State constrained tracking control for nonlinear systems

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Abstract

This work addresses the model reference tracking control problem. It aims to highlight the encountered difficulties and the proposed solutions to achieve the tracking objective.
Based on a literature overview of linear and nonlinear reference tracking, the achievements and the limitations of the existing strategies are highlighted. This motivates the present work to propose clear control algorithms for perfect and approximate tracking controls of nonlinear systems described by Takagi-Sugeno models. First, perfect nonlinear tracking control is addressed and necessary structural conditions are stated. If these conditions do not hold, approximate tracking control is proposed and the choice of the reference model to be tracked as well as the choice of the criterion to be minimized are discussed with respect to the desired objectives. The case of constrained control input is also considered in order to anticipate and counteract the effect of the control saturation.

Keywords: Tracking control, nonlinear systems, constrained input, perfect and approximate state tracking.

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1. Introduction

1.1. Overview of linear model reference tracking control

Considering a plant represented by a linear model

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t),\, x \in \mathbb{R}^{n_x},\, u \in \mathbb{R}^{n_u} \\ y(t) = Cx(t),\, y \in \mathbb{R}^{n_y} \end{cases} \tag{1}$$

and a linear reference model

$$\begin{cases} \dot{x}_r(t) = A_r x_r(t) + B_r u_r(t),\, x_r \in \mathbb{R}^{n_x},\, u_r \in \mathbb{R}^{n_u} \\ y_r(t) = Cx_r(t) \end{cases} \tag{2}$$

representing the desired dynamics of the plant, model reference tracking control consists in determining a feedback law $u(t)$ to achieve dynamic matching between the controlled plant (1) and the desired model reference (2). More precisely, the goal is that the state variables of the plant $x(t)$ (or its output $y(t)$) will closely follow the state variables (or the output) of the model reference $x_r(t)$ (or $y_r(t)$).
In the available literature, the reader may distinguish three main approaches that will be presented and discussed hereafter: perfect, approximate and iterative state tracking.

1.1.1. Perfect state tracking

In the first approach, the well known Erzbergers perfect model matching conditions [1] allow to achieve a null tracking error. Namely, under some structural conditions on the models (1) and (2), it establishes the existence of a state feedback controller ensuring that the plant and the model reference states behave similarly. Unfortunately, these conditions are restrictive matching equations which can only be satisfied for system matrices with great structural similarities, e.g. in the canonical form [2].
Note that one of the main issues for this approach concerns the controller structure. Two procedures may be distinguished.
In *the first procedure*, the controller structure choice is firstly fixed and then the structural matching conditions and the appropriate gains of the controller are deduced. In [3], [4], [5] and [6] for example, a particular state feedback for state tracking is given by:

$$u(t) = Kx(t) + K_r u_r(t),\, K \in \mathbb{R}^{n_u \times n_x},\, K_r \in \mathbb{R}^{n_u \times n_u} \tag{3}$$

where K and K_r are constant parameter matrices so that the plant state vector $x(t)$ can track a reference state vector $x_r(t)$ generated from (2). Such a control design leads to:

$$\begin{cases} \dot{x}(t) = (A+BK)x(t) + BK_r u(t) \\ y(t) = Cx(t),\, y \in \mathbb{R}^{n_y} \end{cases} \tag{4}$$

The comparison between (4) and (2) can be made from two points of view, whether the tracking is limited to the steady state or is also sought during the transient.
For the steady state tracking, one obtains the following sufficient matching conditions:

$$(A+BK)^{-1}BK_r = A_r^{-1}B_r \tag{5}$$

A particular solution of (5) is given by [7]:

$$\begin{pmatrix} K_r & K \end{pmatrix} = B^+PQ^+ \tag{6}$$

with

$$P = AA_r^{-1}B_r, \qquad Q = \begin{pmatrix} I \\ -A_r^{-1}B_r \end{pmatrix} \tag{7}$$

and the consistency condition:

$$BB^+PQ^+Q = P \tag{8}$$

The perfect state matching between (2) and (4), even during the transient, is obtained if there exists K and K_r such that $A+BK = A_r$ and $BK_r = B_r$. This needs the following rank constraints to be fulfilled

$$\begin{cases} rank(B) &= rank([B|A_r - A]) \\ rank(B) &= rank([B|B_r]) \end{cases} \tag{9}$$

The gains K and K_r are then given by:

$$\begin{cases} K_r &= B^+B_r \\ K &= B^+(A_r - A) \end{cases} \tag{10}$$

with B^+ a suitable pseudo-inverse matrix of the full column rank B matrix.
It is also important to highlight the fact that fixing the structure of the controller (3) will not necessarily lead to a solution. In fact, the choice of an inadequate structure may cause some controllability problems, which confirms the importance of the adopted control structure. In [8], the model and controller structures were fixed and the gains were given the basic form corresponding to a PI

controller $u(t) = Kx_r(t) + K_r u_r(t) + K_e \int_0^t (y_r(\tau) - y(\tau))d\tau$ with an asymptotically stable output tracking error.
Imposing the same control gain over the time range may also be subject to criticism. This is why an adaptive control law with updated matrices $K(t)$ and $K_r(t)$ is presented in [2] and [9]. In [10], the authors' aim was to design an adaptive control law ensuring the closed-loop signal boundedness and asymptotic state tracking despite uncertainties.
The second procedure tends to achieve a null tracking error without a pre-requisited structure for the control law nor an assignment of error dynamics. In [11] for example, the adopted approach was to deduce the control structure from the state tracking error $e(t) = x(t) - x_r(t)$ and its dynamics deduced from (1) and (2) as follows

$$\dot{e}(t) = A_r e(t) + (A - A_r)x(t) + Bu(t) - B_r u_r(t) \quad (11)$$

In order that (11) reduces to $\dot{e}(t) = \tilde{A}e(t)$, where $\tilde{A}$ is a prescribed stable matrix, the input control $u(t)$ should be designed such that $(A - \tilde{A})x + (\tilde{A} - A_r)x_r + Bu - B_r u_r$ is null.

1.1.2. Approximate state tracking

As seen previously, perfect state tracking needs to respect strong constraints due to rank conditions ((9) for example). Therefore, in many situations, only approximate state tracking can be obtained, where the goal is to minimize the discrepancy between x and x_r (or y and y_r). For example if (11) cannot be reduced to $\dot{e}(t) = \tilde{A}e(t)$, it is nevertheless possible to find a control $u(t)$ minimizing $\tilde{x}(t) = (A - \tilde{A})x(t) + (\tilde{A} - A_r)x_r(t) + Bu(t) - B_r u_r(t)$ or the transfer from u to $\tilde{x}$.
Approximate state tracking is based on the quadratic optimal control theory and can be applied to arbitrary systems and always yields a feedback configuration which minimizes a quadratic function of the tracking error between the system state and the model reference [1], [12], [13]. Although the quadratic optimal control approach provides a generic framework to design reference tracking controllers, it may be necessary to pay a particular attention to the choice of the weighting matrices of the cost function [1] in order to obtain satisfactory trajectory tracking.
The considered tracking criterion may be expressed in the following terms:

$$\int_0^{t_f} e^T(t)Qe(t)dt \leq \eta^2 \int_0^{t_f} u^T(t)u(t)dt \quad (12)$$

where Q is a positive definite weighting matrix and η the prescribed attenuation level. The matrix Q is chosen accordingly to the state components for which

some specific tracking is desired. In (12), the upper bound of the $\mathscr{L}_2$ gain from $u(t)$ to $e(t)$ given by $\frac{\eta}{\sqrt{\bar{\lambda}(Q)}}$ (where $\overline{\lambda}(Q)$ denotes the largest eigenvalue of the matrix Q) quantifies the effect of the reference input on the weighted tracking error. Obviously, the objective is to maximize the attenuation level $\frac{\sqrt{\bar{\lambda}(Q)}}{\eta}$.
Another point of the optimal control theory that may be subject to criticism is the fact that the linear quadratic regulator synthesis leads to determine a constant feedback gain.
A key point in this approach is the time horizon, which may be either finite and sliding or infinite. In the later case, the $\mathscr{L}_2$ norm of the tracking error is minimized and in the former case, the aim is to minimize the tracking error on a finite sliding horizon. This procedure, known as the Model Predictive Control (also referred as the moving or receding horizon control), consists in solving an optimal control problem, over an horizon of finite length, at each time instant. With a discrete representation, it results in computing a control input sequence at time k, that minimizes, on a time horizon, a criterion mixing the control cost and the tracking error [14]:

$$\Phi_k = \sum_{i=k}^{k+N-1} \left(||x_{i+1} - x_{r,i+1}||_Q^2 + \eta^2 ||u_i||^2 \right) \tag{13}$$

