Spatial Competition in Quality[*]

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Abstract

The well-studied formalism of Hotelling's classic location paradigm does not apply to the case of good quality, which by its very definition requires that all individuals agree on the ranking of goods; therefore, the notion that goods are differentiated by a 'transportation cost' is inept in vertically differentiated markets. Motivated by this observation, we analyze the determinants of product differentiation in a general equilibrium model of monopolistic competition in good quality. The model features many firms, which each hold the monopoly to produce a unique quality level of an otherwise homogenous good, as well as consumers who are heterogeneous in their valuation for quality. We document that the analogue to the transportation cost in the Hotelling model arises if the marginal cost of production is convex with respect to quality. Firms' optimal prices depend on the latter convexity and on the prices of the competitors that are adjacent in the quality space. For given firm entry, average equilibrium markups are decreasing in the density of quality-competition, but are unaffected by average productivity. Endogenizing firm's entry decision, we demonstrate that the density of competition is increasing in the market size and decreasing in average productivity. Last, we apply these insights to analyze the effect of inter- and intra-industry trade on the toughness of quality-competition, firm entry and welfare.

1 Introduction

Hotelling's classic 'location' paradigm is widely used to reflect generic product characteristics.[1] The well-studied formalism of the spacing model, however, does not apply to competition in quality. By its very definition, quality requires that individuals agree on the ranking of varieties and, in particular, their individually preferred "ideal variety". When it comes to vertical differentiation – or differentiation in quality – only the higher price tag of the universally preferred higher quality goods makes different consumers buy distinct qualities.[2]

Aware of the standard spacing model's fundamental misfit to address competition in quality, Shaked and Sutton (1982 and 1983) pioneered research on vertically differentiated markets in which natural oligopolies prevail, i.e. the markets that are dominated by a limited number of "market leaders". The authors call this characteristic, through which vertically differentiated markets fundamentally differ from horizontally differentiated ones, the *finiteness property*. Its key precondition is that the marginal production costs increase only moderately with quality (see also Shaked and Sutton (1984), Sutton (2007 a and 2007b)).

Whenever this condition is violated, consumers differ in their individual ranking of variety-price pairs and Shaked and Sutton (1983) observe that the competition in quality is "reminiscent of the 'location' paradigm" noted above. The latter authors do not analyze this case, since they do not model costly entry and thus a violation of the *finiteness property* would imply that infinitely many firms enter the market and competitive pricing along this set of qualities prevails. In this paper, we analyze the case where the marginal cost of production does increase sufficiently in quality such that the *finiteness property* is violated, but we explicitly model the quality choice by firms, i.e., the fact that market entry is costly and that the cost of market entry may depend on the firm's quality.

The starting observation of the present paper is that models à la Shaked and Sutton (1983) under production costs that do *not* give rise to the *finiteness property* are, still, qualitatively very

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[1] In particular, Lancaster (1966, 1975 and 1980) has coined the term "ideal variety" re-interpreting physical distance as the distance of the good's attribute from the consumer's taste.

[2] There is now ample empirical evidence that quality is an important dimension of product differentiation. At the consumer level, Bills and Klenow (2001) and Broda and Romalis (2009) document that wealthier consumers tend to buy a much larger fraction of high quality goods. At the producer level, Baldwin and Harrigan (2008), Crozet, Head, and Mayer (2007), Hallak and Sivadasan (2009), Johnson (2008), and Kugler and Verhoogen (2008) document that high quality producers tend to be more profitable in equilibrium and are more likely to export than low quality producers. Last, quality differentiation also explains aggregate trade flows: richer countries export goods with high unit value (Schott (2004) and Hummels and Klenow (2005)) and of high quality (Hallak and Schott (2008)), while richer countries also tend to consume high unit-value goods (Hallak (forthcoming)).

different from the traditional spacing literature. The distinctive feature is that while the cost of traveling a physical distance creates a rational for why firms are differentiated horizontally, such a "transportation cost" is – by definition – not present in a vertical framework where higher quality goods are universally preferred. Consequently, when firms compete in the quality space, the degree of product differentiation may only derive from the specifics of the production technology.

We consider a many-firm environment where each firm in the industry has the monopoly to produce a unique quality level of an otherwise homogenous good. As in Mussa and Rosen (1978), Shaked and Sutton (1982) and Auer and Chaney (2008 and 2009), the model features consumers who all value the quality of this good, but who differ in the rate at which they do so (their "valuation" for quality).

We first analyze the static determinants of prices and profits for given industry entry, documenting that it is the very same characteristic of technology – second derivate of the marginal cost of production with respect to quality – that determines both whether the *finiteness property* holds and how high equilibrium markups are when it is violated. Consumer's willingness to pay for increasing quality is proportional to their valuation. If the second derivative of the marginal cost of production with respect to quality (the "marginal cost schedule") is positive, the increasing cost of an additional unit of quality leads consumers with different valuations to buy goods of different quality. Then, if the set of consumer valuations is sufficiently wide, the static equilibrium is characterized by any existing firm selling a positive amount at a positive markup.

The analogue to the transportation cost in the Hotelling model arises from the convexity of the marginal cost of production. Firm's market power arises from the fact that each consumer values quality at a linear rate, while the cost of the good increases more than linear in the good's quality. For example, if there are three firms with $q_1 < q_2 < q_3$ and the marginal cost schedule is convex in quality, there exists a range of valuations where the consumer's marginal willingness to pay for quality exceeds the cost increase of buying q_2 instead of q_1, but it does not exceed the increase of buying q_3 instead of q_2. Consumers with these valuations would receive a surplus if they could buy q_2 at the marginal cost of Firm 2 instead of buying from Firm 1 or 3 at the respective marginal cost. Since Firm 1 and Firm 3 would never sell below their marginal cost, Firm 2 has some market power for all feasible equilibrium prices. This market power is increasing in the convexity of the marginal cost schedule and also, in how different firm's output is, i.e. in the density of quality spacing.

We next endogenize this density of quality spacing and allow for free entry into the industry.

