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PIPELINE SAFETY ASSESSMENT AT INSPECTION TIME

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Abstract
This work presents a reliability model for determining the pipeline safety after obtaining information on the corrosion damage of the piping system by non-destructive inspection. The model is used to estimate pipeline system reliability in every region containing detected corrosion defects and the probability of either global system failure or of a given pipeline segment. The global failure probability incorporates the contribution of defects that are undetectable by the inspection tool. For this, the probability density functions of maximum depths and the number of undetected corrosion defects by the inspection tool are also determined. The failure associated to each corrosion defect is determined from the pipeline resistance and fluid pressure. The model is applied to a pipeline segment of a given length showing that defect measurement errors and the contribution of corrosion defects, undetected by inspection tool, can significantly influence the value of failure probability of the system. To calculate the failure probability of the system the directional simulation technique is utilized.

Keywords: Pipeline, corrosion defect, detected and undetected defect, reliability, inspection.

1. Introduction
One of the main causes of deterioration in pipeline systems used by oil industry is the corrosion, originated by chemical agents of the fluid they transport (internal corrosion) and by external environment (external corrosion). The information obtained by non-destructive inspection gives enough certainty about the corrosion damage of the system, which has an effect on the decisions about future inspection and maintenance strategies. The expensive economic cost of inspections implies their periodical realization at intervals of several years. Inspection results allow knowing the system damage only partially, since inspection tools are incapable of detecting and measuring all defects. The reliability estimates will be influenced by the quality of the inspection tool. Mathematical tools that allow considering the contribution of defects undetected during the inspection need to be developed and thus estimating the system reliability with more likelihood.

In this work a probabilistic model is introduced that allows estimating the pipeline reliability in each defect-containing region. It can also serve to calculate failure probability of the system or of the pipeline segment of a given length considering only detected defects. The model also allows considering the contribution to failure probability of the defects undetectable by inspection tool. The analysis evaluates pipeline performance in each corroded region, which is associated with the stress state in the pipeline wall due to fluid pressure in that region. In addition, the analysis incorporates the defect measurement error.

2. Reliability model
In operating conditions the failure of a pipeline with a corrosion defect will occur when the resisting pressure p_R^d, is smaller than the demand pressure p_D. If these pressures are treated as uncertain, they can be respectively expressed by $P_R^d = P_R^d(\boldsymbol{X}_R, \boldsymbol{z}_R, l, d_{max})$ and $P_D = P_D(\boldsymbol{X}_D, \boldsymbol{z}_D)$, where $\boldsymbol{X}_R$ is a vector of random variables describing the pipeline geometry (diameter, wall thickness) and constitutive function of material (stress-strain relation). $\boldsymbol{X}_D$ is a vector of random variables describing the demand pressure variation at point (x, y), where x describes the pipeline longitudinal position and y the vertical position in relation to an appropriate Cartesian system. $\boldsymbol{z}_R$ and $\boldsymbol{z}_D$ are vectors of deterministic variables. A corrosion defect can be defined using its depth $d(x)$ and position x, which defines a geometrical shape that for practical purposes herein is represented by a function with maximum depth d_{max} and length l. Based on the above assumptions, the pipeline failure can be represented by the safety margin:

$$W(\boldsymbol{X}, \boldsymbol{z}, l, d_{max}) = P_R^d(\boldsymbol{X}_R, \boldsymbol{z}_R, l, d_{max}) - P_D(\boldsymbol{X}_D, \boldsymbol{z}_D) \tag{1}$$

The failure system occur when $W(\boldsymbol{X}, \boldsymbol{z}, l, d_{max}) \leq 0$, where $\boldsymbol{z} = \{\boldsymbol{z}_R, \boldsymbol{z}_D\}$ and $\boldsymbol{X} = \{\boldsymbol{X}_{R,} \boldsymbol{X}_D\}$ with joint probability function $f_{\boldsymbol{X}}(\cdot)$. Thus, the failure probability of the system with a corrosion defect of given length and maximum depth can be estimated by the equation:

$$P_F = \int_{W \leq 0} f_{\boldsymbol{X}}(\boldsymbol{x})\, d\boldsymbol{x} \tag{2}$$

