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REDUCED-ORDER MODELING OF TWO-SIDED FRICTIONAL INTERFACES

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ABSTRACT

We consider a model describing the behavior of a two-sided interface allowing for both elasticity and microslip of the joint. A reduced-order approximation of this system is developed based on a decomposition of the original model into an elastic chain and a dissipative component equivalent to a series-series Iwan chain. The Iwan chain is then solved using a quasi-static complementarity formulation while the order of the elastic chain is reduced using modal analysis. The computational efficiency of the resulting reduced-order model is significantly increased, while the overall response of the interface to realistic forcing conditions is maintained.

1 INTRODUCTION

The importance of predictive simulation tools is rapidly increasing as computational analysis is replacing experimental verification when the appropriate tests are impractical or prohibitively expensive. To develop truly predictive simulations one must overcome both numerical error (discretization errors in space and time) and error arising from modeling assumptions, such as missing physics and uncertain boundary conditions.

In particular, the behavior of mechanical joints and interfaces is often a particularly troubling collection of the "missing physics". The development of tribological models capable of representing the contact between adjacent members is an active area of research and has led to significant advances in our understanding of frictional contact. However, the incorporation of such descriptions into large-scale structural dynamics models is nonetheless problematic due to issues of scale. The time- and length-scales associated with dynamics of the mechanical joints are typically several orders of magnitude smaller that the scales associated with the larger structural simulation.

The nonlinearities and dissipation associated with the interface are often associated with microslip, associated with partial slipping of the contact interface, and contrasted with macroslip, whereby the whole interface undergoes relative displacement. This localized microslip often contributes to a significant fraction of the overall measured damping in a complex structure and is not well represented over a wide range of operating conditions by linear models. Thus truly predictive models for structural dynamics will require an accurate description of the behavior at and near the interface, and in particular the dissipation induced by microslip.

The most straightforward approach to represent microslip in a larger structural model is to resolve directly the interface in a finite element model [1]. Unfortunately, the small length scales required to capture the mechanics of microslip lead to a problem for which the time required to generate a computational solution is prohibitively long [2]. One common resolution to these computational restrictions is to incorporate the observed dissipation and elasticity into a linear joint model with effective mass, damping and stiffness parameters, which must then be estimated to match experimentally observed results. However, as mentioned above, the identified parameters are specific to the operating conditions (load levels, boundary conditions, etc.) of the experimental test. The joint model cannot be extrapolated to markedly different operating conditions—the joint model is no longer predictive.

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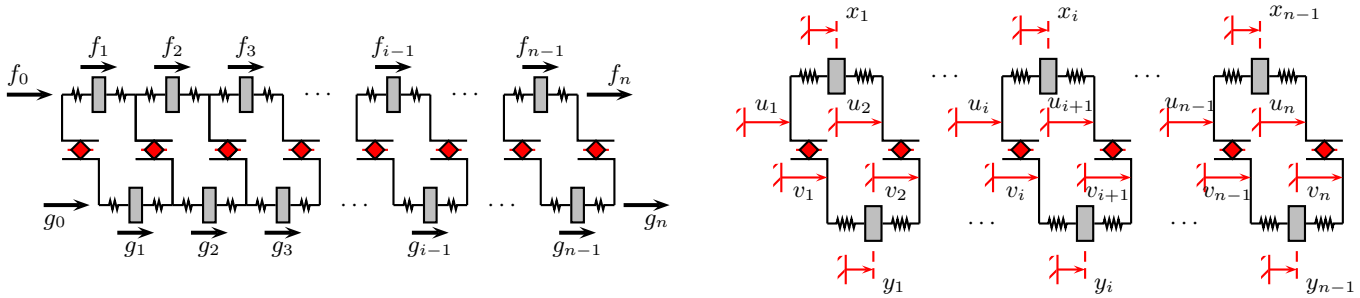

Figure 1: Discrete model.