The common feature of all MPC approaches is to solve, at each sampling time k, a finite horizon optimal control problem by considering the current state as the initial state. Then, only the first element of the computed control sequence is applied and the same problem procedure is repeated at the next sampling times [15].
In the two previous control strategies, the length N of the considered time horizon may affect the transient behavior of the closed-loop system and deserves a particular attention. In fact, for all the tracking methods listed above, the asymptotic behavior is well addressed and the tracking error is ensured to tend to zero in the steady state. An interesting improvement for the controller performance is to consider as well the transient behavior of the tracking error. In [16], the controller is based, not only on the tracking error, but also on its integral in order to achieve transient performance for input and output signals. The control algorithm internally generates a low pass filter, thus preventing high frequency oscillations for the large adaptation rate for a class of MIMO uncertain nonlinear systems. In [17], the control architecture of the adaptive law is not modified, but it is proposed to feed back the reference model with the tracking error signal. It is also important to highlight that these studies are founded on the frequency domain framework.

1.1.3. Iterative state tracking

The third approach refers to the Iterative Learning Control (ILC). The ILC is conducted along both time domain and repetitive trials or iterations and it may be resumed by the following procedure. At the k^{th} iteration, a control $u_k(t)$ is applied:

$$\begin{array}{rcl} \dot{x}_k(t) & = & Ax_k(t) + Bu_k(t) \\ \dot{x}_r(t) & = & A_r x_r(t) + B_r u_r(t) \end{array} \tag{14}$$

At the next iteration, the control law is adapted as follows:

$$\begin{array}{rcl} u_{k+1}(t) & = & u_k(t) + L\dot{e}_k(t) \\ e_k(t) & = & x_r(t) - x_k(t) \end{array} \tag{15}$$

In the ILC approach, the previous control sequence is used to compute the next one and thus improve the tracking performance as k increases by an appropriate choice of the gain L (see [18], [19] and the references therein). Despite its efficiency, the disadvantage of this method in comparison with the ones listed previously, is that the control is generally made off-line, in a finite time interval and supposing that all the data are available. Moreover this approach is mainly devoted to periodic systems. However, there are some recent works dedicated to robust predictive ILC [20] and allowing on-line implementation [21].

1.2. Overview of nonlinear model reference tracking control

In this section, a focus is made on model reference tracking for nonlinear systems, like exact feedback linearization, sliding mode and adaptive control.
The feedback linearization technique has been introduced to deal with nonlinear systems [22]. However, the control algorithm is somewhat complicated, the stability of the controller is not guaranteed for non-minimum phase systems and its application to complex nonlinear systems is tedious. In [23], nonlinear output regulation problem has been formulated and solved by designing a dynamic controller such that the closed-loop system is stable and the tracking error approaches zero asymptotically. Though the Isidori-Byrnes theory is precise and sophisticated, it requires many assumptions. Moreover, in order to synthesize a numerical solution, one has to solve the nonlinear regulator equation described by a system of nonlinear partial differential equations, which is difficult to solve as in the Hamilton-Jacobi-Bellman equation [24].
The sliding mode control (SMC) presents the advantage of the robustness to uncertainties [25], [26]. An output feedback SMC scheme for tracking uncertain

nonlinear plants was adopted in [27]. It is an extension of [28] and is based on a switching algorithm based on a monitoring function for the output tracking error. In [28] and [27], only relative degree one plants were considered. In this case, despite the proved efficiency of the SMC, it appears that the controller is too sensitive to the chattering phenomenon. A solution would be to consider higher order sliding mode control for MIMO nonlinear systems [29], [30], [31], but due to the non-applicability of Lyapunov's direct method, very few results have been presented (see [32]).

The Nonlinear Model Predictive Control, or NMPC, is an extension of the MPC cited previously. It is characterized by the use of nonlinear system models in the prediction [33], [34]. As in linear MPC, NMPC requires the iterative solution of optimal control problems on a finite prediction horizon. While these problems are convex in linear MPC, in NMPC they are not convex anymore. This is challenging for both NMPC stability theory and numerical solution [35], [36].

1.3. The T-S case

Among the several structures of nonlinear models envisaged in tracking control, a focus is made on the Takagi-Sugeno models that are considered in the present paper. The T-S modeling is known to be an efficient way to tackle the problems of nonlinear estimation and control. Originally introduced by [37], the T-S representation allows to exactly describe nonlinear systems, provided that the nonlinearities are bounded. This is reasonable since state variables as well as parameters of physical systems are bounded, and so is the input of the system which may be assumed to be stable, at least in closed-loop (see [38] and the references therein).

Despite an abundant literature on the T-S models, few authors have dealt with the tracking problem. One can refer to some works concerned with state or output feedback with H_∞ performances [39], [12], [40], [41] and [42]. The tracking control is based on the state or output Parallel Distributed Compensation (PDC) structure to minimize the $\mathscr{L}_2$ gain of the tracking error and the controller computation is expressed as an LMI problem [12], [42]. However, in the cited references, a referred "suitable" choice for the reference model is made without any explanations nor details.

The last remark motivated the present study. In fact, either for the linear case or the nonlinear one, few works detail the influence of the reference model choice, which is not a trivial task. In [11] for example, the authors referred to the Erzberger's conditions, but with no further explanations. For these reasons, in the proposed work, a focus is made not only on the control design procedure, but also on the tracking (matching) conditions.

1.4. Paper outline

The paper is organized as follows. In section 2, the structural conditions to achieve exact state tracking are introduced in the T-S case. These conditions are an extension of the well known Erzberger's conditions. As in the linear case, the objective is to achieve a null tracking error. Section 3 deals with the tracking criterion choice. After a short analysis, a quadratic optimal control for T-S model is introduced. As an improvement of this technique, the T-S MPC is presented in section 4. Section 5 is devoted to the tracking problem under input control constraints. In each section appropriate examples are presented. Finally, section 6 summarizes the obtained results.

2. Exact state tracking conditions for T-S systems

2.1. Model and objective

Let us consider the following discrete T-S model [37]:

$$x_{k+1} = A_k\, x_k + B_k\, u_k \tag{16}$$

where $x_k \in \mathbb{R}^{n_x}$ and $u_k \in \mathbb{R}^{n_u}$ with:

$$\begin{aligned} A_k &= \sum_{i=1}^{r} \mu_{i,k}(\xi_k) A_i \\ B_k &= \sum_{i=1}^{r} \mu_{i,k}(\xi_k) B_i \end{aligned} \tag{17}$$

where the weighting functions $\mu_{i,k}(\xi_k)$ depend on the so-called premise variable ξ_k which may be a state, input, or output combination. These weighting functions satisfy the following convex sum property:

$$0 \leq \mu_{i,k}(\xi_k) \leq 1, \sum_{i=1}^{r} \mu_{i,k}(\xi_k) = 1 \tag{18}$$

The considered linear reference model is the following:

$$x_{r,k+1} = A_r\, x_{r,k} + B_{r,}\, u_{r,k} \tag{19}$$

where $x_{r,k} \in \mathbb{R}^{n_x}$ and $u_{r,k} \in \mathbb{R}^{n_u}$.