The integration symbol denotes a multiple integral. If data obtained from an inspection performed at a given time period are available, including geometry of n_D corrosion defects detected or, at least, maximum corrosion depth and length of each defect, the system failure can be expressed as the union of the events:

$$G_D = \bigcup_i^{n_D} (W_i(\cdot) \leq 0)$$

where W_i is the safety margin of defect i. The failure probability is obtained by the equation:

$$P_F = \int_{G_D} f_X(\boldsymbol{x})\, d\boldsymbol{x} \tag{3}$$

Rigorously, at the time of inspection, the system failure will be also associated with an uncertain number $N_{ND} = n$ of undetected defects, whose size is also uncertain, so the system failure can be expressed as:

$$P_F = \sum_{n=1}^{\infty} \int_{G_T} \int_0^{\infty} \int_0^{\infty} f_{\boldsymbol{X}}(\boldsymbol{x})\, f_L(l \mid d)\, f_{Dmax|Detec=0}(d)\, g_{ND}(n)\, dd\, dl\, d\boldsymbol{x} \tag{4}$$

where

$$G_T = \bigcup_{i=1}^{n_D} (W_i(\cdot) \leq 0) \bigcup_{j=1}^{N_{ND}=n} (W_j(\cdot) \leq 0)$$

$g_{ND}(\cdot)$ is the probability mass function of the number of undetected defects, $f_{Dmax\,|Detec\,=0}(\cdot)$ is the probability density function of maximum depths of these defects and $f_L(\cdot)$ is the probability density function of defects length conditional to maximum depths. Unlike the Eq. (3), the number of defects in the Eq. (4), is uncertain and equal to $n_D + N_{ND}$. The main problem to estimate the Eq. (4) consists of evaluating these latter functions, described above, that operate only on undetected defects.

The relation between the length and maximum depth of corrosion defects can be considered independent from the detection or non detection of the defect, $f_L(\cdot)$ can thereby be obtained from a sample of corrosion lengths and depths of each defect measured during inspection. In the following expressions, the events $Detec = 1$ and $Detec = 0$ indicate detection and non detection, respectively.

According to Alamilla et al. [1], the probability density function of maximum depths of undetected defects can be obtained from the quality of the inspection tool as well as from the maximum depths of defects detected and measured by the inspection tool. This function was obtained from a Bayesian analysis and is given by the following equation:

$$f_{Dmax|Detec=0}(d) = \frac{p_D}{1 - p_D} \frac{1 - F_{Detec=1}(d)}{F_{Detec=1}(d)} f''_{Dmax|Detec=1}(d) \tag{5}$$

where the quality of inspection tool is expressed in terms of $F_{Detec=1}(d)$, that is the probability to detect a corrosion defect with maximum depth d. $f''_{Dmax|Detec=1}(d)$ is the Bayesian updated probability density function of maximum depths of detected defects and is obtained from inspection data. p_D is the probability of detecting a defect of uncertain size and is obtained as:

$$p_D = \int_{d_0}^{\infty} F_{Detec=1}(x)\, f_{Dmax}(x)\, dx \tag{6}$$

d_0 is the minimum depth considered in the analysis and is associated with the total number of detected defects n''_D. The probability density function of maximum corrosion depths of detected and undetected defects is given by:

$$f_{Dmax}(d) = \frac{p_D}{F_{Detec=1}(d)} f''_{Dmax|Detec=1}(d)$$

(7)

More details about the development of the last equations are given in Alamilla et al. [1]. According to Shibata [2], in this work the distribution of detected defects is Poisson with rate n''_D. According to Alamilla et al. [1], under assumption that here the number of undetected defects also comes from a Poisson probability mass function with rate:

$$n''_{ND} = n''_D (1 - p_D) / p_D \tag{8}$$

Then the probability mass function $g_{N_T}(\cdot)$ of the sum of detected and undetected defects will be Poisson with rate $n''_D + n''_{ND}$.

According to Eq. (8) if the quality of inspection is excellent so that $p_D = 1$, then $n''_{ND} = 0$ and the total number of defects only corresponds to n''_D. If $p_D = 1/2$, then $n''_{ND} = n''_D$. If the quality of inspection is very bad, then $p_D \to 0$ and $n''_D \to 0$. In general, the quality of inspection can be expected to be neither excellent nor very bad, but at an intermediate level, in which the number of significant defects detected is greater than the number of undetected defects. Since the number of undetected defects corresponds to a pipeline interval of a given length l_S, then the distance between undetected corrosion defects is given by an exponential probability density function with the parameter n''_{ND}/l_S.