In our setup with a clear ranking along the quality line, there is a unique top quality producer, whose first order condition differs from the first order conditions of the rest of the firms facing two competitors each. The latter fact substantially complicates our analysis and we thus do not consider a simultaneous entry game as in Vogel (2008).[3]

Instead, we analyze a sequential entry game that is reminiscent of the "quality ladder" models of endogenous growth of Aghion and Howitt (1992) and Grossman and Helpman (1991). Each firm enters the industry as the new technology leader and successively becomes superseded by newer and higher quality entrants. Since the benefit of entering with a higher quality good and the cost of doing so both grow at constant rates, all entering firms evaluate a scaled but symmetric entry condition. We prove that in this setup, under the conditions that are required for the economy to be on a constant growth path, a unique entry strategy exists where each new technology leader enters the industry with a quality that is a constant percentage higher than the one of the preceding technology leader.[4]

Upon market entry, a firm optimally chooses its quality. Other things equal, higher qualities grant a larger market share and profits, but come at higher fixed and marginal costs. New entrants also aim to distinguish their quality from those of the incumbents, as such isolation in quality increases market power and profits. While this latter effect drives firms to pick 'remote' qualities, the increasing cost of differentiation limits quality dispersion.

We next analyze how equilibrium quality spacing, prices, quantities, and consumer welfare depend on the underlying entry technology, production costs, and market size. Since markups unambiguously increase in the spacing of quality competition, we are primarily interested in how the spacing of competition itself is affected by these parameters. A larger market induces, certeris paribus, higher sales and allows firms to generate more profits. At constant setup costs, larger markets therefore experience more frequent entry of firms at closer distances and lower markups. Similarly, spacing is increasing in the fixed cost of setting up a firm.

Surprisingly, a proportional increase in the marginal cost of production for all firms in the industry by the same proportion is associated with denser spatial competition. This is due to

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[3]In models based on Hotelling (1929) one can avoid such border conditions since one can think of a circle street or the beach surrounding an island. In our setup, however, any attempt to "close the circle" must fail since it would amount to identifying the highest quality good to the lowest quality good.

[4]To our knowledge, the model developed below is the first paper which explains how a large number of seemingly "inferior" low-quality firms can exist alongside the technological leader. The reason for their survival is that, although the highest quality good is preferred by all consumers, it also carries a higher price tag, which is not worth paying for consumers with relatively low valuation for quality. Hence, our entry set up extends the two-firm case of "neck-to-neck" competition in Aghion et al. (2001).

the fact that in equilibrium, markups are proportional to costs. Thus, when costs of production increase for all firms, profits actually increase for any given quality spacing (under the condition that the equilibrium price increase does not lead to consumers stop buying the quality differentiated good). The latter price increase also results in higher profits, which in turn results in increased entry and denser competition.

We next turn to a discussion of consumer surplus and aggregate welfare. As we document, our model highlight a novel mechanism of why firm's market power is socially costly: it drives consumers away from consuming the socially optimal quality. In equilibrium, each consumer compares the increase in good quality to the increase in the goods price. Since also markups are generally increasing along the quality dimension, the increase in the price from one good to the other is higher than the cost increase. Consumers, therefore, tend to choose a too low quality, which is socially costly. The latter mechanism is not present in the Hotelling model, and neither in the Heterogenous firms representation in Vogel (2008).

Before concluding, we apply our analysis to analyse the case of the effect of trade integration on industrial structure.

2 The Model

2.1 General Setup

We develop and solve a model of monopolistic competition in quality. On the supply side, different firms produce different qualities of the same consumption good. Upon market entry, each firm chooses its optimal quality level from an unbounded set of qualities. Other things equal, higher quality typically grants a larger market share and higher profits, but comes at higher fixed and marginal costs.

On the demand side, we postulate that consumers differ in their valuation for quality in the sense that, while all of them strictly prefer higher quality levels over lower ones, individuals differ in their willingness to pay for a marginal increase of quality. This type of consumer heterogeneity can lead to non-degenerate equilibria, where a countable number of firms coexist, each selling to a strict subset of the total market.

The economy we develop features a homogeneous good $\mathcal{A}$, and a range of industries of goods differentiated by quality Q_j $(j \in J)$. With in each such industry goods are offered in different quality levels q_{ik} $(j \in J, i \in I_j)$.

2.1.1 Preferences

On the demand side, we adopt standard preferences that associates higher income with an increased willingness to pay for quality. Thus, as economic growth raises consumer's valuations for higher qualities, new firms will enter at the higher quality spectrum.

We generalize the framework of quality consumption from Shaked and Sutton (1982) to a range of differentiated goods. For each differentiated good Q_j consumers consume either one unit or none. When consuming v units of good A and the vector of qualities $\mathbf{q} = \{q_j\}_{j \in J}$ of the Q_j-goods, an individual derives utility

$$U(v, \mathbf{q}) = a \cdot \int_j \widetilde{v}_j q_j \ dj. \tag{1}$$

This extenion of the standard approach from Shaked and Sutton (1982) to a mulititude of differentiated goods slightly modifies the setup: a single firm's price does not impact the a units consumed of good A and hence is exogenous to the firm's optimization problem. Secondly, these preferences imply that, as standard in the literature, income and quality are complementary. The higher a consumer's income, the higher is v and thus her valuation for quality. For this reason, we will also call v an individual *valuation parameter of quality* or simply the *valuation of quality.*

Normalizing the price of good A to unity and writing $p_j(q_j)$ for the price of quality q_j, the units consumed of good A are

$$v = E - \int_J p_j(q_j) \ dj$$

The mass of individuals totals L. The income distribution is reflected by the uniform density function

$$U([0, v_{\max}])$$

where $0 < v_{\max}$. The distribution of income translates into a pdf for the amount v consumed of good A

$$g(v) \tag{2}$$

2.1.2 Production

The A-type good is produced competitively with constant returns to scale and labor as the only factor. Production technologies of the $\mathcal{Q}$-type good exhibit increasing returns to scale and depend on the quality level produced. Firms that enter the Q-market to produce the quality $q \in (0, \infty)$

need to acquire a blueprint at the fixed cost of

$$F(q) = \phi \left[\min_i (\max\{q/q_i, q_i/q\}) \right]^{2\theta - 1} \qquad (3)$$

labor units. Having acquired this blueprint, the firm then produces at constant marginal cost of

$$c(q) = \varphi q^\theta \qquad (4)$$

labor units. The parameters $\phi, \varphi > 0$ govern the setup costs and marginal production cost. We assume that both the fixed cost of entry as well as marginal cost are increasing and convex in quality $(\theta > 1)$.

We characterize the equilibrium in which firms freely enter the production at the optimal quality level and subsequently engage in monopolistic pricing. As usual, the equilibrium is solved by backward induction: we first determine the prices at given quality levels and subsequently analyze the entry decisions.

2.1.3 Optimal Pricing

We begin by characterizing the general pricing solution for an arbitrary distribution of a countable set of qualities. For notational simplicity, we drop the index j of the Q- industry and in addition use the convention $p_n = p(q_n)$ and $c_n = c(q_n)$.