Remark 1. *For the sake of simplicity, the model reference is chosen to be linear, but the extension to a nonlinear model defined by a T-S system (namely, with $A_{r,k}$ and $B_{r,k}$ instead of A_r and B_r) can readily be done.*

The ideal tracking objective is to adjust, at each instant k, the control u_k in such a way that the system state x_k follows the reference model state $x_{r,k}$ with a null tracking error. The idea is to find, as for the first strategy (section 1), the appropriate structural conditions, but also an analytical expression for the control law. If the ideal tracking is not reachable, some compromises need to be defined such as, for example, the tracking of a subset of the states.
In order to achieve the tracking objective, the following control law with time-varying gains K_k and $K_{r,k}$ is considered:

$$u_k = K_k\, x_k + K_{r,k}\, u_{r,k} \tag{20}$$

Substituting (20) into (16), the closed-loop system is:

$$x_{k+1} = (A_k + B_k\, K_k)\, x_k + B_k\, K_{r,k}\, u_{r,k} \tag{21}$$

The matching conditions for the reference model and the system are then obtained by comparing the closed-loop system (21) and the reference model (19). The perfect transient tracking conditions are given by:

$$\left\{\begin{array}{rcl} A_k + B_k\, K_k & = & A_r \\ B_k\, K_{r,k} & = & B_r \end{array}\right. \tag{22}$$

In order to find the gain K_k and $K_{r,k}$ solution of (22), the following rank conditions have to be fulfilled:

$$\left\{\begin{array}{rcl} rank(B_k) & = & rank([B_k|A_r - A_k]) \\ rank(B_k) & = & rank([B_k|B_r]) \end{array}\right. \tag{23}$$

If conditions (23) are fulfilled, then at each sampling time, the gains K_k and $K_{r,k}$ are given by:

$$\left\{\begin{array}{rcl} K_{r,k} & = & B_k^+ B_r \\ K_k & = & B_k^+ (A_r - A_k) \end{array}\right. \tag{24}$$

with B_k^+ a suitable pseudo-inverse matrix of the full column rank B_k matrix.
Note that in order to satisfy the matching conditions (23), from definitions (17), since the system matrices A_k and B_k depend on the time, a possible sufficient

condition, but not the only one, is to consider the matrices A_i, B_i and A_r, B_r in the following canonical form:

$$\begin{array}{c} A_i = \left(\frac{A_0}{\overline{A}_i}\right), A_r = \left(\frac{A_0}{\overline{A}_r}\right) \\ B_i = \left(\frac{0_{n_x - n_u}}{b_i}\right), B_r = \left(\frac{0_{n_x - n_u}}{b_r}\right) \end{array} \tag{25}$$

with A_0 a matrix of dimension $(n_x - n_u) \times n_x$, $\overline{A}_i$ and $\overline{A}_r$ matrices of dimensions $n_u \times n_x$. b_i and b_r are of dimension $n_u \times n_u$. The structure (25) is equivalent to express that:

1. the $(n_x - n_u)$ first rows of the matrices A_i are equal to the $(n_x - n_u)$ first rows of the matrix A_r
2. the $(n_x - n_u)$ first rows of the matrices B_i are null
3. the $(n_x - n_u)$ first rows of the matrix B_r are null

allowing to fully satisfy the rank conditions (23).
It is important to note that the matching conditions (23) between the reference model and the system depend on the choice of the control law. It means that these conditions have to be adapted when changing the structure of the control law.

2.2. Numerical example

To illustrate the above conditions, let us consider the electro-mechanical model of a motor, with a time varying parameter.
The system model is given by the following equations:

$$\begin{cases} u(t) = e(t) + R(p(t))i(t) + L\frac{di(t)}{dt} \\ J\frac{d\Omega(t)}{dt} = C_m(t) - C_r(t) \end{cases} \tag{26}$$

with $u(t)$ the voltage, $i(t)$ the current, $e(t) = K_e\omega(t)$ the induced EMF with $\omega(t)$ the rotation speed. $C_m(t)$ and $C_r(t)$ are respectively the electromechanical and load torque ($C_m(t) = K_m i(t)$ and $C_r(t) = f\omega(t)$). The inductance L, the inertia J and the parameters K_m, K_c and f are constant.
Taking into account the operating conditions (motor aging, temperature, etc) the resistance is considered to be time varying and bounded: $R(p(t)) \in [R_1\ R_2]$, where $p(t)$ is a known external parameter.

Including the angular position $\theta(t)$, the electro-mechanical model is given by the following state representation:

$$\frac{d}{dt}\begin{pmatrix}\theta(t)\\ \omega(t)\\ i(t)\end{pmatrix}=\begin{pmatrix}0 & 1 & 0\\ 0 & \frac{-f}{J} & \frac{K_m}{J}\\ 0 & \frac{-K_e}{L} & \frac{-R(p_k)}{L}\end{pmatrix}\begin{pmatrix}\theta(t)\\ \omega(t)\\ i(t)\end{pmatrix}+\begin{pmatrix}0\\ 0\\ \frac{1}{L}\end{pmatrix}u(t) \tag{27}$$

For the sampling time T, the discretized form of (27) is given by:

$$\begin{pmatrix}\theta_{k+1}\\ \omega_{k+1}\\ i_{k+1}\end{pmatrix}=\begin{pmatrix}1 & T & 0\\ 0 & 1-\frac{Tf}{J} & \frac{TK_m}{J}\\ 0 & \frac{-TK_e}{L} & 1-\frac{TR(p)}{L}\end{pmatrix}\begin{pmatrix}\theta_k\\ \omega_k\\ i_k\end{pmatrix}+\begin{pmatrix}0\\ 0\\ \frac{T}{L}\end{pmatrix}u_k \tag{28}$$

Defining the weighting functions by:

$$\mu_{1,k}=\frac{R_2-R(p_k)}{R_2-R_1},\ \mu_{2,k}=\frac{R(p_k)-R_1}{R_2-R_1} \tag{29}$$

the following sub-models are obtained (see [38] for procedure details):

$$\begin{aligned}A_1&=\begin{pmatrix}1 & T & 0\\ 0 & 1-\frac{Tf}{J} & \frac{TK_m}{J}\\ 0 & -\frac{TK_e}{L} & 1-\frac{TR_1}{L}\end{pmatrix},B_1=\begin{pmatrix}0\\ 0\\ \frac{T}{L}\end{pmatrix}\\ A_2&=\begin{pmatrix}1 & T & 0\\ 0 & 1-\frac{Tf}{J} & \frac{TK_m}{J}\\ 0 & -\frac{TK_e}{L} & 1-\frac{TR_2}{L}\end{pmatrix},B_2=\begin{pmatrix}0\\ 0\\ \frac{T}{L}\end{pmatrix}\end{aligned} \tag{30}$$

The considered reference model is chosen with different value of the resistance and inductance, denoted R_r and L_r:

$$x_{r,k+1}=\begin{pmatrix}1 & T & 0\\ 0 & 1-\frac{Tf}{J} & \frac{TK_m}{J}\\ 0 & -\frac{TK_e}{L_r} & 1-\frac{TR_r}{L_r}\end{pmatrix}x_{r,k}+\begin{pmatrix}0\\ 0\\ \frac{T}{L_r}\end{pmatrix}u_{r,k} \tag{31}$$

One can easily ensures that the tracking conditions (23) are fulfilled:

$$\begin{cases}rank(B_k) &= rank([B_k|A_r-A_k])=1\\ rank(B_k) &= rank([B_k|B_r])=1\end{cases} \tag{32}$$

Applying the tracking control law (20) with (24), the system and model reference states are depicted in figure 1 (respectively noted x_i and x_{ir}, $i=1,\ldots,3$). From the depicted figures, one can see that the control tracking is efficient for all the three states under the specified structural conditions.