On the other hand, it is possible to relate the probability density function $f_{Dmax|Detec=0}(\cdot)$ to the function describing the rate of undetected defects whose maximum depth exceeds a given value, obtained as follows:

$$\begin{aligned} \nu''_{Dmax>d|Detec=0}(d) &= n''_{ND}\left[1 - F_{Dmax|Detec=0}(d)\right] \\ &= \frac{1-p_D}{p_D}\, n''_D \left[1 - F_{Dmax|Detec=0}(d)\right] \end{aligned} \quad , d \ge d_0 \qquad (9)$$

where $F_{Dmax|Detec=0}(d) = \int_{d_0}^{d} f_{Dmax|Detec=0}(x)\,dx$. Also, the total rate of defects that describes the total number of defects with depths greater than d, can be expressed as

$$\begin{aligned} \nu''_{Dmax>d}(d) &= \frac{n''_D}{p_D}\left[1 - F_{Dmax}(d)\right] \\ &= \left(n''_D + n''_{ND}\right)\left[1 - F_{Dmax}(d)\right] \end{aligned} \quad ; d \ge d_0 \qquad (10)$$

where $F_{Dmax}(d) = \int_{d_0}^{d} f_{Dmax}(x)\,dx$. Using the total probability theorem it is easy to show that Eq. (10) can be expressed as sum of rates of detected and undetected defects as follows:

$$\nu''_{Dmax>d}(d) = \nu''_{Dmax>d|Detec=0}(d) + \nu''_{Dmax>d|Detec=1}(d); d \ge d_0 \qquad (11)$$

Where:

$$\nu_{D\max>d|Detec=1}(d) = n''_D\left(1 - F_{D\max>d|Detec=1}(d)\right) \qquad (12)$$

and $F_{D\max>d|Detec=1}(d) = \int_{d_0}^{d} f''_{D\max>d|Detec=1}(x)\,dx$

3. Failure function

According to the mechanical model proposed by Oliveros et al. [3], if a pipeline has a corrosion defect whose geometric shape can be represented by the function $d(x)$, wherein x is in an appropriate Cartesian system that defines the position in the longitudinal direction and the corresponding depth $d(x)$ in the perpendicular direction; then, the resistant pressure of a pipeline with a corrosion defect can be estimated using the expression:

$$p_R^d = p_{LongGroove}^d + \left(p_{PlainPipe}^d - p_{LongGroove}^d\right) g_d^{\min} \qquad (13)$$

that in accordance with Cronin and Pick [4] $p_{PlainPipe}^d = p_{PlainPipe}^d(\mathbf{z})$ is the resistant pressure of a pipeline without corrosion defects representing an upper limit, whereas the vector $\mathbf{z} = (\mathbf{z}_g, \mathbf{z}_m)$ consist on the vector of geometrical properties of the pipe, $\mathbf{z}_g$, and the vector $\mathbf{z}_m$ of mechanical properties of the steel; $p_{LongGroove}^d = p_{LongGroove}^d(\mathbf{z}, d_{max})$ is the resistant pressure of the pipeline with a corrosion defect whose geometry corresponds to a groove of infinite length and depth $d_{max} = \max_{x \in [a,b]}\{d(x)\}$, where $[a,b]$ is the interval within which the corroded material exists. This pressure represents a lower limit. The parameter $g_d^{\min} = \min_{x[a,b]}\{g^d(x)\}$ quantifies the contribution of the remaining material at resistant pressure. Here, the remaining material is defined as a material existing above the maximum depth threshold. The value of this parameter is the minimum resulting from evaluating, at each point of the defect, the function $g^d(\cdot)$, dependent upon the corrosion depth at the assessed point, i. e., on defect geometry and corrosion adjacent to that point. The error of the model, defined as the quotient of pressure obtained from the model and the value of burst test pressure, has a mean of 0.96 and variation coefficient of 0.08. More details on the model can be found in Oliveros et al. [3].