Firms compete in prices, *i.e.* each firm sets the price for its quality to maximize its operating profits, while taking total demand and the other firms' prices as given. In the equilibrium of the entry game to which we turn later, firms need to cover their setup cost with monopoly rents. Under Bertrand competition and positive setup cost this implies that firms must be located at positive distance to each other. Hence, the number of firms is countable and we can index firms by $n \in \mathcal{N}_0 = \{0, -1, -2, ...\}$. The quality level produced by firm n is denoted by q_n. Without loss of generality we order firms by the quality level they produce so that firm 0 produces the highest quality level q_0 and all further quality levels satisfy $q_{n-1} < q_n$.

Under the preferences (1) a consumer with valuation v is indifferent between two goods q_n and q_{n+1} if and only if their prices p_n and p_{n+1} are such that $vq_{n+1} - p_{n+1} = vq_n - p_n$. Thus, given $v_{\max}$ and given the prices $\{p_n\}_{n \leq 0}$, the nth firm sells to all consumers with valuations v in

the range $[\underline{v}_n, \overline{v}_n]$, where[5]

$$\underline{v}_n = \frac{p_n - p_{n-1}}{q_n - q_{n-1}} \quad \text{and} \quad \overline{v}_n = \begin{cases} v_{\max} & \text{if } n = 0 \\ \dfrac{p_{n+1} - p_n}{q_{n+1} - q_n} & \text{if } n < 0 \end{cases} \tag{5}$$

As a consumer with valuation $v \in (\underline{v}_n, \overline{v}_n)$ demands one unit of the variety produced by firm n, this firm's optimal price p_n solves the maximization problem

$$\max_{p_n} (p_n - c_n) [\overline{v}_n - \underline{v}_n] \frac{L}{v_{\max} - v_{\min}} \quad \text{s.t. (5)} \tag{6}$$

The optimality condition is thus

$$[\overline{v}_n - \underline{v}_n] = (p_n - c_n) \left[-\frac{d\overline{v}_n}{dp_n} + \frac{d\underline{v}_n}{dp_n} \right]. \tag{7}$$

The second order condition is quickly checked to grant a local maximum. Combining conditions (7) and (5) leads to the recursive formulation of prices

$$p_n = \begin{cases} \dfrac{1}{2} \left[c_0 + (q_0 - q_{-1}) v_{\max} + p_{-1} \right] & \text{if } n = 0 \\ \dfrac{1}{2} \left[c_n + \dfrac{q_n - q_{n-1}}{q_{n+1} - q_{n-1}} p_{n+1} + \dfrac{q_{n+1} - q_n}{q_{n+1} - q_{n-1}} p_{n-1} \right] & \text{if } n < 0 \end{cases} \tag{8}$$

These conditions describe the optimal prices only if they exceed marginal costs c_n. A sufficient condition for this to be true is that marginal production costs (4) are increasing and convex in quality.

Lemma 1 *Assume conditions*

$$c(q)|_{q=0} = 0 \qquad \left. \frac{\partial c(q)}{\partial q} \right|_{q=0} = 0 \qquad \left. \frac{\partial c(q)}{\partial q} \right|_{q=q_0} \leq v_{\max} \qquad \frac{\partial^2 c(q)}{(\partial q)^2} > 0 \tag{9}$$

hold. Then, the optimal price for all firms $n \in \mathcal{N}$ is determined by (8).

    Proof. *Consider firm $n < 0$. None of its direct competitors sells below marginal costs: $p_{n-1} \geq c_{n-1}$ and $p_{n+1} \geq c_{n+1}$. Hence*

$$p_n - c_n \geq \frac{1}{2} \frac{q_n - q_{n-1}}{q_{n+1} - q_{n-1}} (c_{n+1} - c_{n-1}) - \frac{1}{2} (c_n - c_{n-1})$$

and firm n has a nonnegative margin if $\frac{c_{n+1} - c_{n-1}}{c_n - c_{n-1}} > \frac{q_{n+1} - q_{n-1}}{q_n - q_{n-1}}$, which is satisfied if $c(q)$ is convex. Further, the lowest quality firm with $q \to 0$ sells its good if $c(q)|_{q=0} = 0$ and $\frac{\partial c(q)}{\partial q}|_{q=0} = 0$

[5] For now, we rule out that firms "undercut", i.e. that firm n sets its quality-adjusted price to take the market share of a directly neighboring firm and compete with second-next firms. As we show below, if the cost structure is such that all firms sell when there is no undercutting, undercutting can never be a profit maximizing strategy.

(compare (9)). Last, consumers with highest valuation $v_{\max}$ prefer to buy the highest quality q_0 rather than the second highest q_{-1} if $v_{\max}q_0 - p_0 \geq v_{\max}q_{-1} - p_{-1}$. Using (8) for $n = 0$ and rearranging this condition is

$$v_{\max}\left(q_0 - q_{-1}\right) \geq c_0 - p_{-1}$$

With $p_{-1} \geq c_{-1}$ the condition $\partial c\left(q\right)/\partial q|_{q=q_1} \leq v_{\max}$ is a sufficient one. Second, it is easily verified that if $c(q)$ is convex, no firms will ever find it profitable to set its price such that the next highest or next lowest firms is completely crowded out of the market. ∎

Apart from imposing conditions on the borders (at $q = 0$ and $q = q_0$) the Lemma states that marginal production costs need to increase more than linearly in quality to generate the interior pricing solution defined by (8). This condition is straightforward given our preference structure (1): any individual who is consuming good $\mathcal{Q}$ at a given quality level $\bar{q}$ prefers an increase of x percent in consumed quality that comes about with an increase in price of less than x percent. When costs are convex, higher quality firms can offer exactly this deal. Consequently, the highest quality firm q_0 can, by charging a price moderately above its marginal costs attract the entire market, in which case all consumers purchase the highest quality available on the market and the equilibrium collapses to a corner (see Figure 1, bottom panel). Ruling out linear costs as the limit case of concavity, we are left with the requirement that costs be convex. In this sense, convexity of quality costs is a natural condition for the market to generate non-degenerate allocations.