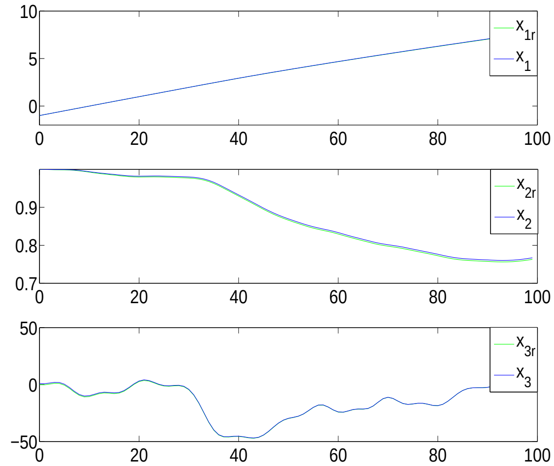

Figure 1: System and model reference states

Remark 2. *If the premise variables ξ_k of the weighting functions $\mu_{i,k}$ depend on the input u_k, the control law (20) will be of the form $u_k = F(u_k)$, since A_k and B_k are input depending (17). A solution may be given by an iterative algorithm with the following recurrence:*

$$u_k^{(j+1)} = \left((B_k^{(j)})^T B_k^{(j)}\right)^{-1} \left(B_k^{(j)}\right)^T (A_r - A_k^{(j)})\, x_k \tag{33}$$

with $B_k^{(j)} = \sum_{i=1}^{r} \mu_{i,k}(u_k^{(j)}) B_i$, $A_k^{(j)} = \sum_{i=1}^{r} \mu_{i,k}(u_k^{(j)}) A_i$, $j = 0, \ldots, N$ *with N the number of iterations and u_k^0 the input initialization (may be taken as $u_{r,k}$ for example).*

Remark 3. *The convergence proof of the proposed iterative algorithm can be locally ensured [43], [44]. But since it is not our study object, the reader may refer to the cited work for more explanations.*

3. Approximate state tracking for T-S systems

This section concerns the optimal control with the introduction of the MPC for T-S models If the needed structural conditions for exact state tracking (23) are not satisfied by the system and reference models, there is a need for an approximate tracking control with less conservative conditions. Instead of ensuring equal state trajectory of the system and reference, the aim should be to minimize the discrepancy between them. The so-called quadratic optimal control for T-S models aims at minimizing a function of the tracking error. As an consequence, the optimization ensures a compromise between the tracking of the different state components.

3.1. Control law design

At each time instant k, the objective is to minimize the following criterion which is the norm of the tracking error:

$$\Phi_k(u_k) = \| B_k\, u_k - x_{r,k+1} + A_k\, x_k \|_W^2 \tag{34}$$

where W is a positive definite weighting matrix chosen accordingly to the state components for which some specific tracking is desired.
The control tracking law is then given by:

$$u_k = (B_k^T W B_k)^{-1} B_k^T W (x_{r,k+1} - A_k\, x_k) \tag{35}$$

where the matrices A_k and B_k have been already defined in (17).

Remark 4. *When the premise variables ξ_k depend on the control as explained previously (remark 2), the control law (35) can be iteratively computed by:*

$$u_k^{(j+1)} = ((B_k^{(j)})^T W B_k^{(j)})^{-1} (B_k^{(j)})^T W\, (x_{r,k+1} - A_k^{(j)}\, x_k) \tag{36}$$

for $j = 0, \ldots, N$ with $B_k^{(j)} = \sum_{i=1}^{r} \mu_{i,k}(u_k^{(j)}) B_i$, $A_k^{(j)} = \sum_{i=1}^{r} \mu_{i,k}(u_k^{(j)}) A_i$.

3.2. Numerical examples

3.2.1. Control independent premise variables

Let us consider a simplified vehicle lateral dynamic model studied in [45] and represented in figure 2.
The system dynamics are given by:

$$\begin{pmatrix} \dot{\beta}(t) \\ \ddot{\Psi}(t) \end{pmatrix} = \begin{pmatrix} -\frac{c_{aV}+c_{aH}}{m v(t)} & \frac{l_H c_{aH} - L_V c_{aV}}{m v^2(t)} - 1 \\ \frac{l_H c_{aH} - l_V c_{aV}}{I_z} & \frac{l_H^2 c_{aH} + l_V^2 c_{aV}}{I_z v(t)} \end{pmatrix} \begin{pmatrix} \beta(t) \\ \dot{\Psi}(t) \end{pmatrix} + \begin{pmatrix} \frac{c_{aV}}{m v(t)} \\ \frac{l_V c_{aV}}{I_z} \end{pmatrix} u_L(t) \tag{37}$$

where β denotes the side slip angle, $\dot{\Psi}$ the yaw rate, u_L the relative steering wheel angle and v the speed of the vehicle.

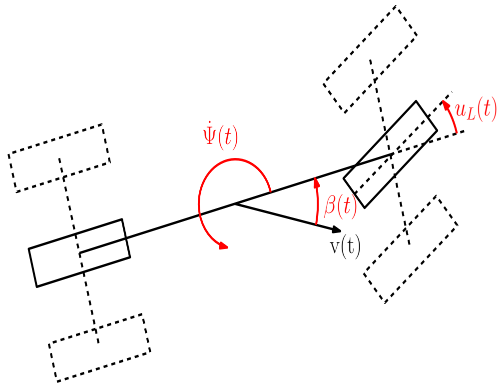

Figure 2: Vehicle lateral dynamic system

$m = 1621Kg$ corresponds to the vehicle total mass, $l_V = 1.15m$ is the distance from the center of gravity (C.G.) to front axle, $I_H = 1.38m$ is the distance from C.G. to rear axle, $I_z = 1975Kgm^2$ is the moment of inertia about the vertical axis, $c_{aV} = 57117Nrad^{-1}$ is the front axle tire cornering stiffness and $c_{aH} = 81396Nrad^{-1}$ is the rear axle tire cornering stiffness.
For small variations of the speed around v_0, the discretized model is then given by:

$$\begin{aligned} \begin{pmatrix} \beta_{k+1} \\ \dot{\Psi}_{k+1} \end{pmatrix} = & \begin{pmatrix} 1-1.71(\frac{1}{v_0}-\frac{\rho_k}{v_0^2}) & 0.58(\frac{1}{v_0^2}-\frac{2\rho_k}{v_0^3})-0.02 \\ 0.47 & 1-2.33(\frac{1}{v_0}-\frac{\rho_k}{v_0^2}) \end{pmatrix} \begin{pmatrix} \beta_k \\ \dot{\Psi}_k \end{pmatrix} \\ & + \begin{pmatrix} 0.7(\frac{1}{v_0}-\frac{\rho_k}{v_0^2}) \\ 0.67 \end{pmatrix} u_{Lk} \end{aligned} \tag{38}$$

where the parameter $\rho_k = \delta v_k$ is time-varying and bounded $\rho_k \in [\rho_1, \rho_2]$.
Following the same procedure as for the previous example, a T-S model with two sub-models is obtained. One can verify that the exact tracking conditions (23) are not fulfilled. Then, a quadratic optimal control is chosen and is implemented.
The weighting matrix is chosen as $\mathrm{diag}(1,1)$. The system and reference model states are depicted in figure 3 (respectively noted x_i and x_{ir}, $i = 1,2$). From the depicted figures, one can see that the control tracking is efficient (especially for the second state) although the structural conditions are not fulfilled.

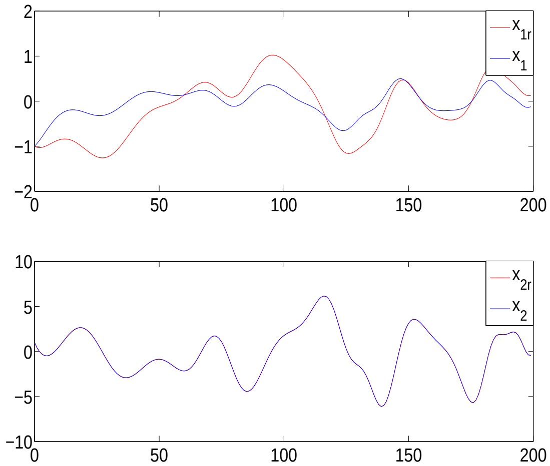

Figure 3: System and model reference states

3.2.2. Control dependent premise variables

Let us now consider the following illustrative example where the premise variables are control dependent. The system and reference model are defined by:

$$
\begin{aligned}
A_r &= \begin{pmatrix} 0.2 & 0.5 & 0 \\ -0.2 & 0.99 & -0.1 \\ 0 & 0 & 0.2 \end{pmatrix}, \quad B_r = \begin{pmatrix} -0.3 & 0 \\ 1 & 0.11 \\ 0 & 1 \end{pmatrix} \\
A_1 &= \begin{pmatrix} 0 & 0.5 & 0 \\ -0.2 & 1.19 & -0.1 \\ 0 & 0 & 0.1 \end{pmatrix}, \quad B_1 = \begin{pmatrix} 0.2 & 0.5 \\ 1.5 & 0.61 \\ 0.5 & 1.5 \end{pmatrix} \\
A_2 &= \begin{pmatrix} 0.6 & 0.5 & 0 \\ -0.2 & 1.09 & -0.1 \\ 0 & 0 & 1.1 \end{pmatrix}, \quad B_2 = \begin{pmatrix} -0.8 & -0.5 \\ 0.5 & -0.39 \\ -0.5 & 0.5 \end{pmatrix}
\end{aligned}
\tag{39}
$$

The weighting functions are control dependent and given by:

$$
\mu_{1,k} = \frac{1 + 2\tanh(u_{1,k})}{2} \qquad \mu_{2,k} = 1 - \mu_{1,k} \tag{40}
$$

From (39), one can verify that the exact tracking conditions (23) are not fulfilled. Then, a quadratic optimal control is chosen and the iterative solution (36) is implemented.