In operating conditions, resistant pressure of the pipeline will be uncertain and therefore, its performance will be uncertain too, not only due to the error characteristic of the model, but also due to variability of geometric characteristics and mechanical properties of the material along the pipeline. Also, due to changes that these properties experience over time as a result of chemical products they continuously transport. According to the above facts, geometrical and mechanical properties are expressed by the random vector $\mathbf{Z} = (\mathbf{Z}_g, \mathbf{Z}_m)$, so that the properties in each region containing a corrosion defect will be given by the vector $\mathbf{Z} = \mathbf{z}$. If these properties are uncertain, then the resistant pressures described above, which represent the upper and lower limit, are random variables and will be respectively expressed as $P_{PlainPipe}^d = P_{PlainPipe}^d(\mathbf{Z})$ and $P_{LongGroove}^d = P_{LongGroove}^d(\mathbf{Z}, d_{max})$.

Another source of uncertainty is that associated with the transported fluid, which involves pressure fluctuations due to pipeline roughness and pipeline position as a result of ground topography and burial depth. Based on the above facts, the limit-state function associated with the failure due to pressure can be specified as:

$$\left(W = \varepsilon_1 \; P_R^d - P_D\right) \leq 0 \tag{14}$$

Where:

$$P_D = \varepsilon_2 \, \bar{P}_0 - \varepsilon_3 \, \gamma \left(y - y_0\right) + \gamma \, f \, \frac{\bar{Q}_c^2}{4 \, g \, \pi^2 \, \bar{R}_0^5} \left(\left(x - x_0\right)^2 + \left(y - y_0\right)^2\right)^{1/2} + \gamma \, h_b \tag{15}$$

and P_R^d is the random variable of resistant pressure, associated with a corrosion defect of known geometry, which is a function of geometrical and mechanical properties of the material. The random variable ε_1 considers the error on mechanical model prediction, whereas the random function P_D, given by Bernoulli's equation (Benedict, [5]), depicts the fluid pressure variability in defect region, associated with the mean point of the defect with coordinates (x, y). According to Eq. (15), the pressure along the pipeline is a function of four factors: the first on the right side of equation quantifies the pressure and its variability at the reference point (x_0, y_0), and ε_2 is a random variable that considers pressure fluctuations around mean pressure $\bar{P}_0$ of the fluid at the reference point (x_0, y_0). The second term quantifies the pressure change due to the height of the reference point; ε_3 is a random variable that considers uncertainty in the pipeline position with regard to its burial depth, and γ is the mean density of the transported fluid. The third term quantifies pressure losses due to the roughness of defects along the pipeline length. This term is a function of: volumetric flow rate $\bar{Q}_c$, gravitational constant g, γ, mean pipe diameter $\bar{R}_0$, distance in relation to the point of reference and friction factor f, dependent upon fluid viscosity and pipe roughness. For the purpose of simplicity, in this work the loss due to roughness is deterministic. Finally, the fourth term of Eq. (15) considers the possible increase or decrease of pressure due to the operation of a pump or turbine in the considered segment. h_b is the height associated to the pressure charge; however, herein it is considered that $h_b = 0$ and the random variables ε are assumed to be distributed in accordance with a Lognormal probability density function, which requires further research.

In the professional practice, a simple geometrical shape is commonly assumed to denote the geometry of corrosion defects, with maximum depth $d_{\max}$ and length $l = b - a$. For this reason, herein is assumed a parabolic geometry. The assumption of such geometry implies that statistical characteristics of the random variable ε_1 change. Oliveros et al. [3] found that this variable possesses a mean value of 1.05 and variation coefficient of 0.11. According to this, the resistant pressure of the system is associated with the point of maximum depth $x_{max} = \frac{b-a}{2}$, so that $g_d^{\min} = g^d(x_{max})$ thus the safety margin is expressed as:

$$W = \varepsilon_1 \left[P^d_{LongGroove} + \left(P^d_{PlainPipe} - P^d_{LongGroove}\right) g_d^{\min}\right] - P_D \tag{16}$$