The role of friction and microslip has been incorporated into several nonlinear reduced order models for the joint based on descriptions of the slip interface in the joint. Menq et al., develop a continuum model representing the microslip that arises in frictional dampers [3; 4]. Quinn and Segalman consider a similar model and show that by varying the spatial distribution of the frictional intensity the predicted dissipation is representative of experimentally observed scalings [5]. Discrete models of the interface are often based on combinations of spring-slider elements, as considered by Iwan [6; 7]. Segalman has developed a four parameter Iwan model that is capable of reproducing the qualitative properties of the joint dynamics [8]. Meanwhile, Song et al., have developed an adjusted Iwan beam element (AIBE) based on a parallel-series Iwan model that can be incorporated naturally into an existing finite element framework [9; 10]. With the proper identification of the model parameters, the AIBE can be used to capture experimentally observed profiles for the response of jointed structures.

2 TWO-SIDED INTERFACE MODEL

To begin consider a single series of Iwan elements, made up of n interfaces and $2\,(n-1)$ masses as shown in Figure 1. In this model each element is assumed to be identical, with a mass m, and a stiffness k respectively. The forces f_i and g_i, $i = 1, \ldots, n-1$ represent the shear loading applied to the masses in the i^{th} component, while f_0 and g_0 (f_n and g_n) describe the forces acting on the left (right) edge of the interface. In addition, each interfaces is described through the frictional force σ_i. For this system the equations of motion can be written as

$$
\begin{aligned}
m\,\ddot{x}_i + 2\,k\,x_i - k(u_i + u_{i+1}) &= f_i, \ \text{Upper masses} \\
m\,\ddot{y}_i + 2\,k\,y_i - k(v_i + v_{i+1}) &= g_i, \ \text{Lower masses}
\end{aligned}
$$

$$
\begin{aligned}
\sigma_1 + k(x_1 - u_1) + f_0 &= 0, \ \text{1-st Slider} \\
-\sigma_1 + k(y_1 - v_1) + g_0 &= 0, \\
\sigma_i + k(x_{i-1} - 2\,u_i + x_i) &= 0, \ i\text{-th Slider} \\
-\sigma_i + k(y_{i-1} - 2\,v_i + y_i) &= 0, \\
\sigma_n + k(x_{n-1} - u_n) + f_n &= 0, \ n\text{-th Slider} \\
-\sigma_n + k(y_{n-1} - v_n) + g_n &= 0,
\end{aligned}
$$

This is essentially a two-sided interface problem, in which deformation can arise on either side of the frictional interface, while the force arising from the interface depends on the relative velocity across the interface. From these equations, the following coordinates can be identified

$$
w_i = \frac{x_i + y_i}{2}, \qquad z_i = \frac{x_i - y_i}{2},
$$
$$
p_i = \frac{u_i + v_i}{2}, \qquad q_i = \frac{u_i - v_i}{2},
$$

where (w_i, z_i) and (p_i, q_i) represent the average and relative displacements across the masses and sliders respectively. With these, the equations describing the evolution

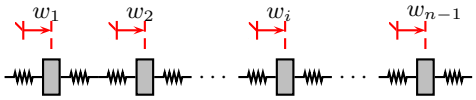

Figure 2: Elastic component.

of w_i decouple from those on z_i to yield

$$\begin{aligned} m\,\ddot{w}_i + 2\,k\,w_i - k\,(p_i + p_{i+1}) &= \frac{f_i + g_i}{2}, \\ 2\,k\,(w_1 - p_1) + (f_0 + g_0) &= 0, \\ (w_{i-1} - 2\,p_i + w_i) &= 0, \\ 2\,k\,(w_{n-1} - p_n) + (f_n + g_n) &= 0, \end{aligned}$$

$$\begin{aligned} m\,\ddot{z}_i + 2\,k\,z_i - k\,(q_i + q_{i+1}) &= \frac{f_i - g_i}{2}, \\ 2\,\sigma_1 + 2\,k\,(z_1 - q_1) + (f_0 - g_0) &= 0, \\ \sigma_i + k\,(z_{i-1} - 2\,q_i + z_i) &= 0, \\ 2\,\sigma_n + 2\,k\,(z_{n-1} - q_n) + (f_n - g_n) &= 0. \end{aligned}$$

In the above equations on w_i and p_i, the interface forces σ_i are absent and the response of this set is independent of the interface. Solving for p_i yields

$$\begin{aligned} p_1 &= w_1 + \frac{f_0 + g_0}{2\,k}, \\ p_i &= \frac{w_{i-1} + w_i}{2}, \\ p_n &= w_{n-1} + \frac{f_n + g_n}{2\,k}. \end{aligned}$$