First, the objective is to ensure a good tracking of the second and third state components. Consequently, the weighting matrix is chosen as $\mathrm{diag}(0.01, 1, 1)$, implying a relaxation of the tracking of the first state component. The system and reference model states are depicted in figure 4 (respectively noted x_i and x_{ir}, $i = 1, \ldots, 3$). From the depicted figures, one can see that the control tracking is

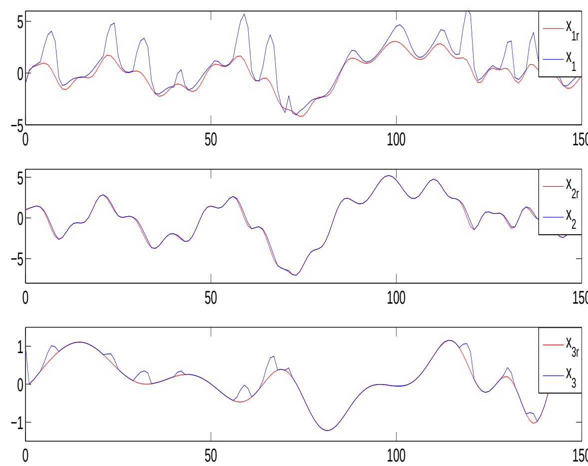

Figure 4: System and model reference states

efficient (especially for the second and third state) although the structural conditions are not fulfilled.
Secondly, if the main objective is an accurate tracking of x_{1r} by x_1, one should set the weighting matrix as $W = \mathrm{diag}(0.1, 0.01, 0.01)$. The system and model reference states are displayed on figure 5 and it can be seen that the first state tracking has been improved when the second and third states tracking have been deteriorated.

Remark 5. *The proposed quadratic method consists in calculating and minimizing the tracking error at each sampling instant k. Another way to consider the problem, is to minimize the norm of the tracking error on an infinite time horizon* $t \to \infty$ *with an* $\mathscr{L}_2$ *attenuation [46].*

4. Toward the Model Predictive Control for T-S models

Minimizing the weighted state tracking error on a finite sliding horizon, instead of doing it at a given sampling time as in (34), allows to take into account

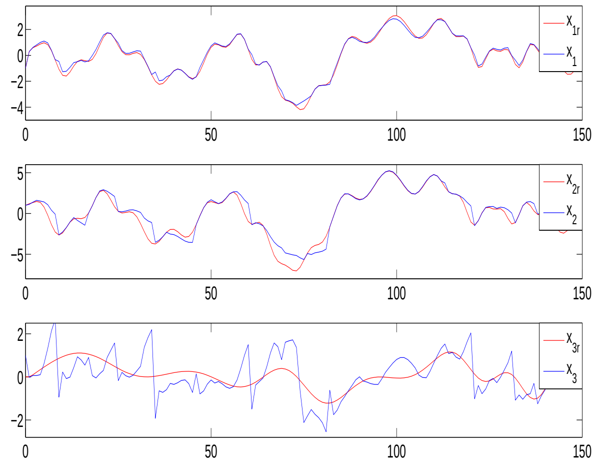

Figure 5: System and reference model states

the known state trajectory of the reference model and to have an anticipation effect of the tracking control. This leads to the extension of the MPC to state tracking control of T-S model. Roughly speaking, the procedure for T-S models is the same as for the conventional MPC. However, some difficulties occur when the premise variables depend on the control.

4.1. Premise variables independent of the input

Using the state equations (16) and (17), it follows at time $k+m+1$

$$\begin{aligned} x_{k+m+1} &= A_{k+m}x_{k+m} + B_{k+m}u_{k+m} \\ &= \left(\prod_{i=0}^{m} A_{k+i}\right) x_k + \sum_{i=0}^{m} \left(\prod_{j=i+1}^{m} A_{k+j}\right) B_{k+i}u_{k+i} \end{aligned} \tag{41}$$

Gathering the states on the time horizon $[k : k+p+1]$, (41) becomes:

$$\bar{x}_{k,p} = \mathscr{A}_{k,p}x_k + \mathscr{B}_{k,p}\bar{u}_{k,p}, \quad \bar{x}_{k,p} \in \mathbb{R}^{n(p+1)} \tag{42}$$

with:

$$\overline{x}_{k,p} = \begin{bmatrix} x_{k+1} \\ x_{k+2} \\ \vdots \\ x_{k+p+1} \end{bmatrix}, \mathscr{A}_{k,p} = \begin{bmatrix} A_k \\ A_{k+1}A_k \\ \vdots \\ \prod_{i=0}^{p} A_{k+p-i} \end{bmatrix}, \overline{u}_{k,p} = \begin{bmatrix} u_k \\ u_{k+1} \\ \vdots \\ u_{k+p} \end{bmatrix}$$

$$\mathscr{B}_{k,p} = \begin{bmatrix} B_k & 0 & \dots & 0 \\ A_{k+1}B_k & B_{k+1} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \prod_{i=0}^{p-1} A_{k+p-i}B_k & \prod_{i=0}^{p-2} A_{k+p-i}B_{k+1} & \dots & B_{k+p} \end{bmatrix} \tag{43}$$

To ensure the reference model tracking on the time horizon $[k : k+p+1]$, the control $\overline{u}_{k,p}$ is adjusted in order to minimize the criterion:

$$\Phi_{k,p}(\overline{u}_{k,p}) = \| \overline{x}_{r,k,p} - \mathscr{A}_{k,p}x_k - \mathscr{B}_{k,p}\overline{u}_{k,p} \|_W^2 \tag{44}$$

with $\overline{x}_{r,k,p} = \begin{bmatrix} x_{r,k+1}^T & x_{r,k+2}^T & \dots & x_{r,k+p+1}^T \end{bmatrix}^T \in \mathbb{R}^{n(p+1)}$ and where W is a weighting matrix.
This leads to:

$$\overline{u}_{k,p} = (\mathscr{B}_{k,p}^T W \mathscr{B}_{k,p})^{-1} \mathscr{B}_{k,p}^T W (\overline{x}_{r,k,p} - \mathscr{A}_{k,p}x_k) \tag{45}$$

where only the first computed value of the input is applied to the system at the k^{th} sampling time

$$u_k = \begin{bmatrix} I_{n_u} & 0 & \dots & 0 \end{bmatrix} \bar{u}_{k,p} \tag{46}$$

At the next sampling time, the horizon is shifted and the criterion $\Phi_{k+1,p}$ is optimized in order to obtain and apply the control u_{k+1}.