Rigorously, $P^d_{PlainPipe}$ and $P^d_{LongGroove}$ are random variables correlated by the random vector of geometrical and mechanical properties $\boldsymbol{Z} = (\boldsymbol{Z}_g, \boldsymbol{Z}_m)$. Nevertheless, it is possible to relate these random variables by means of independent random variable Ψ, as follows:

$$P^d_{LongGroove} = \Psi \; P^d_{PlainPipe} \tag{17}$$

where $\Psi = \Psi(d_{\max})$ depends only on the maximum depth of corrosion, which was proven by Monte Carlo simulations, as shown in figure 1, Ψ is in fact observed to vary with maximum depth of corrosion. In addition, it can be considered independently from the yield stress, since the mean and mean plus one standard deviation obtained from simulations do not vary with yield stress. It can also be seen that the mean plus one standard deviation does not differ significantly from the mean value of each corrosion depth value. For this reason in this work $P^d_{LongGroove}$ can be represented as:

$$P^d_{LongGroove} \approx \psi \; P^d_{PlainPipe} \tag{18}$$

where ψ denotes the expected value of Ψ. Hence, W is expressed as:

$$W = \varepsilon_1 P^d_{PlainPipe}\left[\psi + g_d^{\min}\left(1 - \psi\right)\right] - P_D \tag{19}$$

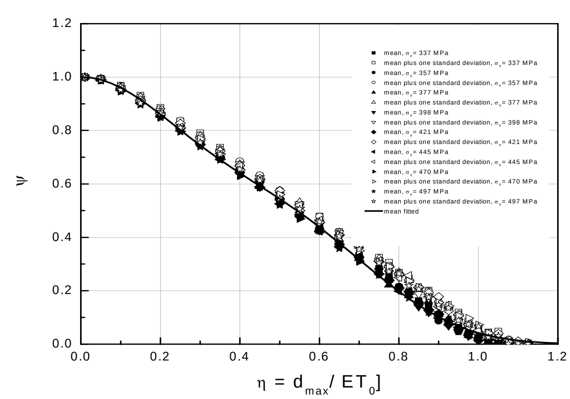

Figure 1. Statistical moments of Ψ for given values of normalized depth η and yield stress σ_Y.

4. Estimation of detected defects rate

In order to illustrate the above formulation, we analyzed a 1000 m long pipeline segment with 10 corrosion defects detected, whose maximum depths and lengths are shown in table 1. The lengths of exhibited defects range between 20 and 70 mm, whereas their maximum depths vary from 1.2 to 3.5 mm. For each defect, the table also shows the corresponding wall thickness, which ranges between 4.65 and 4.77 mm, and the mean of these thicknesses corresponds to $\overline{T}_0 = 4.68$ mm. Furthermore, table 1 shows the defect position in relation to pipeline position, specified at coordinates (x, y), that arises from the topographic configuration of the ground. Here, the lengths of corrosion defects are assumed to be distributed in accordance with a Lognormal probability density function, with a mean 48.0 mm and standard deviation 15.7 mm.

Table 1. Position and geometry factors of corrosion defects.

No.	position		wall thickness	defect geometry	
	x (m)	y (m)	mm	dmax (mm)	long. (mm)
1	50	-0.25	4.77	1.20	20
2	100	-0.50	4.77	1.70	30
3	200	-1.00	4.65	3.20	52
4	500	-2.50	4.65	3.50	37
5	501	-2.51	4.65	3.30	48
6	503	-2.52	4.65	1.50	70
7	650	-3.25	4.65	1.50	40
8	700	-3.50	4.67	2.25	60
9	900	-4.50	4.67	2.00	70
10	901	-4.51	4.67	2.00	55

Figure 2 shows the measured rate of detected defects, which as defined above, decreases with the equivalent maximum depth defined in this work as $d^*_{max_k} = d_{max_k}\left(T_0/t_{0_k}\right)$, where t_{0_k} is the remaining wall thickness associated with k defect. The need to define this equivalent maximum depth lies in the fact that maximum depths of each defect k measured are associated with a specific wall thickness. In general, if wall thicknesses measured correspond to the same value, then $d^*_{max_k} = d_{max_k}$. In short-length segments, wall thicknesses do not vary significantly, thus resulting in $d^*_{max_k} \approx d_{max_k}$, as in defects of the pipeline segment analyzed here.

