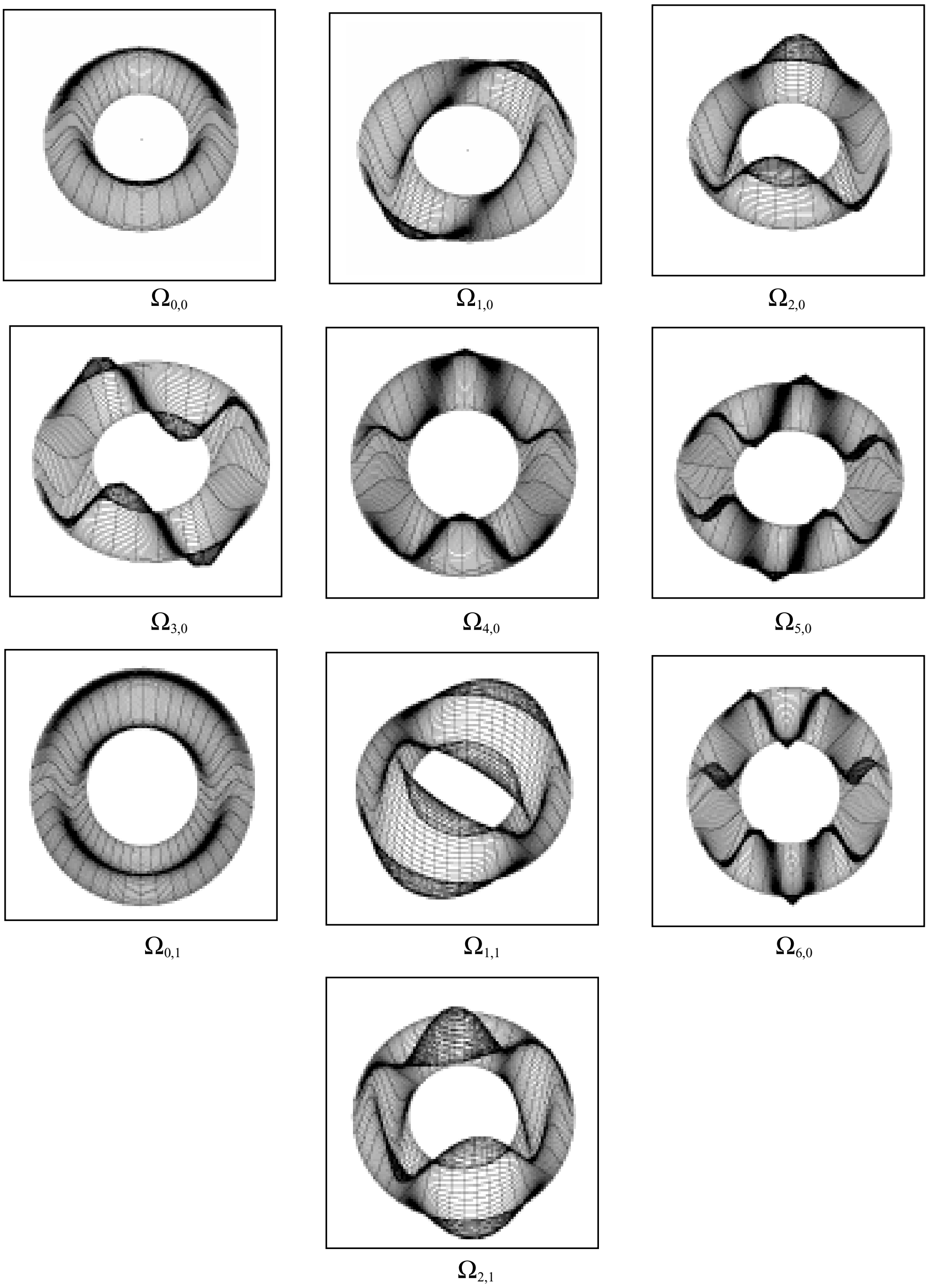


Fig. 5. Three dimensional plots for first ten normal modes of vibration for C-C plate when $\alpha = \beta = 0.4$, $e_r = g_r = 5.0$, $R_0 = 0.5$, $K_f = 500$.

