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RESEARCH ARTICLE

A NEW METHOD FOR SOLVING LINEAR PROGRAMMING PROBLEMS

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ABSTRACT

Anew method which does not depend on the simplex method for solving the linear programming (LP) problems is given. The proposed method is based mainly upon solving this problem algebraically using the concept of duality .The important of this method is that we are not based our work on vertex information which may have difficulties as the problem size increases. A simple example is given to clarify the developed theory of this proposed method.

INTRODUCTION

The problem of linear programming (LP) is one of the earliest formulated problems in mathematical programming where a linear function has to be maximized (minimized) over a convex constraint polyhedron X. The simplex algorithm was early suggested for solving this problem by moving toward a solution on the exterior of the constraint polyhedron X. In 1984, the area of linear programming underwent a considerable change of orientation when Karmarker [1984] introduced an algorithm for solving (LP) problems which moves through the interior of the polyhedron. This algorithm of Karmarker's and subsequent additional variants [6,8,9,16] established a new class of algorithms for solving linear programming problems known as the interior point methods for more details about interior point methods we refer the reader to [17] .Also more works has been made to solve this problem [1-3,10-15]. In this paper a new method which does not depend on the simplex method for solving linear programming problems is given .This method solves this problem algebraically using the concept of duality. The important of this study is that in some certain problems on using the simplex algorithm we have to visit a large number of extreme points since we are moving toward a solution on the exterior of the constraint polyhedron. In Section 2 we give a full description of the problem together with our main results while Section 3 contains the steps of our new algorithm with a simple example to illustrate the given method followed by our conclusion in Section 4.

Definitions and theory

The linear programming problem (LP) arises when a linear function has to be maximized on a convex constraint polyhedron X. this problem can be formulated as follows:

$$\text{Maximize } F(x) = c^T x$$
$$\text{Subject to}$$
$$x \in X = \{x, Ax \le b\} \qquad (1)$$

where c, $x \in R^n$, A is an $(m + n) \times n$ matrix, $b \in R^{m+n}$, we point out that the nonnegative conditions are included in the set of constraints. This problem can also be written in the form:

$$\text{Maximize } F(x) = c^T x$$
$$\text{Subject to}$$
$$a_i^T x \le b_i \qquad i = 1, 2\dots, m + n. \qquad (2)$$

here a_i^T represents the i th row of the given matrix A, then we have to note that in the non degenerate case an extreme point (vertex) of X lies on some n linearly independent subset of X. Now consider the dual problem for the linear program (1) in the form

$$\text{Minimize } w = u^T b$$
$$\text{Subject to } u^T A = c^T \qquad (3)$$
$$u \ge 0$$

since the set of constraints of this dual problem is written in matrix form hence we can multiply both side by a matrix $T = (T_1 \mid T_2)$, where $T_1 = c\,(c^T c)^{-1}$, and the columns of the matrix T_2 constitute the bases of

$$N(c^T) = \{x;\ c^T x = 0\} \qquad (4)$$

Then we have $u^T A T_1 = 1, \quad u^T A T_2 = 0 \quad \text{and } u \ge 0. \qquad (5)$

If we define q x (m+n) matrix P of non-negative entries such that $PAT_2 = 0$, then (5) can be rewritten in the form

$$v^T G = 1, \quad \text{and } v \ge 0. \qquad (6)$$

where $G = P\,AT_1$ and $v^T P = u^T$, we have to note that equation (6) will play an important rule for specifying the dual values needed for solving the linear programming (1) . This dual values satisfied the well known Kuhn-Tucker condition [4,5]such that for a given point x^0 to be an optimal solution of the linear program we must find

$u \ge 0$ satisfy $A_r^T u = c$, this equation can be written as

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$u = (A_r A_r^T)^{-1} A_r c$ (7)

here A_r is a submatrix of the given matrix A containing only the coefficients of the set of active constraints at the current point x^0 . Also from the complementary slackness theorem for the above set of active constraints the corresponding dual variables must be positive. Using equation (6) then the equivalent linear programming of the linear programming (3) can be written in the form

$$\begin{aligned} &\text{Minimize } w=v^T g \\ &\text{Subject to} \\ &v^T G = 1, \\ &v \geq 0 \end{aligned} \quad (8$$

with $G=P\,AT_1$, $g=Pb$, $v^T P= u^T$,the linear programming (8) has the dual linear programming problem in just one unknown Z in the form

Maximize Z

Subject to

$G Z \leq g$ (9)

Remark

The set of constraint of the above linear programming problem will give the maximum value Z^0 and also will define only one active constraint for this optimal value.

We have to note that from the complementary slackness theorem the corresponding dual value will be positive and the remaining dual variables will be zeros for the corresponding non active constraints

3- New method for solving linear programming (LP) problems

Our method for solving (LP) problems summarize as follows

Step1-Compute $T_1 = (c^T c)^{-1} c$, and the matrix T_2 as in (4).