According to Alamilla et al [1], the measured rate of detected defects can b e represented by the function

$$\nu_{D\max>d|Detec=1}(d) = n''_D \exp\left(-q_0(d-d_0)\right) \quad (20)$$

with parameters $q_0 = 1.0114$, $n''_D = 14.38$ obtained from a Bayesian analysis. In this work $d_0 = 1.0$. In the Fig. 1, it is observed that Eq. (20) describes adequately the behavior of the measured rates.

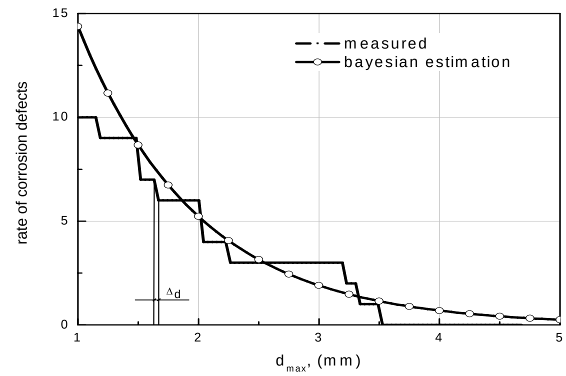

Figure 2. Rates of corrosion detected defects.

5. Estimation of total defect rate and undetected defect rate

It is assumed that measured defects, shown in table 1, come from an inspection tool whose capacity of detection is similar to that in Rodriguez and Provan [6], which is expressed in terms of the probability of detection and represented as:

$$F_{Detec=1}(d) = 1 - exp\left[-\alpha_0(d-\alpha_1)\right] \quad (21)$$

where $\alpha_0 = 0.655$ is a factor that defines the quality of detection and $\alpha_1 = 0.4$ mm the threshold of maximum depth above which the tool detects and measures defects with certain likelihood. This equation indicates that the probability of detection increases with the depth of corrosion. In this work, it was considered that the natural logarithm of the error in the measurement of detected defect dimensions, defined as quotient of real dimension divided by measured dimension, is distributed normally. So the error in the maximum depth and length of the defect has unit mean and standard deviation of 0.05 and 0.1, respectively.

As shown in Fig. 3, the function that describes the rate of undetected defects decreases with d_{max}. This is due to the fact that detection capacity of inspection tool is low for small depth defects and increases when $d_{max} \to T_0$. In the depths of our interest, the rates of detected defects are greater than that corresponding to the undetected ones. In addition, it is shown that for maximum depths above 3 mm, the likelihood for the presence of undetected defects above that threshold is very low. For this reason, the total rate of defects above this threshold is almost equal to the rate of detected defects, since according to Eq. (11) and as shown in Fig. 3, the sum of rates of detected and undetected defects is equal to the rate of total defects. It is important to note that the model can quantify more precisely the number and size of the defects really present at the pipeline.

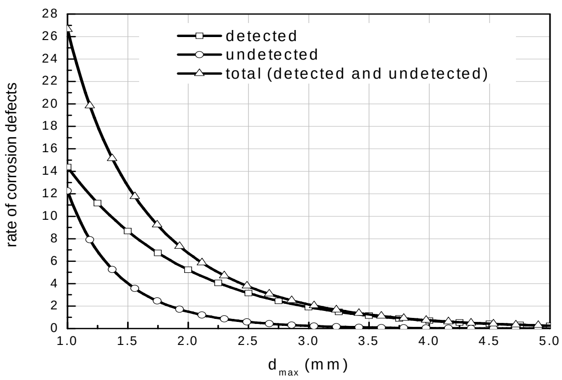

Figure 3. Rates of detected, undetected and total of corrosion defects.

In order to demonstrate the influence of the detection capacity of inspection tool on the number of undetected defects, Fig. 4 shows the rates of total and undetected defects for three inspection tools referred to as I, II and III, respectively, with quality factors α_0: 0.517, 0.655 and 1.091. These rates were considered to correspond to the pipeline segment described above and the three inspection tools were used to detect and measure the defects shown in table 1. It can be seen that for the same number of detected defects, the number of corrosion defects undetected by the inspection tool decreases with the factor of quality. These numbers are not directly comparable since the rate of detected defects in the analysis is the same for each example. Under practical conditions, we would expect from this rate to increase with the quality of the inspection tool. This kind of analysis, however, is out of the scope of this work. Nevertheless, the figure in question shows that the rates of undetected defects and the total rate are equal for maximum depths above 3 mm, which means that the possibility of existence of corrosion defects above this depth is practically zero.