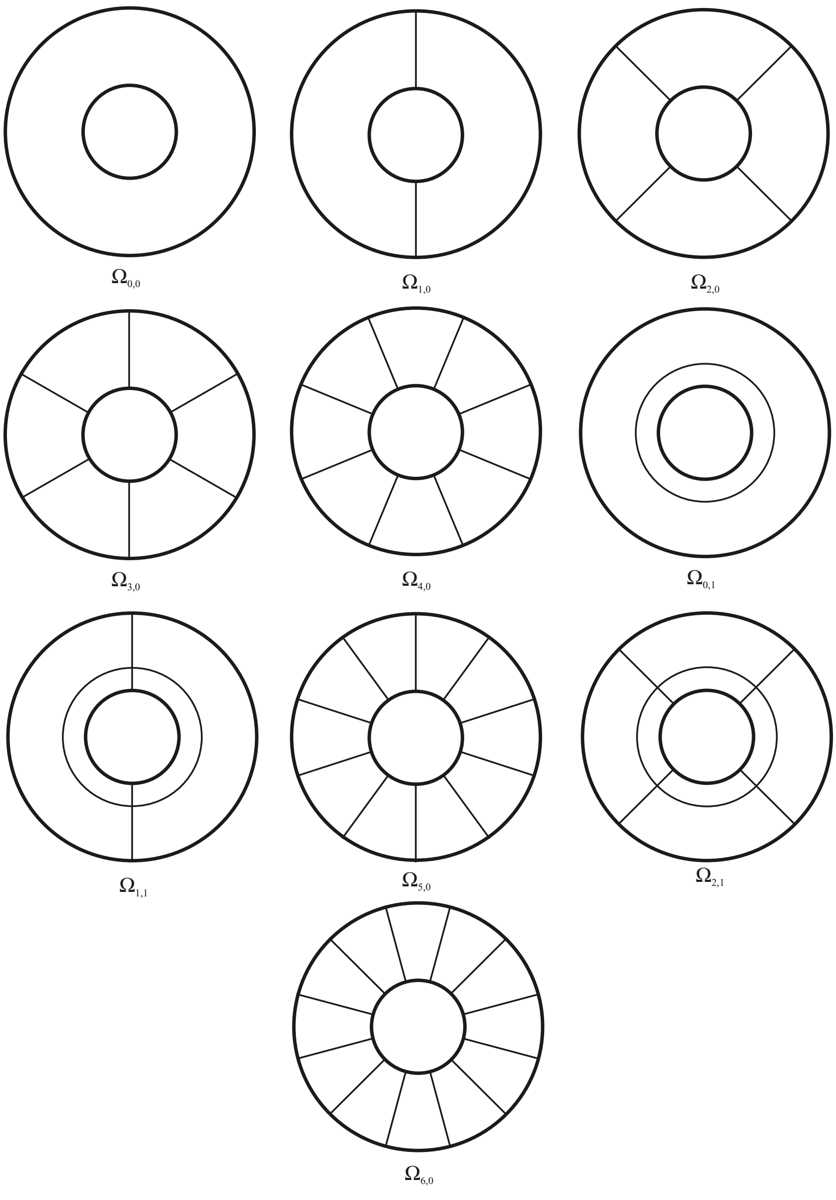


Fig. 6. For first ten nodal lines of S-S plates when $\alpha = \beta = 0.4$, $e_r = g_r = 5.0$, $R = 0.5$, $K_f = 500$.