With the cutoff levels (5) and the recursive pricing formula (8) we can write equilibrium profits in the following convenient form

$$\pi_n = \begin{cases} \dfrac{L}{v_{\max}} \dfrac{1}{q_0 - q_{-1}} \left(p_0 - c_0\right)^2 & \text{if } n = 0 \\[2em] \dfrac{L}{v_{\max}} \dfrac{q_{n+1} - q_{n-1}}{\left(q_{n+1} - q_n\right)\left(q_n - q_{n-1}\right)} \left(p_n - c_n\right)^2 & \text{if } n < 0 \end{cases} \tag{10}$$

With these expressions for the operating profits it is quick to show some regularities of equilibrium profits that arise in our model.

Lemma 2 *Let $\{p'_n\}_{n\leq 0}$ and $\{\pi_n\}_{n\leq 0}$ be the prices and the operating profits of the system given by $v_{\max}$, $\{q_n\}_{n\leq 0}$, $\{c_n\}_{n\leq 0}$ and (8).*
(i) For $\chi > 0$ let $v'_{\max} = \chi^{\theta-1}v_{\max}$, $q'_n = \chi q_n$, $c'_n = c_n$ and the corresponding equations (8) define a transformed system. Then, prices $\{p'_n\}_{n\leq 0}$ and profits $\{\pi'_n\}_{n\leq 0}$ of the transformed system satisfy

$$p'_n = \chi^\theta p_n \qquad \text{and} \qquad \pi'_n = \chi^\theta \pi_n \quad \forall n.$$

(ii) Similarly, prices $\{p''_n\}_{n\leq 0}$ and profits $\{\pi''_n\}_{n\leq 0}$ the transformed system $v''_{\max} = \chi v_{\max}$, $q''_n = q_n$, $c''_n = \chi c_n$ and (8) satisfy

$$p''_n = \chi p_n \qquad and \qquad \pi''_n = \chi \pi_n \quad \forall n.$$

Proof. *(i) Use $c(\chi q) = \chi^\theta c(q)$ from (4) together with (8) to confirm that $p'_n = \chi^\theta p_n$ for all $n \leq 0$. With (10) this completes statement (i).*

(ii) The system (8) implies $p''_n = \chi p_n$, which, together with (10), proves statement (ii). ∎

We will be particularly interested in equilibria under a regular spacing. Thus, under constantly growing valuations and free entry a certain set of equilibria emerges, in which each new entrant chooses to distinguish its quality from existing qualities by a constant proportion. In these cases of equal relative spacing we can solve the pricing rule explicitly. Before turning to the first stage of the entry game where firms chose the quality levels, we will therefore provide the equilibrium pricing under equal relative spacing.

2.1.4 Equal Relative Spacing

In this subsection we assume that the ratio of neighboring quality levels is constant, *i.e.* that

$$\gamma q_{n+1} = q_n \qquad \gamma > 1 \tag{11}$$

holds. Under this condition equilibrium prices are determined as follows.

Proposition 1 *Assume equal relative spacing in quality, i.e. (11) holds. Then prices are*

$$p_n = \left(A \left(\lambda / \gamma^\theta \right)^n + \alpha \right) c_n \qquad \forall \ n \leq 0 \tag{12}$$

where

$$\alpha = \frac{\gamma + 1}{2(\gamma + 1) - \gamma^\theta - \gamma^{1-\theta}} \tag{13}$$

$$\lambda = \gamma + 1 + \sqrt{\gamma^2 + \gamma + 1} \tag{14}$$

$$A = \frac{\lambda}{2\lambda - 1} \left(1 - \alpha \left(2 - \gamma^{-\theta} \right) + \frac{\gamma - 1}{\gamma} \frac{q_0 v_{\max}}{c_0} \right). \tag{15}$$

Proof. Substitution $u_n = p_n - \alpha c_n$ and recursive formulation (8) of the prices gives

$$2 \left[u_n + \alpha c_n \right] = c_n + \frac{1}{\gamma + 1} \left[u_{n+1} + \alpha c_{n+1} \right] + \frac{\gamma}{\gamma + 1} \left[u_{n-1} + \alpha c_{n-1} \right]$$

for $n > 1$. With $\alpha = (1 + \gamma)/\left[2(1 + \gamma) - \gamma^{\theta} - \gamma^{1-\theta}\right]$ this is $2(\gamma + 1)u_n = u_{n+1} + \gamma u_{n-1}$. The equation

$$X^2 - 2(\gamma + 1)X + \gamma = 0 \tag{16}$$

has two roots, $\lambda = \left[\gamma + 1 + \sqrt{\gamma^2 + \gamma + 1}\right]$ larger than unity and $\mu = \left[\gamma + 1 - \sqrt{\gamma^2 + \gamma + 1}\right]$, smaller than unity. The general solution to the recursive series is thus

$$p_n = \tilde{A}\lambda^n + \tilde{B}\mu^n + \alpha c_n \tag{17}$$

where $\tilde{B} = 0$ because of $\mu < 1$ and the transversality condition $\lim_{n \to -\infty} p_n = 0$. Equation (8) for $n = 0$ is $2p_0 = c_0 + (q_0 - q_{-1})v_{\max} + p_{-1}$ and implies

$$2\left[\tilde{A} + \alpha c_0\right] = c_0 + q_0 \left(1 - 1/\gamma\right)v_{\max} + \tilde{A}/\lambda + \alpha c_{-1}.$$

Solving for $\tilde{A}$ and replacing $A = \tilde{A}/c_0$ this proves (15). ∎

Notice that the term α from (12), which is common to firms' markups, might be positive or negative, depending on whether or not $\gamma^{\theta} + \gamma^{1-\theta} < 2(\gamma + 1)$ holds. Nevertheless, expression (12) defines positive markups in either of the cases provided that the highest quality firm is active in the market. To verify this statement, observe first that[6]

$$\alpha > 0 \qquad \Leftrightarrow \qquad \lambda/\gamma^{\theta} < 1. \tag{18}$$

Next, consider the case $\lambda > \gamma^{\theta}$ and $A \geq 0$, in which case $A\left(\lambda/\gamma^{\theta}\right)^n + \alpha > 1$ holds since $\alpha > 1$. If, instead, $\lambda > \gamma^{\theta}$ and $A < 0$ then $A\left(\lambda/\gamma^{\theta}\right)^n + \alpha > 1$ holds for all $n \leq 0$ if it does so for $n = 0$. Similarly, in the case $\lambda < \gamma^{\theta}$ we have, by the above considerations, $\alpha < 0$ and hence $A > 0$ so that, again, $A\left(\lambda/\gamma^{\theta}\right)^n + \alpha > 1$ holds for all $n \leq 0$ if it does so for $n = 0$.