These can then be returned to the equations for $\ddot{w}_i$ to yield

$$\begin{aligned} m\,\ddot{w}_1 + \frac{k}{2}\,(w_1 - w_2) &= \left(\frac{f_0 + g_0}{2}\right) + \left(\frac{f_1 + g_1}{2}\right), \\ m\,\ddot{w}_i + \frac{k}{2}\,(-w_{i-1} + 2\,w_i - w_{i+1}) &= \left(\frac{f_i + g_i}{2}\right) \\ & \qquad i = 2, \ldots, n-2 \\ m\,\ddot{w}_{n-1} + \frac{k}{2}\,(-w_{n-2} + w_{n-1}) &= \left(\frac{f_n + g_n}{2}\right) \\ & \quad + \left(\frac{f_{n-1} + g_{n-1}}{2}\right). \end{aligned}$$

Therefore these equations are equivalent to those describing the response of an elastic chain, as represented in Figure 2.

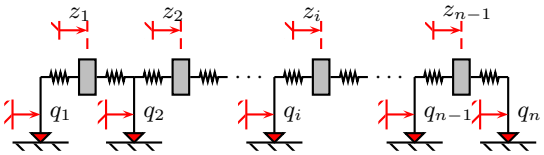

Figure 3: Dissipative component.

The response described by w_i is conservative, so that the dissipation in the system arises solely from the equations on z_i, for which the equations of motion can be written as

$$m\,\ddot{z}_i + k\,((z_i - q_i) + (z_i - q_{i+1})) = \frac{f_i - g_i}{2}, \qquad (1)$$

with

$$\begin{aligned} \sigma_1 &= -\left(\frac{f_0 - g_0}{2}\right) - k\,(z_1 - q_1), \\ \sigma_i &= -k\,\left((z_{i-1} - q_i) + (z_i - q_i)\right) \\ & \qquad i = 2, \ldots, n-1, \\ \sigma_n &= -\left(\frac{f_n - g_n}{2}\right) - k\,(z_{n-1} - q_n). \end{aligned}$$

Therefore, these equations represent the dissipative structure shown in Figure 3, a series-series Iwan system [5]. The decoupling of the governing equations does not rely on the form of the dissipative model used for σ_i. In addition, these forces need be neither uniform nor constant in time.

For this component, the dissipative power can be determined as

$$P_d(t) = \sum_{i=1}^{n} \sigma_i(t) \cdot \dot{q}_i(t),$$

so that the work done by the dissipative component, and therefore the total interface model, becomes

$$-D(t) = \int_0^t \sum_{i=1}^{n} \sigma_i(\tau) \cdot \dot{q}_i(\tau)\; d\tau.$$

Finally, the displacement across the interface can be described from these decoupled components as

$$\begin{aligned} \Delta_1 &= u_n - v_1 \sim (w_{n-1} - w_1) + (q_n + q_1), \\ \Delta_2 &= v_n - u_1 \sim (w_{n-1} - w_1) - (q_n + q_1). \end{aligned}$$

In [5] a spatially non-uniform frictional intensity is considered and is shown to yield measures for the dissipation that is consistent with experimental data.

We see from the above analysis that the behavior of a two-sided interface model can be represented as an elastic component, combined in an appropriate manner with a dissipative chain, which is represented as a series-series Iwan network of Jenkins elements. However, such an approach still suffers from the same restrictions on the computational timesteps that plagued the original fine discretization of the finite element model. Specifically, for $2\,(n-1)$ elements in the original element, the equivalent longitudinal stiffness of the overall structure is

$$K_{\rm eq} = \frac{k}{n-1},$$

where k is the stiffness of each individual spring. In addition, if the total mass of the interface is $M_{\rm eq}$ then each individual element has mass $m = M_{\rm eq}/(2\,(n-1))$. Therefore if the lowest characteristic frequency of the interface scales as $\omega_{\rm c} = \sqrt{K_{\rm eq}/M_{\rm eq}}$, then the largest characteristic frequency scales as

$$\omega_{\rm max} = \sqrt{\frac{2\,k}{m}} = 2\,(n-1)\,\sqrt{\frac{K_{\rm eq}}{M_{\rm eq}}} = 2\,(n-1)\,\omega_{\rm c}.$$

Thus if $\omega_{\rm c}$ describes the characteristic frequency of the interface and the computational timescale of a larger structural model, the computational timescale with the inclusion of this joint model must be increased by a factor of n.