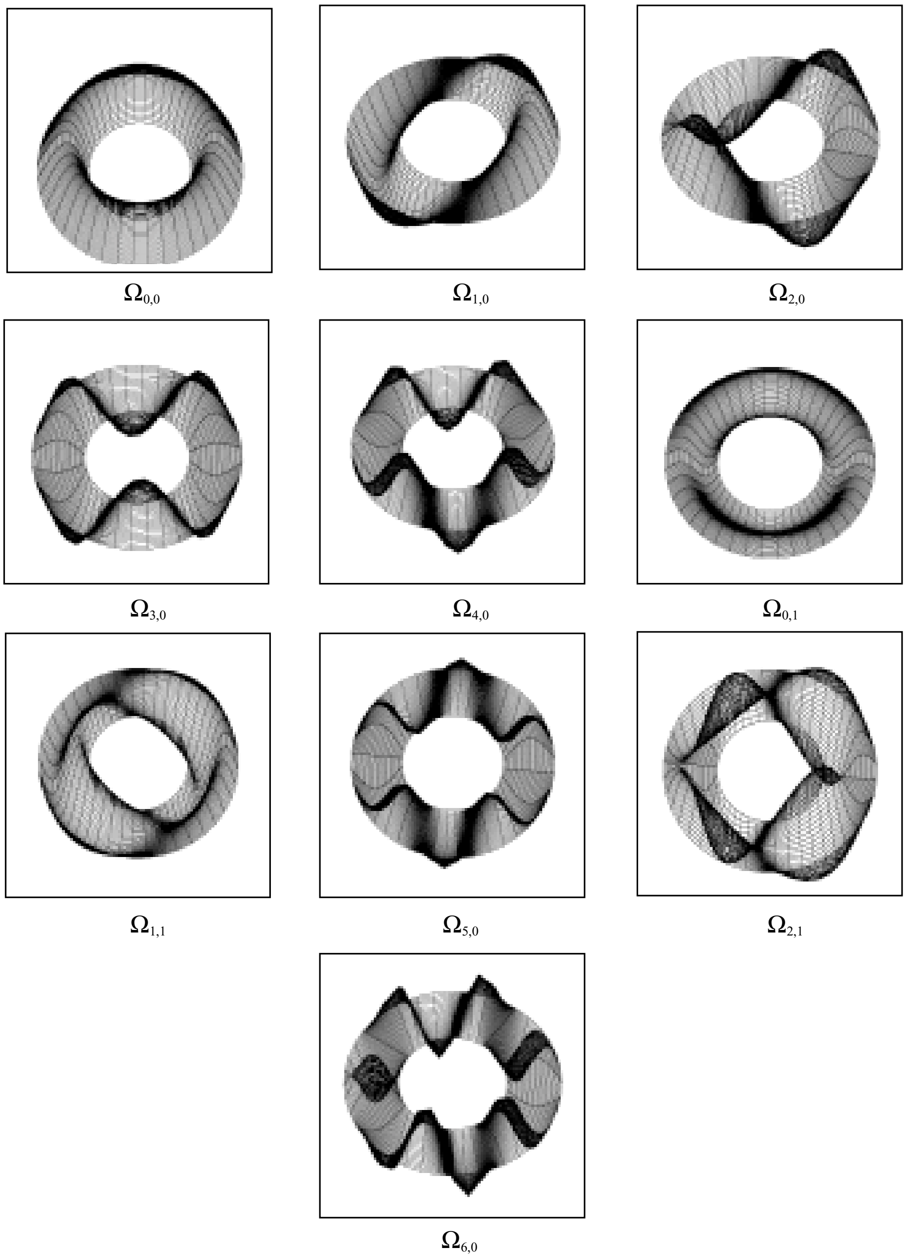


Fig. 7. Three dimensional plots for first ten normal modes of vibration for S-S plate when $\alpha = \beta = 0.4$, $e_r = g_r = 5.0$, $R_0 = 0.5$, $K_f = 500$.

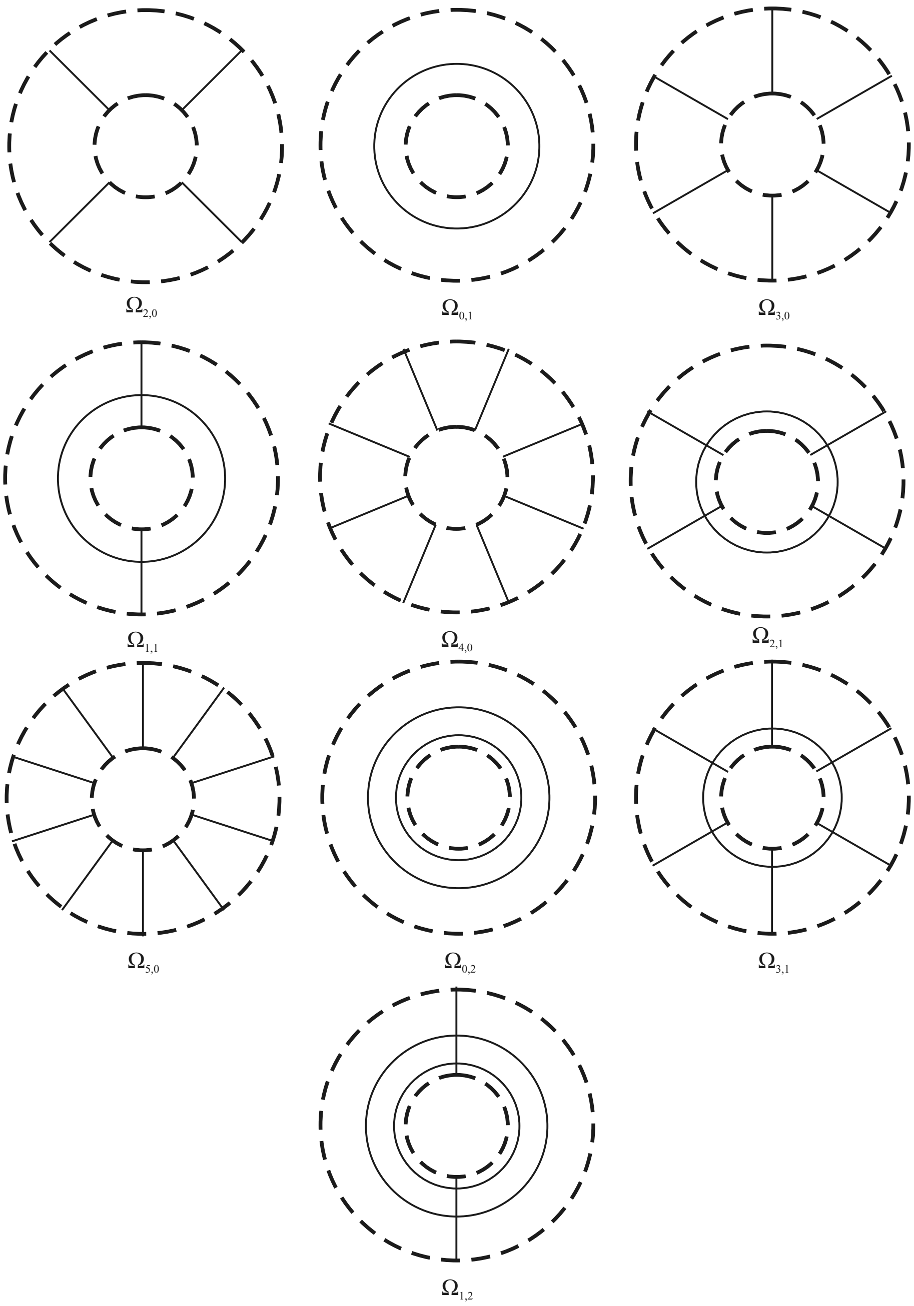


Fig. 8. First ten nodal lines of F-F plates when $\alpha = \beta = 0.4$, $e_r = g_r = 5.0$, $R = 0.5$, $K_f = 500$.