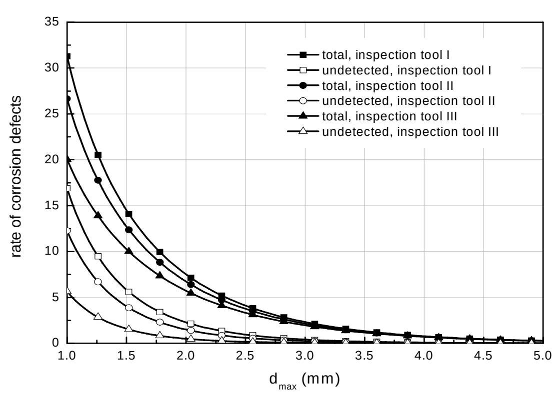

Figure 4. Rates of undetected and total of corrosion defects for three inspection tools.

The probability p_D of detecting a defect of uncertain size in the pipe for I, II and III tool, respectively was: 0.4598, 0.5397 and 0.7178. This is consistent with the corresponding quality factor, which indicates that p_D increases with this factor.

6. Reliability analysis

Next is the calculation of reliability of the pipeline segment from figure 3, X52 steel, with mean outer diameter of $254\ mm$, and mean yield stress $\bar{\sigma}_y = 422\ MPa$. Figure 8 shows in logarithmic scale the mean and mean plus standard deviation of resistant pressure of the pipeline without defects for given values of yield stress. These statistical parameters were obtained using Monte Carlo simulations. For given values of yield stress, simulated values of resistant pressure of the pipeline were obtained; they were determined by substituting in equation (13) the simulated random function that describes the behavior of the material (stress-strain relation) and the mean values of wall thickness and diameter. The stress-strain relation was represented by Ramberg-Osgood function [4] with uncertain parameters. The joint probability density function of these parameters is described in detail in Alamilla et al. [7]. Since the variability of wall thickness and pipe diameter is small as compared to the variability of material behavior, we have worked only with their mean values.

According to Alamilla et al. [1] the mean resistant pressures follow a linear behavior with yield stress and are represented by the following equation:

$$\ln P_{PlainPipe} = A_0 + A_1 \sigma_Y \tag{22}$$

where $A_0 = 2.16$, $A_1 = 0.0022$ were obtained from a linear fit. The standard deviation of logarithm $\sigma_{Ln\ PlainPipe} = 0.06$ is invariant with respect to yield stress. A statistical analysis of prior simulations showed that resistant pressure of a pipeline without defects can be appropriately represented by a Lognormal probability function.

Figure 1 shows mean values of the random variable Ψ obtained by simulation for given values of yield stress. It is observed that mean value of this variable is invariant with yield stress and decreases with maximum normalized corrosion depth $\eta = d_{\mathbf{max}} / \bar{T}_0$. These values were obtained by Monte Carlo simulations similarly to the random variable $P^d_{PlainPipe}$. However, in this case, for a set of simulated parameters and a given yield stress a stress-strain relation were obtained and for a given maximum corrosion depth, the following simulated values were obtained: $P^d_{PlainPipe} = p^d_{PlainPipe}$ and $P^d_{LongGroove} = p^d_{LongGroove}$. So, the simulated values of Ψ variable were obtained as quotient of $p^d_{LongGroove} / p^d_{PlainPipe}$. Fig. 3 shows the fitted mean ψ of the variable Ψ, obtained upon assuming that the relation between ψ and η is given by the sum of two exponentials in the following way:

$$\psi = \frac{1}{2}\exp\left(B_0\eta^2 - B_1\eta^3\right) + \frac{1}{2}\exp\left(-B_2\eta^2\right) \tag{23}$$

where the coefficients $B_0 = 2.61$, $B_1 = 5.09$ and $B_2 = 10.58$ were obtained from a nonlinear fit.

In this work, the behavior of the material is considered to be independent between corroded regions, because there is not enough information to consider spatial correlation of the parameters that describe the stress-strain function of material between corroded regions; however, the formulation proposed here allows incorporating this correlation with relatively little effort. It would be sufficient to know the statistical variation of coefficient correlation of yield stress with the distance to obtain correlated simulated stress-strain functions. Finally, according to Eq. (17), correlated values of variables $P^d_{PlainPipe}$ $P^d_{LongGroove}$ could be obtained.