To serve as a basis for capturing the contribution of the interface on the overall response, computationally efficient methods must be developed for the solution of this interface representation. The computational effort required to efficiently represent the elastic component in the above interface model can be significantly reduced with the application of a straightforward modal decomposition, retaining only the lowest order modes that are commensurate with the timescale of the larger structural model. However, one must still reduce the computational effort directed toward the dissipative one-sided interface model described by Eqs. (1). One such approach is described below.

3 COMPLEMENTARITY FORMULATION FOR THE DISSIPATIVE CHAIN

In the series-series Iwan chain, the dissipation can be accurately captured by neglecting the mass in each Iwan element, again provided that the timescale of the external loading is sufficiently long—effectively solving for the response of the system quasistatically. Therefore, a finer mesh of dissipative Iwan elements can be used to describe partial slip states without the computational penalty of prohibitively small time steps required for numerical stability.

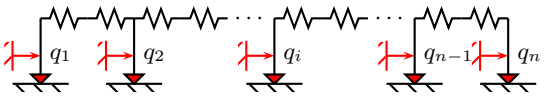

Figure 4: Massless dissipative component.

With $m = 0$ the displacement of z_i is

$$z_i = \frac{f_i - g_i}{4\,k} + \frac{q_i + q_{i+1}}{2},$$

so that the equations of motion for the massless dissipative chain can be written as

$$\begin{aligned}
k\,(q_1 - q_2) &= (f_0 - g_0) + \left(\frac{f_1 - g_1}{2}\right) + 2\,\sigma_1, \\
k\,(-q_{i-1} + 2\,q_i - q_{i+1}) &= \\
&\left(\frac{f_{i-1} - g_{i-1}}{2}\right) + \left(\frac{f_i - g_i}{2}\right) + 2\,\sigma_i \\
&\qquad i = 2, \ldots, n-1, \\
k\,(-q_{n-1} + q_n) &= (f_n - g_n) + \left(\frac{f_{n-1} - g_{n-1}}{2}\right) + 2\,\sigma_n,
\end{aligned}$$

and are representative of the dissipative structure in Figure 4, a massless series-series Iwan model. The above equations can be written in a compact matrix form as

$$[\boldsymbol{K}] \cdot \{\boldsymbol{q}\} = \{\boldsymbol{R}\}, \tag{2}$$

and solved quasistatically at each time step. A complementarity approach was implemented to solve the matrix equations. An initial assumption array on the slip state of each slider is created apriori to each time step. The slider is assumed to be in a state of stiction, slipping right, or slipping left. Based on the initial assumption, the displacements of the slider coordinates and displacements and velocities of the elastic coordinates are calculated for that time step. A solution array is then formulated based on the state of each slider after the time step. If the solution and assumption arrays agree the time step is then advanced, else the assumption array is updated according to the solution array and the time step is repeated. This process continues until convergence. However, if the iteration fails to converge, the magnitude of the time step is reduced and the process repeats.

In what follows we assume that the interface is subject only to forces applied at the edges of the interface, of the

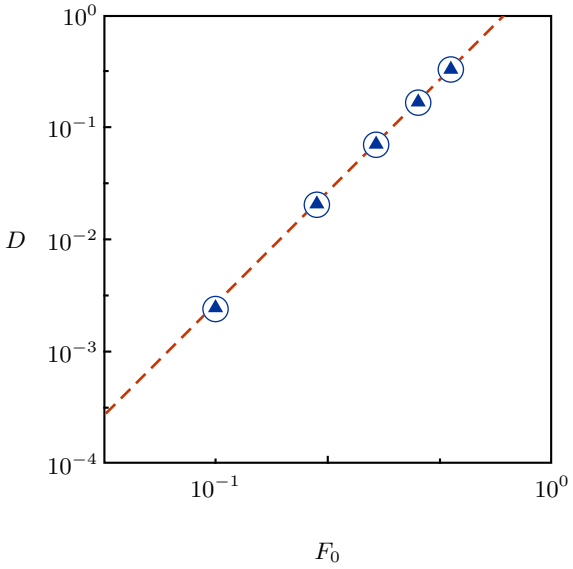

Figure 5: Numerical simulations of the discrete dissipative component ($n = 40$, $\omega = \pi/15$). The open circles represent original Iwan with mass while the filled triangles describe the complementarity formulation. Finally, the solid line is the continuum prediction $D = 8/3\,(F_0)^3$, based on [5].