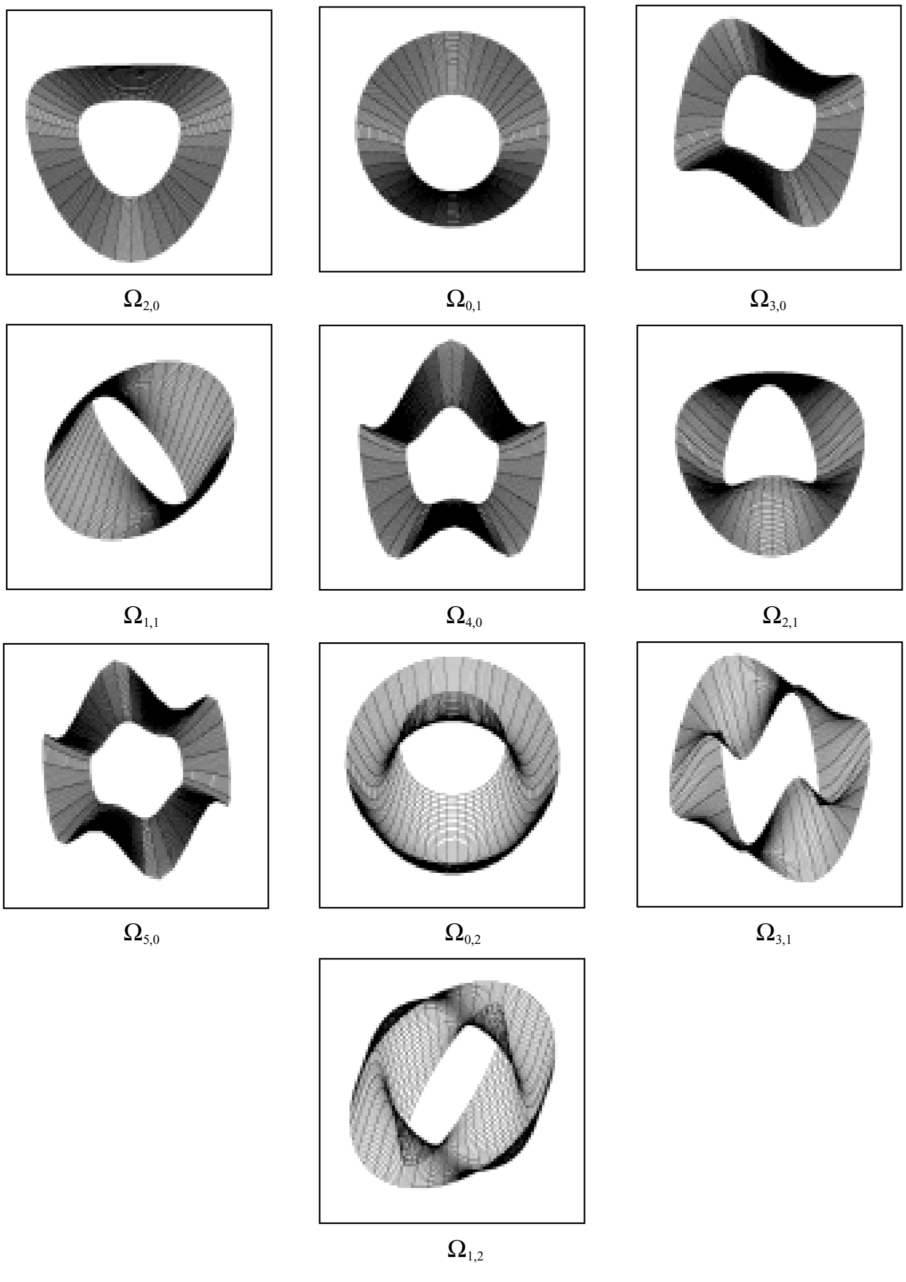


Fig. 9. Three dimensional plots for first ten normal modes of vibration for F-F plate when $\alpha = \beta = 0.4$, $e_r = g_r = 5.0$, $R_0 = 0.5$, $K_f = 500$.

Table 9
Variation in $\Omega_{m,n}$ with α and β for C-C plate when $e_r = g_r = 5.0$, $R_0 = 0.5$ and $K_f = 500$