The crucial condition for markups to be positive is thus $A + \alpha - 1 > 0$. This condition is satisfied as long as $v_{\max}$ is large enough to generate positive demand for the firm producing the highest quality. In sum, if under equal relative spacing (11) a firm with quality q sells positive quantities, then all firms with minor qualities do so.

Obviously, the highest quality firm does not produce under all circumstances, *e.g.*, if the highest valuation is small so that $v_{\max}q_0 < c_0$ demand for the highest quality is zero even if q_0 is sold at marginal cost (compare (9)). In this case, however, the highest quality firm remains idle and we can drop it from the set of firms considered. Doing so successively for all idle top

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[6]This relation is quickly done by verifying that both inequalities $\gamma^{\theta} + \gamma^{1-\theta} < 2(\gamma + 1)$ and $\lambda/\gamma^{\theta} < 1$ hold (are violated) for small (large) $\gamma > 1$ and that, moreover, $\lambda = \gamma^{\theta}$ if and only if γ solves (16), i.e. $\gamma^{\theta} + \gamma^{1-\theta} = 2(\gamma + 1)$.

firms, the ratio $q_0 v_{\max}/c_0$ increases up to the point where $A + \alpha - 1$ is positive, which then defines positive markups throughout.

Finally, we write for the relative markup of the highest quality firm

$$A + \alpha - 1 = \frac{\lambda}{2\lambda - 1} \left\{ \frac{\gamma - 1}{\gamma} \frac{q_0 v_{\max}}{c_0} + \left(\frac{\alpha}{\gamma^\theta} - 1 \right) - \frac{\alpha - 1}{\lambda} \right\}$$

and observe with (13) that the limit

$$\frac{A + \alpha - 1}{\gamma - 1} \to \frac{1}{\sqrt{3}} \left\{ \frac{q_0 v_{\max}}{c_0} - \theta \right\} \qquad (\gamma \to 1)$$

is finite. With the explicit formula for the prices (12), the operating profits from (10) are

$$\pi_n = \begin{cases} \gamma(\gamma - 1) \left(\dfrac{A + \alpha - 1}{\gamma - 1} \right)^2 \dfrac{c_0^2}{q_0} \dfrac{L}{v_{\max}} & \text{if } n = 0 \\[3ex] (\gamma^2 - 1) \left(\dfrac{A\left(\lambda/\gamma^\theta\right)^n + \alpha - 1}{\gamma - 1} \right)^2 \dfrac{c_n^2}{q_n} \dfrac{L}{v_{\max}} & \text{if } n < 0 \end{cases} \qquad (19)$$

In the case of equal relative spacing, the operating profits are thus, by (13) - (15) and the limit above, continuously differentiable for all $\gamma \geq 1$ and satisfy, moreover

$$\pi_n \to 0 \qquad (\gamma \to 1).$$

Consequently, we can conclude that for small $\gamma > 1$, operating profits are increasing in γ.

Having a good understanding of the equilibrium prices (12) and profits (19) for the case of equal relative spacing, we can turn to the first stage of the entry game.

2.1.5 Endogenous Spacing Under Free Entry

We next show that in a dynamic version of our model where individual income grows at constant rates, free entry makes firms enter at constant time-intervals and supply qualities of constant ratios. We thereby endogenize the spacing postulated in Auer and Chaney (2008), which should encourage the use of the very convenient pricing rule developed therein.

In our setup, as the economy grows, the distribution of consumer valuations grows at a constant rate, thus making it profitable for a new entrepreneur to enter the economy with higher quality goods. In this dynamic setting, a particular form of symmetric quality level arises under our model specification, thus allowing us to solve explicitly for equilibrium prices. The corresponding comparative statics show that larger markets and lower setup costs induce tighter competition and

lower markups. Conversely, higher marginal productivity is associated with looser competition and higher markups, while overall prices are lower.

The nature of the entry game is the following.

1. Consumer valuations grow at exogenous rate a.

2. Denote by $\Pi(q_m, t_m)$ the "shadow profit", i.e. the discounted flow of profits of when enter at time t_m with the optimal quality q_m. entering the discounted Over time, as the upper range of valuations grows, so does the shadow profit of inventing becoming the new technology leader.

3. At each point in time, there is a mass of potential entrepreneurs who can pay a fixed cost $F(q)$ (see 3) to enter the industry. Each entrant, where she to enter would choose the quality that would maximize the expected sum of future profits minus, i.e, she maximizes

$$\max_{q_m} \left[\Pi(q_m, t_m) - F(q_m)\right] s.t. q_m > q_{m-1}$$

4. Given that entry is contested, the timing of entry is such that whenever the maximum shado profit of entering is equal to or exceeds the entry cost, i.e.

$$t_m = \inf_t \left(\max_{q_m} \left[\Pi(q_m, t_m) - F(q_m)\right] s.t. q_m > q_{m-1} \geq 0 \right)$$

The entry game at hand also relates to is the literature on industry dynamics with heterogeneous firms deriving from Hopenhayn (1992) and, in the case of the open economy, from Melitz (2003) and Bernard et al. (2003).[7] It is noteworthy that there are fundamental differences in both the nature of firm heterogeneity itself and the origin of this heterogeneity in our model and the latter studies. Regarding the nature of firm heterogeneity, the existing literature assumes that each firm's type is drawn from an exogenous distribution and then analyses how the equilibrium survival "cutoff" is dependent on market conditions. In contrast, we analyze how firms choose their level of quality in a given market environment and we then analyze how the spacing – i.e. the distribution of firm-qualities itself – depends on market environment.

Also, the origin of firm heterogeneity is different than in the existing literature on firm heterogeneity and industry dynamics. In our setup, all firms are ex ante symmetric in terms of the

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[7] While Melitz (2003), Bernard et al. (2003), and - in a model allowing for variable markups - Melitz and Ottaviano (2008) assume that the intrinsic difference of firms is in physical productivity, Baldwin and Harrigan (2008) and Johnson (2007) assume that the key heterogeneity of firm survival is the difference in the quality of the goods these firms produce.

technology they could potentially use, but the market conditions they face when entering the industry are not. Each firm maximizes profits from the moment of entry and then discounted into the future. Each industry entrant is hence optimizing the quality of its output under different market conditions than did past entrants or than future ones will do. Thus, in our setup, it is the distribution of consumer valuations at the moment of entry that determines the ex-post firm heterogeneity.