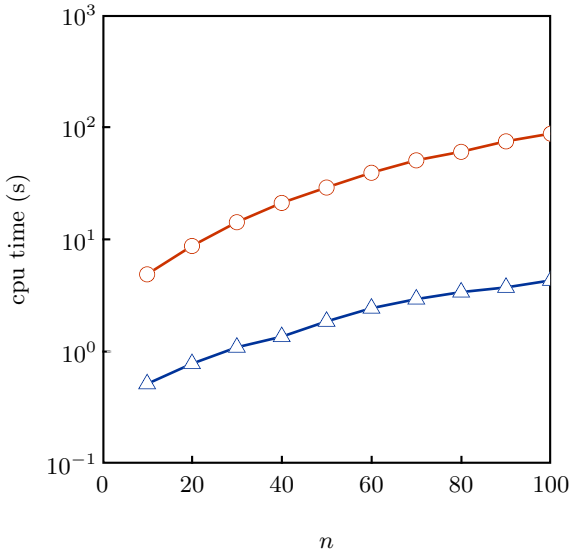

Figure 6: CPU time required for solution of the Iwan dissipative chain as n varies ($F_0 = 0.30$, $\omega = \pi/15$). The open circles indicate the original differential formulation while the triangles denote the CPU time required for the complementarity approach.

form

$$f_0(t) = 0, \qquad f_n(t) = F_0 \sin(\omega\, t),$$
$$g_0(t) = -F_0 \sin(\omega\, t), \qquad g_n(t) = 0.$$

The dissipative friction forces at the interface are represented with Coulomb friction. When the interface is in a state of slip, $\sigma_i = -\mu_i\, N_i\, \mathrm{sgn}(\dot{u}_1 - \dot{v}_1)$, where μ_i and N_i are the coefficient of friction and the normal load acting on the i^{th} interface. In a state of sticking, the interfacial force σ_i satisfies the inequality $|\sigma_i| \leq \mu_i\, N_i$ and this constraint must be satisfied for the dissipative network, regardless of the numerical scheme employed to solve the equations.

From the discrete dissipative system given by Eqs. (1), the dissipation per forcing cycle is illustrated in Figure 5 for $n = 40$ as the forcing amplitude F_0 varies. Here the equivalent stiffness of the interface is chosen to be $K_{\mathrm{eq}} = 1$, as is the total mass $M_{\mathrm{eq}} = 1$. The excitation frequency $\omega = \pi/15$ is chosen well below the characteristic frequency of the interface $\omega_{\mathrm{c}} = 1$. Nonetheless, the computational timescale is limited by the characteristic frequency $\omega_{\mathrm{max}} = 2\,(n-1)$ imposed by the discretization, although we expect the interface to respond almost quasi-statically to the applied load. The large open circles describe the numerical simulations of the original model (with mass) using the built-in numerical solver `ode45` within MATLAB. The smaller filled triangles, which are almost coincident with the original model, represent the dissipation arising from the complementarity formulation of the dissipative chain. Finally, for comparison, the prediction from the quasi-static continuum limit, described by [5], is shown as the solid line and can be expressed as

$$D_{\mathrm{continuum}} = \frac{8}{3}\,(F_0)^3.$$

Thus the dissipation of the Iwan chain is well represented by the complementarity approach, in which the mass is neglected. However, the computational effort required for the later approach is significant reduced, compared to the formulation of the dissipative chain with mass. Figure 6 shows the computational time required to simulate 3 complete forcing cycles with $\omega = \pi/15$ and for a forcing amplitude of $F_0 = 0.30$, which is equivalent to 30% of the load required to initiate macroslip. The computational time required for the complementarity approach is at least an order of magnitude less than that required for the differential formulation. Note that all simulations in this paper were carried out on a Dell Latitude D610 with a 1.60 GHz Intel(R) Pentium(R) M processor.