α	β	$\Omega_{0,0}$	$\Omega_{1,0}$	$\Omega_{2,0}$	$\Omega_{3,0}$	$\Omega_{4,0}$	$\Omega_{5,0}$	$\Omega_{0,1}$	$\Omega_{1,1}$	$\Omega_{2,1}$	$\Omega_{3,1}$
−0.4	−0.4	106.36	113.18	132.12	160.49	196.61	239.84	283.52	294.00	323.55	368.14
	−0.2	104.56	111.34	130.15	158.30	194.14	237.03	279.14	289.52	318.79	362.95
	0.0	102.74	109.48	128.16	156.10	191.64	234.19	274.69	285.10	313.97	357.69
	0.2	100.92	107.61	126.16	153.88	189.13	231.33	270.19	280.38	309.09	352.36
	0.4	99.070	105.73	124.14	151.63	186.59	228.43	265.63	275.71	304.13	346.96
−0.2	−0.4	102.07	108.69	127.03	154.50	189.45	231.28	271.60	281.68	310.10	352.49
	−0.2	100.25	106.82	125.03	152.27	186.93	228.42	267.11	277.09	305.22	347.67
	0.0	98.404	104.93	123.01	150.03	184.39	225.52	262.56	272.43	300.28	342.27
	0.2	96.544	103.03	120.97	147.76	181.82	222.58	257.93	267.71	295.26	336.78
	0.4	94.666	101.11	118.92	145.47	179.22	219.61	253.24	262.91	290.56	331.22
0.0	−0.4	97.401	104.14	121.88	148.42	182.18	222.59	259.47	269.14	296.40	337.56
	−0.2	95.884	102.24	119.84	146.15	179.61	219.65	254.86	264.42	291.39	332.08
	0.0	94.010	100.32	117.79	143.86	177.01	216.67	250.86	259.63	286.30	326.52
	0.2	92.117	98.385	115.71	141.55	174.38	213.66	245.42	254.77	281.13	320.87
	0.4	90.203	96.428	113.61	139.21	171.71	210.60	240.57	249.81	275.86	315.11
0.2	−0.4	93.358	99.538	116.66	142.25	174.79	213.72	247.10	256.34	282.43	321.81
	−0.2	91.469	97.604	114.58	139.93	172.15	210.70	242.35	251.48	277.26	316.17
	0.0	89.560	95.650	112.48	137.59	169.48	207.64	237.52	246.54	272.01	310.42
	0.2	87.630	93.674	110.36	135.22	166.78	204.53	232.59	241.51	266.65	304.56
	0.4	85.677	91.675	108.21	132.82	164.07	201.38	227.57	236.38	261.19	298.58
0.4	−0.4	88.922	94.876	111.36	135.97	167.25	204.67	234.44	243.26	268.14	305.71
	−0.2	86.997	92.904	109.24	133.60	164.54	201.55	229.53	238.24	262.79	299.86
	0.0	85.050	90.909	107.09	131.20	161.80	198.39	224.53	233.12	257.34	293.89
	0.2	83.079	88.891	104.92	128.76	159.01	195.17	219.42	227.89	251.77	287.79
	0.4	81.083	86.846	102.72	126.30	156.17	191.89	214.19	222.54	246.08	281.55

Table 10
Variation in $\Omega_{m,n}$ with α and β for S-S plate when $e_r = g_r = 5.0$, $R_0 = 0.5$ and $K_f = 500$

α	β	$\Omega_{0,0}$	$\Omega_{1,0}$	$\Omega_{2,0}$	$\Omega_{3,0}$	$\Omega_{4,0}$	$\Omega_{0,1}$	$\Omega_{1,1}$	$\Omega_{5,0}$	$\Omega_{2,1}$	$\Omega_{6,0}$
−0.4	−0.4	51.711	63.558	89.969	123.35	162.51	182.40	197.27	207.60	236.18	258.81
	−0.2	51.218	62.769	88.736	121.74	160.56	179.99	194.60	205.29	232.88	256.14
	0.0	50.722	61.982	87.506	120.13	158.59	177.55	191.89	202.96	229.56	253.44
	0.2	50.222	61.197	86.279	118.52	156.61	175.08	189.16	200.61	226.20	250.70
	0.4	49.720	60.415	85.055	116.90	154.62	172.58	186.39	198.24	222.80	247.94
−0.2	−0.4	50.073	61.277	86.582	118.83	156.77	174.73	174.73	200.49	188.91	250.18
	−0.2	49.578	60.493	85.352	117.22	154.79	172.27	172.27	198.14	186.19	247.44
	0.0	49.081	59.712	84.126	115.60	152.80	169.78	169.78	195.76	183.43	244.68
	0.2	48.580	58.933	82.903	113.98	150.79	167.25	167.25	193.37	180.63	241.88
	0.4	48.076	58.157	81.685	112.35	148.77	164.69	164.69	190.94	177.80	239.04
0.0	−0.4	48.415	59.017	83.204	114.30	150.98	166.96	180.45	193.29	215.92	241.40
	−0.2	47.957	58.240	81.980	112.67	148.97	164.45	177.67	190.89	212.49	238.60
	0.0	47.460	57.467	80.759	111.04	146.95	161.89	174.84	188.46	209.02	235.75
	0.2	46.959	56.696	79.543	109.42	144.91	159.30	171.98	186.01	205.51	232.87
	0.4	46.455	55.929	78.331	107.78	142.86	156.67	169.08	183.53	201.95	229.94
0.2	−0.4	46.853	56.786	79.843	109.74	145.13	159.09	171.87	185.97	205.59	232.46
	−0.2	46.360	56.019	78.625	108.11	143.08	156.51	169.02	183.52	202.08	229.57
	0.0	45.865	55.255	77.412	106.48	141.04	153.89	166.13	181.03	198.52	226.64
	0.2	45.366	54.495	76.204	104.84	138.97	151.22	163.19	178.52	194.92	223.66
	0.4	44.864	53.739	75.002	103.19	136.89	148.51	160.20	175.97	191.26	220.63
0.4	−0.4	45.287	54.594	76.503	105.18	139.21	151.09	163.16	178.53	195.09	223.32
	−0.2	44.798	53.839	75.295	103.54	137.13	148.44	160.24	176.01	191.45	220.33
	0.0	44.307	53.089	74.093	101.89	135.04	145.74	157.26	173.46	187.84	217.29
	0.2	43.813	52.343	72.092	100.24	132.94	142.99	154.23	170.87	184.12	214.20
	0.4	43.316	51.602	71.706	98.589	130.81	140.18	151.14	168.24	180.35	211.04