4.2. Extension to control dependent premise variables

Since the weighting functions $\mu_{i,k}$ of the T-S model that appear in the matrices $\mathscr{A}_{k,p}$ and $\mathscr{B}_{k,p}$ (43) may depend on the control $\bar{u}_{k,p}$, the analytical solution (45) needs to be slightly adapted, as follows:

1. define a threshold δ, a time window width p and set $k = 0$.
2. for the time horizon $[k : k+p+1]$ and for $j = 0$, define $\overline{u}_{k,p}^{(j)}$

3. compute $\mathscr{A}_{k,p}^{(j)}$ and $\mathscr{B}_{k,p}^{(j)}$ with:

$$\left\{\begin{array}{l}
\mathscr{A}_{k,p}^{(j)} = \begin{bmatrix} A_k^{(j)} \\ A_{k+1}^{(j)}A_k^{(j)} \\ \vdots \\ \prod_{i=0}^{p} A_{k+p-i}^{(j)} \end{bmatrix} \\
\mathscr{B}_{k,p}^{(j)} = \begin{bmatrix} B_k^{(j)} & 0 & \dots & 0 \\ A_{k+1}^{(j)}B_k^{(j)} & B_{k+1}^{(j)} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \prod_{i=0}^{p-1} A_{k+p-i}^{(j)}B_k^{(j)} & \prod_{i=0}^{p-2} A_{k+p-i}^{(j)}B_{k+1}^{(j)} & \dots & B_{k+p}^{(j)} \end{bmatrix} \\
A_k^{(j)} = \sum_{i=1}^{r} \mu_{i,k}(u^{(j)}(k))A_i \\
B_k^{(j)} = \sum_{i=1}^{r} \mu_{i,k}(u^{(j)}(k))B_i
\end{array}\right. \tag{47}$$

4. $j \leftarrow j+1$, compute $\mathscr{A}_{k,p}^{(j+1)}$, $\mathscr{B}_{k,p}^{(j+1)}$ and $\overline{u}_{k,p}^{(j+1)}$

$$\overline{u}_{k,p}^{(j+1)} = \left((\mathscr{B}_{k,p}^{(j)})^T W \mathscr{B}_{k,p}^{(j)}\right)^{-1} (\mathscr{B}_{k,p}^{(j)})^T W (\bar{x}_r - \mathscr{A}_{k,p}^{(j)} x_k) \tag{48}$$

5. if $||\overline{u}_{k,p}^{(j)} - \overline{u}_{k,p}^{(j-1)}|| < \delta$, then go to step 3, else apply the k^{th} control input defined by

$$u_k = \left[\begin{array}{cccc} I_{n_u} & 0 & \dots & 0 \end{array}\right] \overline{u}_{k,p} \tag{49}$$

6. $k \leftarrow k+1$ and go to step 2.

4.3. Numerical example

Let us consider the same example (39) as in the previous section with the weighting matrix $W = \mathrm{diag}(0.01, 1, 1)$. The MPC is performed for three steps forward ($p = 2$). Implementing the algorithm (47), the results represented on figure 6 are obtained.

Comparing the results displayed on figure 6, with those on figure 4, it can be

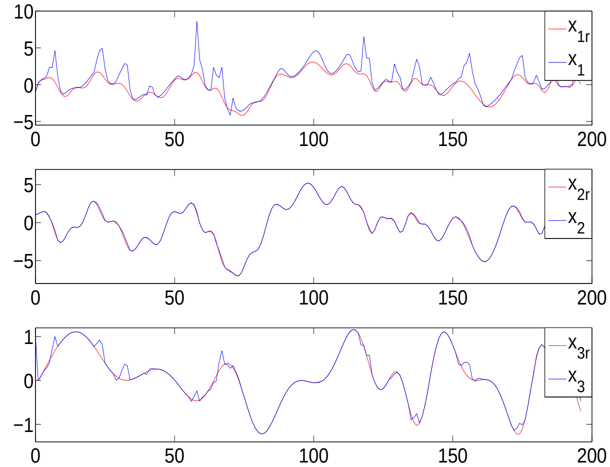

Figure 6: System and reference model states

seen that the anticipation introduced by the MPC approach improves the tracking. In order to quantify this improvement, let us consider the following criterion for each state:

$$\phi_i = \sum_{k=0}^{N} |x_{r,k,i} - x_{k,i}|, \; i = 1, \ldots, n_x \tag{50}$$

where i is the component number of a vector, N is the simulation horizon and $x_{k,i}$ is obtained from non predictive control. The criteria Φ_{ip} are analogously defined with $x_{k,i}$ obtained with MPC. Finally, the performance gain τ_i due to MPC is obtained from $\tau_i = 100\frac{\phi_i - \phi_{ip}}{\phi_i}$. For the considered example, the following improvement is obtained (for each state): $\tau_1 = 12.3\%$, $\tau_2 = 31.2\%$, and $\tau_3 = 30.1\%$.

In order to highlight the influence of the time horizon length p on the tracking performances, the improvement of ϕ_i, namely τ_i, is computed for different time horizons defined by $p \in \{2,3,4,5\}$. The results are gathered in table 1.

One can conclude, for the presented example, that a horizon of length $p = 3$ gives the best results. From the obtained results, it is important to highlight the contribution of the horizon length in the predictive control performances. A too short, as well as a too long horizon may not give the best expected results depending also on the dynamic characteristic of the reference on this time horizon. A compromise is then needed. To quantify the best horizon length, a comparative

	$p=2$	$p=3$	$p=4$	$p=5$
τ_1	10.2%	12.3%	11.5%	12.5%
τ_2	27.4%	31.2%	27.4%	28.5%
τ_3	25.4%	30.1%	25.4%	27.6%

Table 1: Improvement gains for different horizon lengths

study as the one presented may be a good solution.

5. Model Predictive Control for T-S systems with saturated control

In this section, the MPC for T-S systems with control saturation is considered. The tracking objective is maintained, even if each control component of the control input is upper and lower bounded. The objective is to show that, to some extent, the predictive aspect of the control will allow to compensate the saturation effect. Taking into account the control bounds in the control computing allows to anticipate and counteract the saturation effect and thus, improve the reference tracking.

5.1. Proposed strategy

The adopted strategy is the following:

1. On a given horizon of length p, synthesize the nominal control ensuring the tracking without considering the saturation constraints (equation (45) or (48) depending on whether or not the weighting functions depend on the control input)
2. Detect the components of the control $\overline{u}_{k,p}$ that will exceed the saturation levels (n_{sM} and n_{sm} components for respectively the upper and lower limits). Define then two constraint matrices denoted $F^1 \in \mathbb{R}^{n_u(p+1)\times n_{sM}}$ and $F^2 \in \mathbb{R}^{n_u(p+1)\times n_{sm}}$.
   All the entries of F^1_{ij} are null except one entry by column equal to 1 where $F_{ij} = 1$ indicates that the j^{th} upper saturation phenomenon on the time horizon $[k : k+p]$ affects the ℓ^{th} component of u_{k+m}, where $i = mn_u + \ell$. Same construction goes for the matrix F^2.

3. Modify the control depending on the previous equality constraints represented by the matrix $F = \begin{bmatrix} F^1 & F^2 \end{bmatrix}$. Knowing which inputs must be modified due to the saturation, the new criterion to minimize takes into consideration the control constraints with help of the Lagrange's parameter λ:

$$\begin{array}{c} \Phi = \| \overline{x}_{r,k+p+1} - \mathscr{A}_{k+p} x_k - \mathscr{B}_{k+p} \overline{u}_{k,p} \|_W^2 \\ +\lambda^T (F^T \overline{u}_{k,p} - \overline{U}) \end{array} \tag{51}$$

where the matrices $\mathscr{A}_{k+p}$, $\mathscr{B}_{k+p}$ and $\overline{u}_{k,p}$ have been already defined in (43), $\overline{x}_{r,k+p+1}$ corresponds to the gathering of the reference state on the considered horizon, $\overline{U} = \begin{bmatrix} U_{max}^T & U_{min}^T \end{bmatrix}^T$, where $U_{max} \in \mathbb{R}^{n_{sM} \times 1}$ with $U_{max,i} = u_{max}$ for $i = 1, \ldots, n_{sM}$ and $U_{min} \in \mathbb{R}^{n_{sm} \times 1}$ with $U_{min,j} = u_{min}$ for $j = 1, \ldots, n_{sm}$.
4. Test the new control obtained after minimizing Φ, if some control input components still exceed the saturation levels, then go to step (3).