4 REDUCED-ORDER ELASTIC INTERFACE MODEL

The elastic and dissipative chains, when reduced in the manner described above, can be combined to formulate a computationally efficient model for the two-sided interface. The elastic chain is subjected to a modal reduction, retaining only the lowest M linear modes for the elastic chain, denoted as $\boldsymbol{\phi}_i$, $i = 1, \ldots, M$, so that the response of this component is then given as

$$w_i(t) = \sum_{j=1}^{M} A_j(t) \, [\boldsymbol{\phi}_j]_i$$

Therefore the elastic component is approximated by a reduced-order model of the form

$$\hat{\boldsymbol{M}} \, \ddot{\boldsymbol{A}} + \hat{\boldsymbol{K}} \, \boldsymbol{A} = \hat{\boldsymbol{f}},$$

with

$$[\hat{\boldsymbol{M}}]_{jk} = \boldsymbol{\phi}_j^T \boldsymbol{M} \boldsymbol{\phi}_k, \quad [\hat{\boldsymbol{K}}]_{jk} = \boldsymbol{\phi}_j^T \boldsymbol{K} \boldsymbol{\phi}_k, \quad [\hat{\boldsymbol{f}}]_j = \boldsymbol{\phi}_j^T \boldsymbol{f},$$

while the dissipative component is incorporated with the complementarity formulation described above.

For $n = 40$, so that there exist 40 slip locations across the interface, the total steady-state response across the two-sided interface model, Δ_1, is shown in Figure 7. Specifically, the response for the original differential formulation is shown in Figure 7a while the response of the reduced-order model is depicted in Figure 7b. In each panel the contribution from each component is shown, in addition to the total response across the interface. The time interval shown corresponds to a single forcing cycle after the decay of the transient response.

Comparing the difference between the two formulations, the error in the total displacement between the differential and reduced-order formulation is shown in Figure 8 as the model size n increases. In the reduced-order formulation $M = 10$ modes have been retained in the elastic component. As the resolution of the interface increases the reduced-order model approaches the differential formulation. As seen in Figure 9, as n increases the error in dissipation D between the differential and reduced-order formulation decreases. Finally, the computational time required for the numerical solution of these two formulations is illustrated in Figure 10. The computational time required for the original differential formulation appears to grow at a much faster rate than that of the reduced-order model and for $n = 100$ requires two orders of magnitude longer to simulate. In addition to the computational efficiency of the complementarity formulation, the modal analysis applied to the elastic chain is significantly more efficient than the simulation of the original n-degree-of-freedom elastic chain.

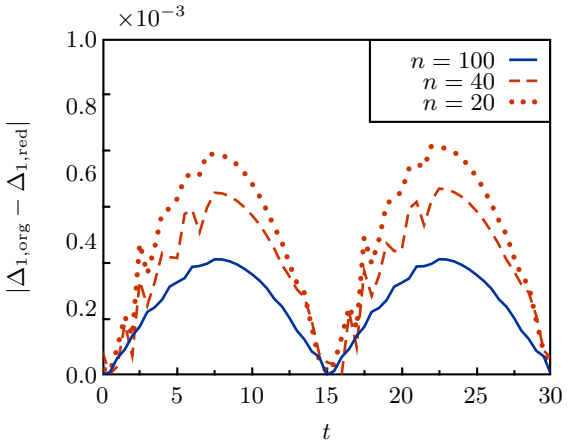

Figure 8: Difference in total displacement Δ_1 between the differential and reduced-order formulation of the interface model ($M = 10$, $F_0 = 0.30$, $\omega = \pi/15$).