Table 11
Variation in $\Omega_{m,n}$ with α and β for F-F plate when $e_r = g_r = 5.0$, $R_0 = 0.5$ and $K_f = 500$

α	β	$\Omega_{2,0}$	$\Omega_{0,1}$	$\Omega_{3,0}$	$\Omega_{1,1}$	$\Omega_{4,0}$	$\Omega_{5,0}$	$\Omega_{2,1}$	$\Omega_{6,0}$	$\Omega_{3,1}$	$\Omega_{7,0}$
−0.4	−0.4	24.059	30.190	36.178	54.988	57.946	86.761	89.081	121.20	129.56	160.66
	−0.2	24.035	29.912	35.828	54.164	57.256	85.797	88.044	120.03	128.17	159.33
	0.0	24.016	29.657	35.488	53.347	56.574	84.838	87.001	118.86	126.76	158.00
	0.2	24.002	29.426	35.157	52.539	55.901	83.885	85.952	117.69	125.34	156.67
	0.4	23.994	29.221	34.836	51.742	55.238	82.938	84.896	116.52	123.90	155.34
−0.2	−0.4	24.093	29.510	35.228	52.572	55.837	83.547	85.386	116.95	124.37	155.36
	−0.2	24.082	29.284	34.902	51.776	55.169	82.599	84.347	115.79	122.95	154.11
	0.0	24.077	29.085	34.586	50.992	54.510	81.657	83.303	114.63	121.52	152.78
	0.2	24.076	28.915	34.280	50.220	53.863	80.722	82.252	113.47	120.08	151.45
	0.4	24.080	28.779	33.985	49.464	53.225	79.794	81.196	112.31	118.62	150.12
0.0	−0.4	24.165	28.954	34.345	50.246	53.792	80.386	81.717	112.74	119.15	150.24
	−0.2	24.168	28.792	34.044	49.488	53.149	79.457	80.679	111.59	117.71	148.91
	0.0	24.175	28.662	33.755	48.746	52.518	78.535	79.635	110.44	116.26	147.59
	0.2	24.188	28.570	33.476	48.022	51.898	77.621	78.586	109.29	114.78	146.26
	0.4	24.205	28.520	33.210	47.320	51.290	76.715	77.533	108.15	113.30	144.92
0.2	−0.4	24.277	28.553	33.534	48.036	51.820	77.285	78.083	108.58	113.91	145.07
	−0.2	24.293	28.470	33.262	47.327	51.206	76.378	77.048	107.44	112.44	143.74
	0.0	24.314	28.430	33.001	46.641	50.605	75.478	76.009	106.30	110.96	142.42
	0.2	24.967	28.439	32.753	45.982	50.158	74.588	74.967	105.17	109.46	141.09
	0.4	24.370	28.501	32.516	45.354	49.440	73.709	73.924	104.04	107.94	139.76
0.4	−0.4	24.429	28.348	32.804	45.971	49.931	74.253	74.495	104.47	108.63	139.92
	−0.2	24.458	28.370	32.562	45.330	49.349	73.370	73.468	103.34	107.14	138.60
	0.0	24.493	28.448	32.332	44.720	48.781	72.498	72.441	102.22	105.63	137.28
	0.2	24.533	28.586	32.116	44.150	48.228	71.637	71.415	101.10	104.10	135.95
	0.4	24.578	28.545	31.912	43.625	47.689	70.788	70.394	99.987	102.55	134.62

(a) higher elastic property in circumferential direction tends to produce higher stiffness with respect to lateral vibration of annular plate on elastic foundation for all three cases of $\boldsymbol{C}$-$\boldsymbol{C}$, $\boldsymbol{S}$-$\boldsymbol{S}$ and $\boldsymbol{F}$-$\boldsymbol{F}$ **plates**;
(b) Lateral vibration characteristics of $\boldsymbol{F}$-$\boldsymbol{F}$ **plates** is more sensitive towards parametric changes in material orthotropy and foundation stiffness than $\boldsymbol{C}$-$\boldsymbol{C}$ and $\boldsymbol{S}$-$\boldsymbol{S}$ **plates**;
(c) Effect of quadratical thickness variation on fundamental frequencies is more significant in cases of $\boldsymbol{C}$-$\boldsymbol{C}$ and $\boldsymbol{S}$-$\boldsymbol{S}$ **plates** than that of $\boldsymbol{F}$-$\boldsymbol{F}$ **plates**. Thickness profile which is convex relative to plate center-line tends to result in higher stiffness of annular plates against lateral vibration than the one which is concave.
(d) Fundamental mode of vibration of $\boldsymbol{C}$-$\boldsymbol{C}$ and $\boldsymbol{S}$-$\boldsymbol{S}$ **plates** is axisymmetrical while that of $\boldsymbol{F}$-$\boldsymbol{F}$ **plates** is asymmetrical.