Step 2-Find the matrix P of non-negative entries such that $PAT_2 = 0$,

Step 3-Use the linear programming (9) to find the optimal value Z^0 and also determine the corresponding active constraint and use the constraint of (8) to compute v^T

Step 4- Find the dual variables $u^T = v^T P$, for each positive variable u_i ,i=1,2,3,…,n

find the corresponding active set of constraint of the matrix A

Step 5- Solve an nxn system of linear equations for these set of active constraints

(a subset from an m+n constraints) to get the optimal solution.

Example: Consider the linear programming problem

$$\begin{aligned} &\text{Maximize } F(x) = x_1 + x_2 \\ &\text{Subject to:} \\ &2x_1 + 3x_2 \leq 6 \\ &x_1 \leq 2 \\ &x_2 \leq 1 \\ &-x_1 \leq 0,\ -x_2 \leq 0 \end{aligned}$$

Step1- compute $T_1 = \begin{pmatrix} 1/2 \\ 1/2 \end{pmatrix}$, and the matrix T_2 is given by

$T2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$,

we go to step2 to compute the nonnegative matrix P

Step2 -

$$P = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{pmatrix}$$

Step3- For the linear programming problem

$$\begin{aligned} &\text{Max } Z \\ &\text{Subject to} \\ &3Z \leq 8 \\ &2Z \leq 6 \\ &Z \leq 3 \\ &0Z \leq 2 \\ &0Z \leq 1 \\ &0Z \leq 0 \end{aligned}$$

The first constraint is the only active constraint for the above linear programming , this active constraint shows that the maximum optimal value Z^0=8/3 . Use the constraint of (8) to get the dual variables v^T =(1/3 0 0 0 0 0) and go to step 4

Step 4- Compute $u^T = v^T P$ = (1/3 1/3 0 0 0). And this Indicates that in the original set of constraints the first and the second constraint are the only active constraints

Step5-Solve the system of linear equations in the form

$$\begin{aligned} 2x_1 + 3x_2 &= 6 \\ x_1 &= 2 \end{aligned} ,$$

to get the optimal solution x^{T*}= (2 2/3), of the above linear programming problem with optimal value F* =8/3

Conclusion

In this paper we gave a new method to find the optimal extreme point for the linear programming problem .Our method is based on solving this problem algebraically and doesn't depend on the simplex version of linear program .The important of this study is that in using the simplex algorithm we are moving toward a solution on the exterior of the constraint polyhedron and in some certain problems we have to visit a large number of extreme points of the constraint polyhedron.

References

1. A.R.L.Oliveira and D.C. Sorensen, A new class of preconditioners for large-scale linear systems from interior point methods for linear programming Linear Algebra and Its Applications, 394 (1-3), (2005) 1-24.
2. C.Durazzi, V. Ruggiero and G. Zanghirati, Parallel interior-point method for linear and quadratic programs with special structure Journal of Optimization Theory and Applications, 110 (2), (2001) 289-313.
3. C.Durazzi and V. Ruggiero, Numerical solution of special linear and quadratic programs via a parallel interior-point method Parallel Computing, 29, (2003) 485-503.
4. D.M. Greig, Optimization. Longman.london and New York. (1980)
5. F.S. Hillier and G.J. Lieberman, Introduction to

operations research. , 5 th edition, McGraw-Hill publishing Company. (1990)

6. I.Adler, M.G.C. Resende, G. Veiga and N.K. Karmarkar, An implementation of karmarkar's algorithm for linear programming Math.Program.44, (1989) 297-335.
7. N.K. Karmarkar, Anew polynomial time algorithm for linear programming, combinatorica 4, (1984) 373-395.
8. N.K. Karmarkar and K.G. Ramakrishnan, Computational results of an interior point algorithm for large scale linear programming, Mathematical Programming, Series B 52, (3), (1991) 555-586
9. R. J. Vanderbei, M. S. Meketon and B.A. Freedman, A modification of Karmarkar's linear programming algorithm, Algorithmica 1, (1986)395-407
10. R.E .Bixby, Progress in linear programming ORSA Journal on Computing, 6, (1994) 15-22.
11. S.F. Tantawy, Using feasible direction to find all alternative extreme optimal points for linear programming problem, Journal of Mathematics and Statistics 3 (3)(2007) 109-111
12. S.F. Tantawy and A.M. Hariri , An O ((m+n) Max {m+n,n3}) procedure for solving the linear programming problem, Journal of Applied Sciences Research, 5(4) (2009) 329-333
13. T.F.Coleman and A. Verma, A preconditioned conjugate gradient approach to linear equality constrained minimization Computational Optimization and Applications, 20 (1), (2001) 61-72.
14. W. Li, A new method to achieve primal feasibility for linear programming ,Operations Research and Management Science, 13, . (2004) 7-10.
15. W. Wang ,The convergence of an interior-point method using modified search directions in final iterations Computers and Mathematics with Applications,44 (3-4), (2002) 3 47-356.
16. W.Wang, Final iterations in interior point methods - Preconditioned conjugate gradients and modified search directions Journal of the Australian Mathematical Society Series B-Applied Mathematics, 41 (3) (2000) 312-328
17. Y. Ye, Interior point algorithms, John Wiley & Sons, New York(1997)

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