Let us now explicit the control expression in step 3 of the strategy. Derivating Φ (51), the optimality equations with respect to $\overline{u}_{k,p}$ and λ give:

$$\left\{ \begin{array}{l} \mathscr{B}_{k,p}^T W (\mathscr{B}_{k,p} \overline{u}_{k,p} + \mathscr{A}_{k,p} x_k - \overline{x}_{r,k,p+1}) + F\lambda = 0 \\ F^T \overline{u}_{k,p} - \overline{U} = 0 \end{array} \right. \tag{52}$$

or equivalently

$$\left\{ \begin{array}{l} \overline{u}_{k,p} = (\mathscr{B}_{k,p}^T W \mathscr{B}_{k,p})^{-1} \\ \qquad \times (\mathscr{B}_{k,p} W (\overline{x}_{r,k,p+1}) - \mathscr{A}_{k+p} x_k - F\lambda) \\ \lambda = (F^T (\mathscr{B}_{k,p}^T W \mathscr{B}_{k,p})^{-1} F)^{-1} (F^T (\mathscr{B}_{k,p}^T W \mathscr{B}_{k,p})^{-1} \\ \qquad \times \mathscr{B}_{k,p}^T W (\overline{x}_{r,k,p+1} - \mathscr{A}_{k,p} x_k) - \overline{U}) \end{array} \right. \tag{53}$$

Recall that without constraints, the control input is given by:

$$\overline{u}_{k,p}^0 = (\mathscr{B}_{k,p}^T W \mathscr{B}_{k,p})^{-1} \mathscr{B}_{k,p}^T W (x_{r,k,p+1} - \mathscr{A}_{k,p} x(k)) \tag{54}$$

thus (53) becomes:

$$\left\{ \begin{array}{l} \overline{u}_{k,p} = \overline{u}_{k,p}^0 - (\mathscr{B}_{k,p}^T W \mathscr{B}_{k,p})^{-1} F\lambda) \\ \lambda = (F^T (\mathscr{B}_{k,p}^T W \mathscr{B}_{k,p})^{-1} F)^{-1} (F^T \overline{u}_{k,p}^0 - \overline{U}) \end{array} \right. \tag{55}$$

allowing to express the control

$$\left\{ \begin{array}{l} \overline{u}_{k,p} = (I - Q_{k,p}^{-1} F (F^T Q_{k,p}^{-1} F)^{-1} F^T) \overline{u}_{k,p}^0 \\ \qquad + Q_{k,p}^{-1} F (F^T Q_{k,p}^{-1} F)^{-1} \overline{U} \\ Q_{k,p} = \mathscr{B}_{k,p}^T W \mathscr{B}_{k,p} \end{array} \right. \tag{56}$$

Remark 6. *As in the previous section, if the weighting functions depend on the control input, the control law (56) may be computed iteratively.*

5.2. Illustrative example

Let us consider the following example:

$$
\begin{aligned}
A_r &= \begin{pmatrix} 0.8 & 0.4 \\ -0.2 & 0.4 \end{pmatrix}, \quad B_r = \begin{pmatrix} 0 \\ 0.2 \end{pmatrix} \\
A_1 &= \begin{pmatrix} 0.8 & 0.4 \\ -0.3 & 0.9 \end{pmatrix}, \quad B_1 = \begin{pmatrix} 0 \\ 0.17 \end{pmatrix} \\
A_2 &= \begin{pmatrix} 0.8 & 0.4 \\ 0.2 & 0.7 \end{pmatrix}, \quad B_2 = \begin{pmatrix} 0 \\ 0.23 \end{pmatrix}
\end{aligned}
\tag{57}
$$

The weighting functions are control dependent and given by:

$$
\mu_{1,k} = \frac{1+2\tanh(u_k)}{2} \qquad \mu_{2,k} = 1-\mu_{1,k} \tag{58}
$$

The considered saturation is defined as $u_{min} = -1$ and $u_{max} = 0.5$. The simulations were done for different lengths of the prediction horizons p.

In order to quantify the improvement obtained with the proposed approach, the tracking errors between the reference model and the saturated system are compared with an without taking into account the saturation when computing the MPC law.

Let us define the criteria

$$
\begin{aligned}
\phi_{i,s} &= \sum_k |x_{r,k,i} - x_{s,k,i}| \\
\phi_{i,sc} &= \sum_k |x_{r,k,i} - x_{sc,k,i}|
\end{aligned}
\tag{59}
$$

where $\phi_{i,s}$ corresponds to the nominal saturated control (without considering the saturation in the control synthesis) and $\phi_{i,sc}$ for the control law considering the saturation constraint, i.e. the proposed approach (56).

The comparative criterion for each state variable between the two strategies is $\tau_i = 100\frac{\phi_{i,s}-\phi_{i,sc}}{\phi_{i,s}}$. For the considered example, the obtained results are given in table 2 where different values of the prediction horizon are considered.

One can conclude, for the presented example, that a horizon of length $p = 3$ gives the best results. Figure 7 represents the state trajectories when the nominal control is applied without saturation (x_n), when the nominal control is saturated (x_s) and when the proposed MPC law is applied with saturation (x_{sc}). Figure 8 depicts the control input with and without saturation.

One can observe from the obtained results, that the tracking objective is significantly improved (36% for the first state and 21% for the second).

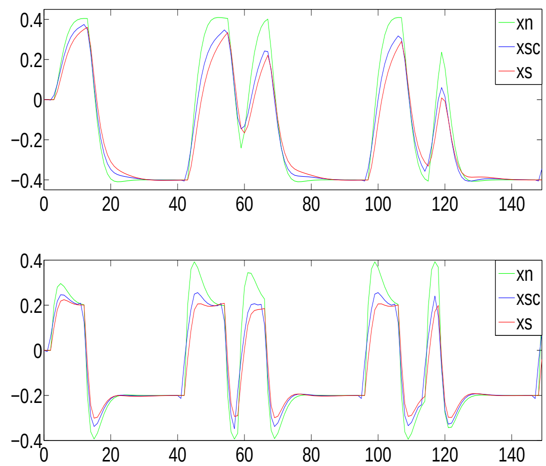

Figure 7: System states

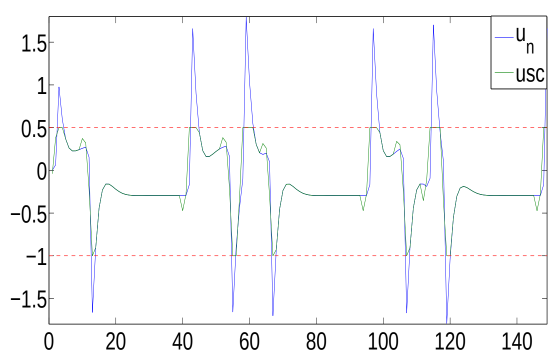

Figure 8: Control inputs

	$p=2$	$p=3$	$p=4$	$p=5$
τ_1	28.8%	36.4%	29.5%	24.6%
τ_2	16.8%	21.2%	16.0%	13.7%

Table 2: Improvement gains for different horizon lengths

6. Conclusion

In this paper, reference model tracking for nonlinear T-S models was considered. Structural conditions for perfect state tracking were established. When these conditions cannot be satisfied, relaxed solutions were proposed, based on quadratic optimal control. The Model Predictive Control for finite time horizon was developed and extended to the case of saturated control inputs.
A first perspective for the present work is to generalize the matching conditions and the structure proposed in section 2 for a general control structure law and establish the relation between the reference model (A_r, B_r) and the system matrices (A_i, B_i). During the study, a strong correlation between the time horizon length and the model reference dynamics was pointed, in fact, a second interesting perspective will be to present a choice criterion that optimizes the time horizon length according to the model reference dynamics.

References

[1] H. Erzberger, *Analysis and design of model following control systems by state space techniques*. NASA techdocs, 1967.

[2] J. Guo, Y. Liu, and G. Tao, "Multivariable MRAC with state feedback for output tracking," in *American Control Conference*, St. Louis, USA, 2009.

[3] R. Curran, "Equicontrollability and the model-following problem," Stanford University, California, USA, Tech. Rep., 1971.

[4] S. Sastry and M. Bodson, *Adaptive control: Stability, Convergence and Robustness*. New Jersey, USA: Prentice Hall, 1989.

[5] H. Khalil, *Nonlinear Systems*. New Jersey, USA: Prentice Hall, 1992.

[6] G. Tao, *Adaptive Control Design and Analysis*. New York, USA: John Wiley and Sons, 2003.

[7] D. Cvetkovic-Ilic, "Re-nnd solutions of the matrix equation $AXB = C$," *The Journal of the Australian Mathematical Society*, vol. 84, pp. 63–72, 2008.

[8] R. Patton, "Fault-tolerant control systems: The 1997 situation," in *IFAC Symposium on Fault Detection Supervision and Safety for Technical Processes*, Hull, UK, 1997.