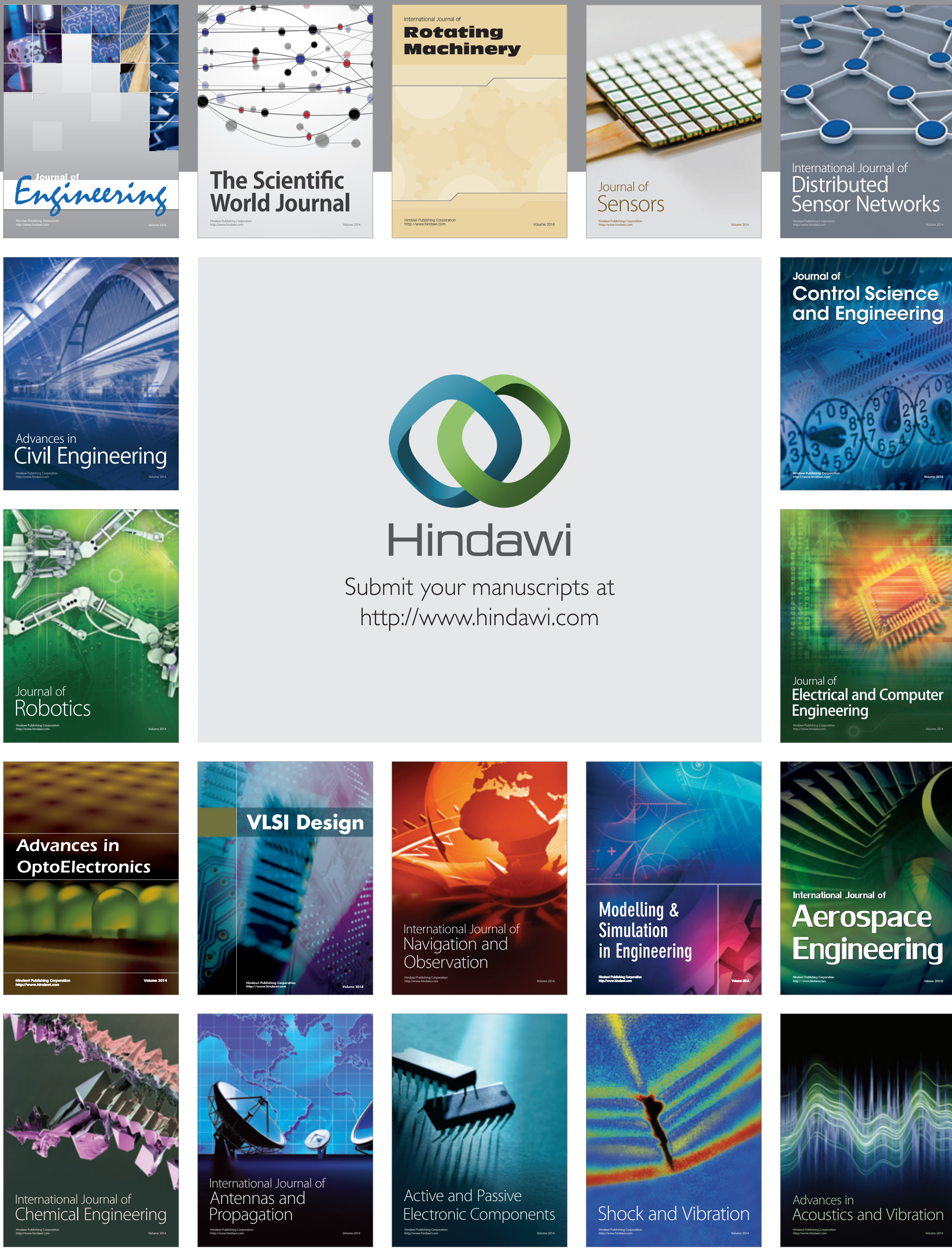
Acknowledgements

This paper is acknowledged to Ms. Garima Dalal (Lecturer, Applied Science and Humanities Department, Institute of Technology and Management, Sector 23-A, Gurgaon, Haryana).

References

[1] A.W. Leissa, Vibration of plates. *Washington: Office of Technology Utilization*, NASA, SP-160, 1969.
[2] A.W. Leissa, Recent research in plate vibrations: Classical theory, *The Shock and Vibration Digest* **9**(10) (1977), 3–24.
[3] A.W. Leissa, Recent research in plate vibrations, 1973–1976: Complicating effects, *The Shock and Vibration Digest* **9**(11) (1977), 1–35.
[4] A.W. Leissa, Plate vibration research, 1976–1980: Classical theory, *The Shock and Vibration Digest* **13**(9) (1981), 11–22.
[5] A.W. Leissa, Plate vibration research, 1976–1980: Classical theory, *The Shock and Vibration Digest* **13**(10) (1981), 19–36.
[6] A.W. Leissa, Recent studies in plate vibrations: 1981-1985 part I, Classical theory, *The Shock and Vibration Digest* **19**(3) (1987), 11–18.
[7] A.W. Leissa, Recent studies in plate vibrations: 1981–1985 part II, Complicating effects, *The Shock and Vibration Digest* **19**(3) (1987), 10–24.
[8] A.W. Leissa, Vibration of Plates, *Acoustic Society of America*, Sewickley, 1993.