This subsection shows that in a dynamic version of the general setup described above, free entry supports equilibria with equal relative spacing of firms, endogenously generating quality levels that satisfy (11). We introduce a dynamic dimension to our model, by assuming that time is continuous and that maximal valuation $v_{\max}$ grows exogenously at the constant rate a so that

$$\bar{v}(t) \equiv v_{\max,t} = e^{at} v_{\max,0}. \tag{20}$$

Initially, the set of active firms is $\{0, -1, -2, ...\}$ and firms are ordered according to ascending qualities, as described in the previous subsections. These initially active firms produce qualities $\{q_n\}_{n \leq 0}$ that satisfy (11). As demand for goods at the top-end of the quality spectrum grows new firms gradually establish at the upper end of the quality spectrum. We assume that a plant established at quality level q_m automatically holds the blueprints for all qualities between q_{m-1} and q_m, where q_{m-1} is the next-lower quality level. This assumption restricts entry of additional firms to quality levels above the pre-existing ones $(q_{m+1} \geq q_m)$.

Now, for $m \geq 1$ let t_m denote the entry date of the mth *additional* firm (implying $0 \leq t_1 \leq t_2 \leq ..$) and let further q_m stand for its quality level (implying $0 \leq q_1 \leq q_2 \leq ..$). It will prove convenient to express the quality choice of the mth entrant relative to the previously highest quality (q_{m-1}) as

$$\gamma_m = q_m/q_{m-1} \qquad m \geq 1.$$

Finally, we assume that no firm can enter the market before it makes marginally positive profits.[8] Using (11) and (20) we thus impose the condition $\bar{v}(t_m)q_m - c_m \geq \bar{v}(t_m)q_{m-1} - p_{m-1}(q_{m-1}, q_{m-2}, ..)$, where $p_{m-1}(q_{m-1}, q_{m-2}, ..)$ is the price of quality q_{m-1} given the set of firm locations $\{q_{m-1}, q_{m-2}, ..\}$. Dropping the arguments of p_{m-1} we write with (4) and (20) the constraint as

$$e^{at_m} \bar{v}(0) (q_m - q_{m-1}) \geq \varphi q_m^\theta - p_{m-1}. \tag{21}$$

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[8] Relaxing this assumption would be straightfoward. We assume this to be the relevat case since firms generally delay invention of a new product at least up to the point in time when the product faces some positive demand.

By (17) through p_{m-1} the constraint (21) depends implicitly on the set of all preexisting quality levels $\{q_{m-1}, q_{m-2}, ...\}$. Applying Lemma 2 (i), however, we observe that it is invariant under the transformation $q'_n = \chi q_n$ and $v'_{\max} = \chi^{\theta-1} v_{\max}$ (or with (20) $q'_n = \chi q_n$ and $t' = t + (\theta-1)a^{-1}\ln(\chi)$).

Together with the requirement $q_m \geq q_{m-1}$, the constraint (21) effectively puts a lower bound on t_m, the time of the mth firm's entry. Technically, this lower bound impedes firms from entering very early in time in order to effectively block a certain range of blueprints and wait – possibly long periods – before starting production.

At time $\tau \in [t_{m+k}, t_{m+k+1})$ the set of quality levels supplied to the market is $\{q_n\}_{n \leq m+k}$. Current prices are determined by (8) and depend on the currently produced quality levels as well as on current maximum valuation $\bar{v}(t)$. Consequently, at time $\tau \in [t_{m+k}, t_{m+k+1})$ the operating profits (10) of the mth additional firm are a function of qualities $\{q_n\}_{n \leq m+k}$ and valuation parameter $\bar{v}(t)$. We can therefore write them formally as

$$\pi_m \left(\bar{v}(t), q_{m+k}, \gamma_{m+k}, \gamma_{m+k-1}, \gamma_{m+k-2}, ..., \gamma_1, \gamma\right) \qquad \tau \in [t_{m+k}, t_{m+k+1}).$$

Defining now the product

$$\Gamma_{m,k} = \prod_{j=1}^{k} \gamma_{m+j} \tag{22}$$

we have $q_{m+k} = \Gamma_{m,k} q_m$ so that at time t_m the present value of the flow of operating profits for a potential entrant is

$$\Pi(\gamma_m, t_m) = \sum_{k \geq 0} \int_{t_{m+k}}^{t_{m+k+1}} e^{-\rho(\tau - t_m)} \pi_m \left(\bar{v}(\tau), \Gamma_{m,k}\gamma_m \Gamma_{0,m-1} q_0, \gamma_{m+k}, \gamma_{m+k-1}, ..., \gamma_1, \gamma\right) d\tau. \tag{23}$$

The parameter ρ is the constant rate at which firms discount future profits.

We are now ready to formulate the entry decision of firms. The mth firm makes two simultaneous choices: first, it chooses its entry date (t_m), second, it decides its location on the quality range (γ_m). With the second choice it maximizes the present value of profits (23) net of costs (3). Given the spacing $\gamma_{m-1}, \gamma_{m-2}, ..., \gamma_1, \gamma$, and conditional on the entry date t_m the mth optimal quality choice is

$$\tilde{\gamma}_m \left(\gamma_{m-1}, ..., \gamma_1, \gamma\right) = \underset{\hat{\gamma} \geq 1}{\arg\max} \left\{ \sum_{k \geq 0} \int_{t_{m+k}}^{t_{m+k+1}} e^{-\rho(\tau - t_m)} \pi_m \left(\bar{v}(\tau), \tilde{\Gamma}_{m,k}\hat{\gamma}\Gamma_{0,m-1} q_0, \tilde{\gamma}_{n+k}, \tilde{\gamma}_{n+k-1} \cdots \right. \right.$$
$$\left. \left. \cdots, \tilde{\gamma}_{n+1}, \hat{\gamma}, \gamma_{n-1}, ..., \gamma_1, \gamma\right) d\tau - F(\hat{\gamma}\Gamma_{0,m-1} q_0) \quad s.t. \ (21) \right\} \tag{24}$$

where, $\tilde{\Gamma}_{m,k}$ stands for

$$\tilde{\Gamma}_{m,k} = \prod_{j=1}^{k} \tilde{\gamma}_{m+j} \left(\tilde{\gamma}_{m+j-1}, \tilde{\gamma}_{m+j-2}, ... \tilde{\gamma}_m, \gamma_{m-1}, ..., \gamma_1, \gamma\right).$$