[9] L. Yang, S. Neild, D. Wagg, and D. Virden, "Model reference adaptive control of a nonsmooth dynamical system," *Nonlinear Dynamics*, vol. 46, no. 3, pp. 323–335, 2006.

[10] S. Joshi, G. Tao, and P. Patre, "Direct adaptive control using an adaptive reference model," *International Journal of Control*, vol. 84, no. 1, pp. 180–196, 2011.

[11] C. Nie and R. Patton, "Fault estimation and MRC-based active FTC," in 18^{th} *IFAC World Congress*, Milano, Italy, 2011.

[12] C. Tseng, B. Chen, and H. Uang, "Fuzzy tracking control design for nonlinear dynamic systems via T-S fuzzy model," *IEEE Transactions on Fuzzy Systems*, vol. 9, no. 3, pp. 381–392, 2001.

[13] S. Tong, T. Wang, and H. Li, "Fuzzy robust tracking control for uncertain nonlinear systems," *International Journal of Approximate Reasoning*, vol. 30, no. 2, pp. 73–90, 2002.

[14] M. Nikolaou, "Model predictive controllers: A critical synthesis of theory and industrial needs," *Advances in Chemical Engineering*, vol. 26, no. 34, pp. 131–204, 2001.

[15] M. Balandat, "Constrained robust optimal trajectory tracking: Model predictive control approaches," Ph.D. dissertation, Technische Universität Darmstadt, Institute of Flight Systems and Automatic Control, Darmstadt, Germany, 2010.

[16] V. Stepanyan, K. Krishnakumar, and N. Nguyen, "Transient performance and asymptotic tracking with filtering robust adaptive control," in *IEEE Aerospace Conference*, Bigsky, Montana, USA, 2009.

[17] V. Stepanyan and K. Krinshnakumar, "MRAC revisited: guaranteed performance with reference model modification," in *American Control Conference*, Maryland, Baltimore, USA, 2010.

[18] H. Yu, M. Deng, T. C. Yang, and D. H. Owens, "Model reference parametric adaptive iterative learning control," in 15^{th} *IFAC World Congress*, Barcelona, Spain, 2002.

[19] F. Chen, J. Tsai, Y. Liao, S. Guo, M. Ho, F. Shaw, and L. Shieh, "An improvement on the transient response of tracking for the sampled-data system based on an improved PD-type iterative learning control," *Journal of The Franklin Institute*, vol. 351, no. 2, pp. 1130–1150, 2014.

[20] M. Phan, R. Longman, and B. a. Panommruttanarug, "Robustification of iterative learning control and repetitive control by averaging," *International Journal of Control*, vol. 86, no. 5, pp. 855–868, 2013.

[21] P. Pipatpaibul and P. Ouyang, "Application of online Iterative Learning Tracking control for quadrotors UAVs," *ISRN Robotics*, vol. 2013, article ID 476153, p. 20, 2013.

[22] B. Chen, H. Uang, and C. Tseng, "Robust tracking enhancement of robot systems including motor dynamics : a fuzzy-based dynamic game approach," *IEEE Transactions on Fuzzy Systems*, vol. 6, no. 4, pp. 538–552, 1998.

[23] A. Isidori and C. Byrnes, "Output regulation of nonlinear systems," *IEEE Transactions on Automatic Control*, vol. 35, no. 2, pp. 131–140, 1990.

[24] K. Shimizu, S. Ito, and S. Suzuki, "Tracking control of general nonlinear systems by a direct gradient descent method," *Nonlinear Dynamics and Systems Theory*, vol. 5, no. 1, pp. 91–105, 2005.

[25] W. Wang and H. Lin, "Fuzzy control design for the trajectory tracking on uncertain nonlinear system," *IEEE Transactions on Fuzzy Systems*, vol. 7, no. 1, pp. 53–62, 1999.

[26] X. Hu, L. Wu, C. Hu, and H. Gao, "Adaptive sliding mode tracking control for a flexible air-breathing hypersonic vehicle," *Journal of The Franklin Institute*, vol. 349, no. 2, pp. 559–577, 2012.

[27] L. Yan and J. Xu, “A model reference robust control with unknown high frequency gain,” in 43^{th} *Conference on Decision and Control*, Atlantis, Bahamas, USA, 2004.

[28] L. Yan, L. Hsu, R. R. Costa, and F. Lizarralde, “Variable structure model reference adaptive control for systems with unknown high frequency gain,” in 42^{th} *Conference on Decision and Control*, Hawaii, USA, 2003.

[29] C. Edwards and S. Spurgeon, “On the development of discontinuous observers,” *International Journal of Control*, vol. 59, no. 1, pp. 1211–1229, 1994.

[30] C. Edwards, S. Spurgeon, and R. Patton, “Sliding mode observers for fault detection and isolation,” *Automatica*, vol. 36, no. 4, pp. 541–553, 2000.

[31] L. Fridman and A. Levant, *Higher Order Sliding Modes*. In Sliding Mode Control in Engineering (W. Perruquetti and J.P. Barbot, eds.), ch. 3, Marcel Dekker, 2002.

[32] L. Hsu, M. Oliveira, and A. Peixoto, “Sliding mode control of uncertain nonlinear systems with arbitrary relative degree and unknown control direction,” in *International Workshop on Variable Structure Systems*, Alghero, Sardinia, Italy, 2006.

[33] L. Grüne and J. Pannek, *Nonlinear Model Predictive Control*, Springer, Ed. Springer, 2011.

[34] A. Alessio and A. Bemporad, “A survey on explicit model predictive control.” in *International Workshop on Assessment and Future Directions of Nonlinear Model Predictive Control*, Pavia, Italy, 2008.

[35] F. Allgower and A. Zhen, *Nonlinear Model Predictive Control*. Boston, Massachusetts, USA: Birkhauser Verlag, 2000.

[36] F. Allgower, R. Findeisen, and L. Biegler, *Assessment and Future Directions of Nonlinear Model Predictive Control*, Springer, Ed., 2007.

[37] T. Takagi and M. Sugeno, “Fuzzy identification of systems and its applications to modeling and control,” *IEEE Transactions on Systems, Man and Cybernetics*, vol. 15, no. 1, pp. 116–132, 1985.

[38] K. Tanaka and H. Wang, *Fuzzy Control Systems Design and Analysis: A Linear Matrix Inequality Approach*. John Wiley & Sons, Inc., 2001.

[39] T. Taniguchi, K. Tanaka, K. Yumafuji, and H. Wang, "A new PDC fuzzy reference models," in *IEEE International Fuzzy Systems Conference*, Seoul, South Korea, 1999.

[40] J. Qiu, G. Feg, and H. Gao, "Static output feedback H_∞ control of continuous-time T-S fuzzy affine systems via piecewise Lyapunov functions," *IEEE Transactions on Fuzzy Systems*, vol. 21, no. 2, pp. 245–261, 2013.

[41] J. Qiu, G. Feng, and H. Gao, "Fuzzy model based piecewise networke nonlinear systems," *IEEE Transactions on Fuzzy Systems*, vol. 18, no. 5, pp. 919–934, 2010.

[42] B. Mansouri, N. Manamanni, K. Guelton, A. Kruszewski, and T. Guerra, "Output feedback LMI tracking control conditions with H_∞ criterion for uncertain and disturbed T-S models," *Information Sciences*, vol. 179, no. 4, pp. 446–457, 2009.

[43] O. Axelsson, *Iterative Solution Methods*, C. U. Press, Ed. Cambridge, Australia: Cambridge UniversityPress, 1994.

[44] C. T. Kelley, *Iterative Methods for Linear and Nonlinear Equations*. Philadelphia, Pennsylvannia, USA: Society for Industrial and Applied Mathematics, 1995.

[45] S. Varrier, D. Koenig, and J. Martinez, "Robust fault detection for uncertain unknown inputs LPV systems," *Control Engineering Practice*, vol. 22, pp. 125–134, 2014.

[46] S. Bezzaoucha, B. Marx, D. Maquin, and J. Ragot, "Model reference tracking control for nonlinear systems described by Takagi-Sugeno structure," in *IEEE International Conference on Fuzzy Systems*, Hyderabad, India, 2013.