[9] K.V. kumar and G.K. Ramaiah, On the use of a coordinate transformation for analysis of axisymmetric vibration of polar orthotropic annular plates, *Journal of Sound and Vibrations* **24** (1972), 165–175.
[10] Y. Narita, Natural frequencies of completely free annular and circular plates having polar orthotropy, *Journal of Sound and Vibration* **93** (1984), 503–511.
[11] Y. Narita, Free vibration analysis of orthotropic elliptic plates resting on arbitrary distributed point supports, *Journal of Sound and Vibration* **108**(1) (1986), 1–10.
[12] R.H. Gutierrez, P.A.A. Laura, D. Felix and C. Pistonesi, Fundamental frequency of transverse vibration of circular, annular plates of polar orthotropy, *Journal of Sound and Vibration* **230**(5), (2000), 1191–1195.
[13] J.B. Greenberg and Y. Stavsky, Flexural vibrations of certain full and annular composite orthotropic plates, *Journal of the Acoustical Society of America* **66** (1979), 501–508.
[14] F. Ginesu, B. Picasso and P. Priolo, Vibration analysis of polar annular discs, *Journal of sound and Vibration* **65** (1979), 97–105.
[15] D.G. Gorman, Frequencies of polar orthotropic uniform annular plates, *Journal of Sound and Vibration* **80** (1982), 145–154.
[16] D.G. Gorman, Natural frequencies of transverse vibration of polar orthotropic variable thickness annular plates, *Journal of Sound and Vibration* **86** (1983), 47–60.
[17] S.R. Soni and C.L. Amba Rao, Axisymmetric vibrations of polar orthotropic annular plates of variable thickness, *Journal of Sound and Vibration* **38** (1975), 465–473.
[18] U.S. Gupta and R. Lal, Axisymmetric vibrations of linearly tapered annular plates under an in-plane forces, *Journal of Sound and Vibration* **64** (1979), 269–276.
[19] R. Lal and U.S. Gupta, Axisymmetric vibrations of annular plates of variable thickness, *Journal of Sound and Vibration* **231**(1) (1982), 246–257.
[20] U.S. Gupta, R. Lal and C.P. Verma, Buckling and vibration of polar orthotropic annular plates of variable thickness, *Journal of Sound and Vibration* **104** (1986), 357–369.
[21] I.S. Raju, Axisymmetric vibrations linearly tapered annular plates, *Journal of Sound and Vibration* **32** (1974), 507–512.
[22] H.D. Conway, Vibration frequencies of tapered bars and circular plates, *Journal of, Applied Mechanics* **31** (1964), 329–331.
[23] T.A. Lenox and H.D. Conway, An exact, closed form, solution for the flexural vibration of a thin annular plate having a parabolic thickness, *Journal of Sound and Vibration* **68** (1980), 231–239.
[24] C.S. Kim and S.M. Dickinson, On the lateral vibration of thin annular and circular composite plates subjected to certain complicated effects, *Journal of Sound and Vibration* **130**(3) (1989), 363–377.
[25] X. Wang, J. Yang and J. Xiao, On the free vibration analysis of circular annular plates with non-uniform thickness by the differential quadrature method, *Journal of Sound and Vibration* **194** (1995), 547–551.
[26] P.A.A. Laura and E. Romanelli, Vibration of an annular circular plate of polar anisotropy with one edge supported, the other one free and an intermediate concentric circular support, *Journal of Sound and Vibration* **235**(3) (2000), 521–529.
[27] D.Y. Chen and B.S. Ren, Finite element analysis of the lateral vibration of thin annular and circular plates with variable thickness, *Journal of Acoustic Society of America* **120**(3) (1998), 747–752.
[28] S.M. Vogel and D.W. Skinner, Natural frequencies of transversely vibrating annular plates, *ASME Journal of Applied Mechanics* **32** (1965), 926–931.
[29] N. Sankaranarayanan, K. Chandrasekhran and G. Ramaiyan, Axisymmetric vibration of layered annular plates with linear variation in thickness, *Journal of Sound and Vibration* **99** (1985), 351–360.
[30] T.S. Chihara, *An Introduction to Orthogonal Polynomials*, New York, Gordon Breach Science Publishers, 1978.
[31] E. Winkler, 1867. Theory of Elasticity and Strength (in Russian) Dominicus, Prague.
[32] N. Bhardwaj, A.P. Gupta and K.K. Choong, Effect of elastic foundation on the vibration of orthotropic elliptic plates with varying thickness, in press, Meccanica, 2007.


Journal of Engineering
The Scientific World Journal
International Journal of Rotating Machinery
Journal of Sensors
International Journal of Distributed Sensor Networks
Advances in Civil Engineering
Journal of Control Science and Engineering
Hindawi
Submit your manuscripts at http://www.hindawi.com
Journal of Robotics
Journal of Electrical and Computer Engineering
Advances in OptoElectronics
VLSI Design
International Journal of Navigation and Observation
Modelling & Simulation in Engineering
International Journal of Aerospace Engineering
International Journal of Chemical Engineering
International Journal of Antennas and Propagation
Active and Passive Electronic Components
Shock and Vibration
Advances in Acoustics and Vibration