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similarly to (22). Notice that all future locations choices $\tilde{\gamma}_{m+j}$ (and $\tilde{\Gamma}_{m,j}$) as well as future entry dates t_{m+j} are functions of the mth firm's choice. For expositional purposes, however, the arguments $\tilde{\gamma}_{m+j}(\hat{\gamma})$, $\tilde{\Gamma}_{m,j}(\hat{\gamma})$, $t_{m+j}(\hat{\gamma})$ are suppressed in (24) and further down. The mth firm's entry date is determined by the free entry condition, *i.e.*, the requirement $\Pi(\gamma_m, t_m) \geq F(\gamma_m q_{m-1})$. Formally, we write

$$t_m = \inf_t \left\{ t \geq t_{m-1} \mid \sup_{\hat{\gamma} \geq 1} \left[\sum_{k \geq 0} \int_{t_{m+k}}^{t_{m+k+1}} e^{-\rho(\tau - t_m)} \pi_m \left(\bar{v}(\tau), \tilde{\Gamma}_{m,k} \hat{\gamma} \Gamma_{0,m-1}^* q_0, \tilde{\gamma}_{m+k}, \tilde{\gamma}_{m+k-1}, \cdots \right.\right.\right.$$
$$\left.\left.\left. ..., \tilde{\gamma}_{m+1}, \hat{\gamma}, \gamma_{m-1}^*, \gamma_{m-2}^* ..., \gamma_1^*, \gamma \right) d\tau - F(\hat{\gamma} \Gamma_{0,m-1}^* q_0) \right] \geq 0 \;\; s.t. \;\; (21) \right\} \tag{25}$$

where γ_m^* denotes the equilibrium locations

$$\gamma_1^* = \tilde{\gamma}_1(\gamma) \qquad \text{and} \qquad \gamma_m^* = \tilde{\gamma}_m(\gamma_{m-1}^*, \gamma_{m-2}^*, ..., \gamma_1^*, \gamma) \tag{26}$$

and, finally, $\Gamma_{0,k}^*$ is defined parallel to the above definitions as

$$\Gamma_{0,k}^* = \prod_{j=1}^{k} \gamma_j^*.$$

Optimal quality choices (24) and the free entry conditions (25) of all entrants ($m \geq 1$) determine the equilibrium of the entry game. The first important result of this section concerns the solution of the system (24) - (25) and is formulated in the following Proposition.

Proposition 2 *For any combination of positive parameters $(\theta, \phi, \varphi, L, \rho, a)$ there exists a $\bar{\gamma} > 1$ so that the equilibrium of the entry game (24) - (25) sustains*

$$q_m = \bar{\gamma} q_{m-1} \qquad\qquad m \in \mathbb{Z}.$$

In this equilibrium the time intervals between consecutive entries are constant and equal to

$$\Delta = (\theta - 1) a^{-1} \ln(\bar{\gamma}). \tag{27}$$

Proof. See Appendix. ∎

For the parameters $(\theta, \phi, \varphi, L, \rho, a)$ and an according parameter $\bar{\gamma}$ we will call the corresponding equilibrium the Equal Relative Spacing Equilibrium (ERSE). Notice that Proposition 2 establishes existence of the ERSE but is silent about uniqueness. For the time being, we can observe with (19) that in any ERSE with spacing parameter $\bar{\gamma}$ the profits of the mth entrant at time $\tau \geq t_m$ are

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$$
\pi_m(\bar{v}(\tau)) = \begin{cases} \bar{\gamma}(\bar{\gamma}-1)\left(\dfrac{A+\alpha-1}{\bar{\gamma}-1}\right)^2 \dfrac{c_m^2}{q_m}\dfrac{L}{\bar{v}(\tau)} & \text{if } \tau \in [t_m, t_{m+1}) \\[2em] (\bar{\gamma}^2-1)\left(\dfrac{A\left(\lambda/\bar{\gamma}^\theta\right)^n+\alpha-1}{\bar{\gamma}-1}\right)^2 \dfrac{c_m^2}{q_m}\dfrac{L}{\bar{v}(\tau)} & \text{if } \tau \in [t_{m+k}, t_{m+k+1}) \end{cases} \tag{28}
$$

where

$$
A = \frac{\lambda}{2\lambda-1}\left(1-\alpha\left(2-\gamma^{-\theta}\right)+\frac{\gamma-1}{\gamma}\frac{q_{m+k}\bar{v}(\tau)}{c_{m+k}}\right)
$$

and α and λ from (13) and (14).

Notice that the constant time intervals (27) together with (20) imply

$$
\frac{q_{m+k}\bar{v}(\tau)}{c_{m+k}} = \frac{q_{m+k+1}\bar{v}(\tau+\Delta)}{c_{m+k+1}}
$$

so that the function A is periodic in time to the interval Δ (*i.e.* $A(t+\Delta) = A(t)$ holds).

We will next turn to the second important result of the present section and infer uniqueness for an important parameter range. Preparatory observations to prove uniqueness and to conduct comparative statics are formulated in the following Lemma.

Lemma 3

(i) For any sequence $\{\phi_k\}_{k\geq 0}$ with $\lim_{k\to\infty}\phi_k = 0$ the spacings $\bar{\gamma}_k$ of corresponding sequence of ERSEs satisfies

$$
\lim_{k\to\infty}\bar{\gamma}_k = 1
$$

(ii) If $\rho < 1/2$ there is a $\bar{\phi} > 0$ so that for all $\phi \leq \bar{\phi}$ the constraint (21) binds for all ERSE.

Proof. See Appendix. ■

Part (i) of Lemma 3 states that, as entry costs become negligible, equilibrium quality supply is arbitrarily dense, making competition is arbitrarily tough. This result seems perfectly intuitive: as setup costs approach zero, the operating profits, with which firms cover their setup costs must vanish, and hence $q_m \to q_{m-1}$ or $\gamma \to 1$. Notice, however, that firms may reduce the net present value of entry not only by moving closer together and thus decreasing contemporary operating profits but, alternatively, by entering the market at earlier dates. In particular, advancing entry to earlier dates implies that the bulk of operating profits accrues in the more distant future and is therefore more heavily discounted, thus depressing the net present value of entry. This way of reacting to decreases in setup costs, however, is limited by the constraint (21), which puts a lower

bound on the date of firm entry. Therefore, in presence of this constraint firms must necessarily decrease the present value of operating profits by reducing the relative spacing.

These considerations related to part (i) of Lemma 3 suggest that the constraint (21) might be binding the equilibrium allocation. Part (ii) of the Lemma states that it actually does so. For an important range of discount factors the constraint is shown to bind if setup costs are small. The parameter range relevant for Lemma 3 to apply is, at the same time, the one that induces a relatively dense spacing and tough competition, which the present paper aims to analyze.