Figure 5 shows the position of the pipeline in question in relation to the ground surface as well as the change of pressure with distance. To estimate this change, a mean pressure of $\bar{P}_0 = 10.0$ MPa was considered together with a mean fluid density of $\gamma = 608.0$ kg/m^3, a mean volumetric flow rate $\bar{Q}_c = 0.292$ m^3/s and a mean friction factor $f = 0.032$, which assumed that flow is in transition. The pipeline was considered to have a mean burial depth $\bar{\varepsilon}_3 = 1.8$ m and a standard deviation of 0.25 m, with a mean negative slope of 0.5 %.

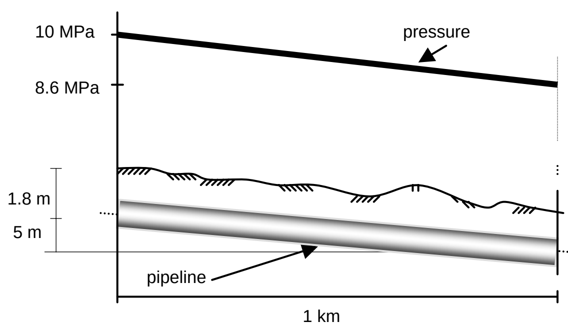

Figure 5. Pipeline position in relation with the ground surface.

The failure probability P_{F_k} corresponding to each detected defect k ,$\{k : 1,...,10\}$, specified in table 1 were obtained using directional simulation technique (Melchers [8]). The highest failure probability corresponds to the defect 4, whose maximum depth is 3.5 mm. Failure probability P_F of the segment analyzed as a series system is higher than failure probability of the individual defect 4, and lower than the probability considered as a series system with independent events (Melchers, [8]), i. e.:

$$\left\{\max\left(P_{F_k}\right) = 0.0384\right\} \leq \left\{P_F = 0.0429\right\} \leq \left\{1 - \prod_{k=1}^{10}\left(1 - P_{F_k}\right) = 0.1070\right\}$$

Failure probability of the system considering only defects detected by inspection tool is observed to be approximately 1.12 times greater than the greatest failure probability resulting from analyzing each defect individually. This means that failure functions of each corrosion defect are strongly correlated. The calculation of that correlation is out of the scope of this work. The failure probability of the system considering both detected and undetected defects was $P_F = 0.060$. This probability is 1.56 times greater than failure probability associated with the defect of greatest depth and 1.40 times greater than failure probability of the system when considering only defects detected by inspection tool. Considering the contribution of undetected defects to failure probability of the system can be important as shown herein. However, there may be systems with different distribution of defect dimensions in which undetected defects do not contribute significantly. Anyway, in a rigorous reliability analysis, undetected defects must be taken into account.

On the other hand, the overall failure probability of the segment considering detected and undetected defects associated with inspection tools I, II and III, proved to be 0.0712, 0.060 and 0.0478, respectively. As the quality of the inspection tool increases, the failure probability converges to the failure probability of detected defects, implying decreased contribution of undetected defects. According to the issues discussed above, prior failure probabilities are not totally comparable between each other, since the total number of defects is not the same in each analysis.

7. Concluding remarks

This work presented a reliability model that allows determining the safety of an inspected pipeline system. Reliability estimates indicate that failure regions associated with each defect are strongly correlated and the hypothesis of independence of failure regions is not convenient in reliability analyses of this kind of systems. It was concluded that failure probability of the system, when considering only detected defects, is slightly higher than the highest failure probability corresponding to individual defects. However, failure probability of the system considering detected and undetected defects is greater than the failure probability resulting from considering only detected defects, which is principally the result of uncertainty in size and detection of defects. Future studies need to consider the spatial correlation of constitutive functions between corroded regions, as well as to better characterize changes of fluid pressure along the pipe. Also, it is necessary to develop models that would consider the evolution of corrosion over time, as a function of internal and external environmental characteristics, which would allow improving the reliability estimates and performing adequate inspection polices and maintenance in this kind of systems.

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