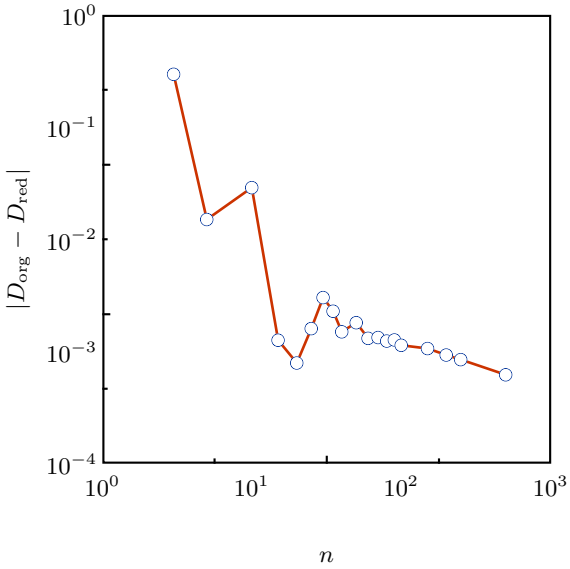

Figure 9: Error in the total dissipation per cycle $|D_{\text{org}} - D_{\text{red}}|$ with increasing model size ($M = 10$, $F_0 = 0.30$, $\omega = \pi/15$).

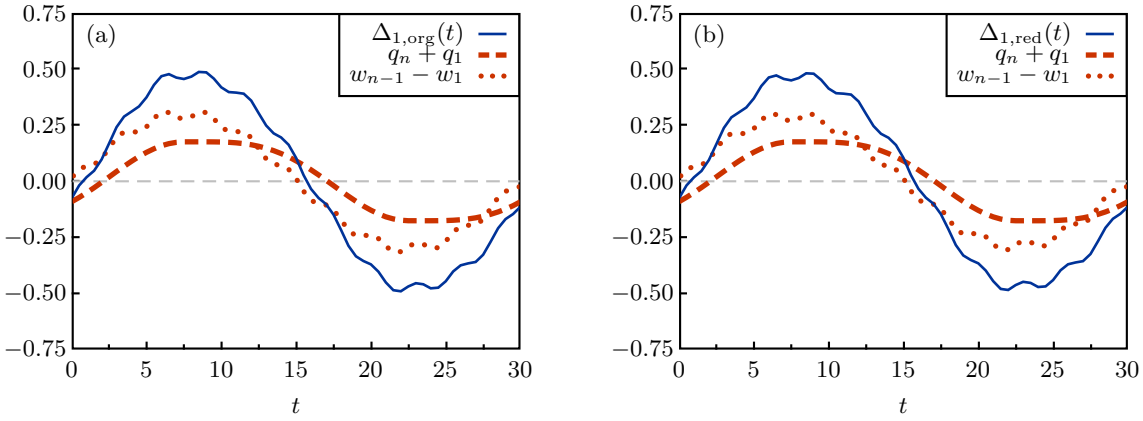

Figure 7: Numerical response across the interface to harmonic loads ($n = 40$, $F_0 = 0.50$, $\omega = \pi/15$) simulations of the discrete dissipative component. In each panel the solid curve describes the total displacement across the interface; (a) original model, (b) complementarity formulation.

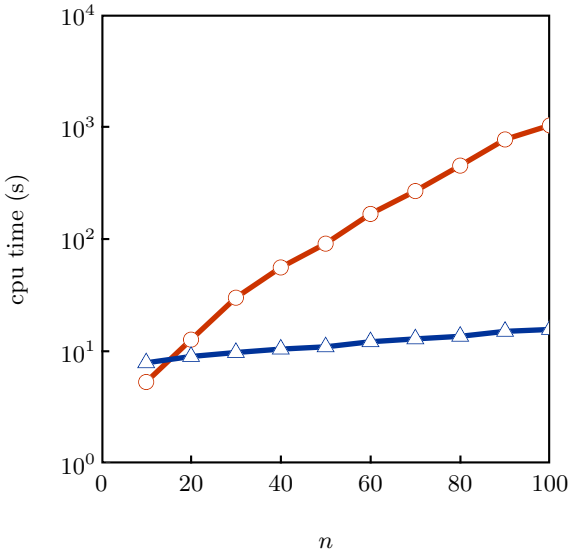

Figure 10: CPU time required for solution of the full interface model as n varies ($F_0 = 0.30$, $\omega = \pi/15$). The open circles indicate the original differential formulation while the triangles denote the CPU time required for the reduced-order model ($M = 10$).

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