The parameter range of small ϕ is all the more important, since it grants uniqueness of the equilibrium. We formulate this second central result in the following proposition.

Proposition 3 $\exists \, \tilde{\phi} > 0$ *so that for $\rho < 1/2$ and $\phi \leq \tilde{\phi}$ the ERSE is unique.*

Proof. Considering an ERSE we need to consider the first firm's quality choice only. With Lemma 3 (ii) we focus on $\phi \in (0, \bar{\phi})$ where the constraint (21) binds for all ERSE.

Observe next with (27) and (28) that, at constant q_1 (and hence c_1), operating profits and entry dates are continuously differentiable functions in $\bar{\gamma}$. This implies that, holding q_1 constant, the discounted flow of profits Π_1 for firm $m = 1$, defined by (23) is continuously differentialble in $\bar{\gamma}$. Since further $\Pi_1 \to 0$ at $\gamma \to 1$ we can conclude that, holding q_1 constant, Π_1 is increasing in γ for $\gamma \geq 1$ small enough. Therefore, defining

$$\bar{\bar{\gamma}}(\phi) = \sup \{\gamma \geq 1 \mid \gamma \text{ belongs to ERSE under } \phi\}$$

and applying Lemma 3 (i), we can find $\phi_o > 0$ that (at constant q_1) Π_1 is increasing in γ for all $\gamma \leq \bar{\bar{\gamma}}(\phi)$ and hence for all $\bar{\gamma}$ belonging to ERSE with $\phi < \phi_o$.

Now assume that for a thus limited ϕ there are two ERSE characterized by $\bar{\gamma}$ and $\bar{\gamma}'$ (wlog $\bar{\gamma} < \bar{\gamma}'$). The free entry condition implies

$$\frac{\Pi(\bar{\gamma}, \bar{t}_1)}{F(\bar{\gamma})} = \frac{\Pi(\bar{\gamma}', \bar{t}_1')}{F(\bar{\gamma}')} \tag{29}$$

Since the constraints (21) bind, we observe that for given q_0 the respective entry dates satisfy $\exp(a(\bar{t}_1' - \bar{t}_1)) = (q_1'/q_1)^{\theta - 1}$ or $\bar{v}(\bar{t}_1') = (q_1'/q_1)^{\theta - 1} \bar{v}(\bar{t}_1)$. Hence, with (3), (23) and (28) the ratio

$$\frac{\Pi(\bar{\gamma}, \bar{t}_1)}{F(\bar{\gamma})}$$

is independent of q_1 and q_1' but increasing in $\bar{\gamma}$. This contradicts (29) and proves the proposition.
∎

Having thus established uniqueness of the ERSE, we can now conduct meaningful comparative statics within the specified parameter range. We do so in the following Lemma.

Lemma 4 *Let $\bar{\gamma}(\phi)$ and $\tilde{\phi}$ be from Proposition 3. Then,*
(i) $\bar{\gamma}(\phi)$ of the ERSE is increasing in ϕ for $\phi \leq \tilde{\phi}$.
(ii) $\bar{\gamma}(\phi, \varphi, L)$ is a function of $\phi/(\varphi L)$ only.

Proof. (i) The proof of Proposition shows that the ratio

$$\frac{\Pi(\bar{\gamma}, \bar{t}_1)}{F(\bar{\gamma})}$$

is increasing in $\bar{\gamma}$ for $\phi \leq \tilde{\phi}$. Together with the free entry condition and $dF(\bar{\gamma})/d\phi > 0$, this proves the claim.

(ii) Notice that operating profits π are linear in L while the setup cost F is linear in ϕ. Thus, when replacing $\phi' = \phi/L$ population L factors out of the slanted brackets in (24) and the square brackets in (25), while the solution and hence the solution of problem (24) - (26) and thus $\bar{\gamma}$ depends on $\phi' = \phi/L$ only. Similarly, by Lemma 2 (ii) operating profits are linear in φ under the time transformation $\bar{v}'(t) = \bar{v}(t)/\varphi$ (or $t' = t - \ln(\varphi^{1/a})$). Thus, replacing $\phi' = \phi/\varphi$ in (24) and (25), the spacing parameter of the ERSE depends on ϕ' only. This completes the proof of (ii). ∎

Part (i) of the Lemma 4 implies that for the range of ϕ within which constraint (21) binds the unique $\bar{\gamma}(\phi)$ is strictly increasing: firms compensate increases in setup costs by increasing profits through wider relative spacing, which brings about a larger market share as well as higher markups due to less intense competition.

Together with observation (i), part (ii) of the Lemma determines the impact of market size (L) and marginal production costs (φ) on the spacing $\bar{\gamma}$ of the ERSE. In particular, increases in L and φ have identical effects as reductions in setup costs, and hence decrease the equilibrium relative spacing $\bar{\gamma}$. Intuitively, a larger market induces, *certeris paribus*, higher profits and allows firms to generate more profits. At constant setup costs, larger markets therefore experience more frequent entry of firms at closer distances – the competitive pressure among firms rises. Conversely, productivity growth at the margin (a decrease in marginal production costs φ) *increases* relative spacing and *reduces* the toughness of competition. This adverse effect of marginal productivity growth on competitive pressure may seem a somewhat puzzling. To understand the forces operating to its effect, observe with (12) that the utility (1) generates, just as ordinary CES preferences, relative firm markups $p_n/c_n - 1$ that are independent of costs. Put differently, at given relative

spacing, operating profits constitute a constant share of revenues. Hence, when quality levels are constant, an increase in marginal productivity tends to curb revenues and thereby depresses operating profits.[9] As firms must cover their setup costs, however, the productivity gains that curb profits per consumer must come about with increases in market share, *i.e.*, with a wider equilibrium spacing. This widening of relative spacing does, at the same time, increase relative markups. Hence, competitive pressure decreases as marginal productivity grows.

Notice that a crucial role comes to the assumption that demand does not react along an intensive margin as consumers choose to consume one or zero units of a given good with a certain quality level (consumers do not react to price changes by consuming marginally more or less but by switching to other firms).

3   International Trade

We now turn to the effects of international trade, considering a world of two countries, Home (no *) and Foreign (*), which are populated, respectively, by L and L^* individuals. The homogenous good $\mathcal{A}$ is costlessly traded, thus equalizing wages in both countries, since the according technology is assumed to be equal worldwide. Trade in the $\mathcal{Q}$-type good is subject to standard gross iceberg trade costs $\tau \geq 1$. The monopolistically competitive firms price discriminate between the export and domestic market. Preferences in both countries are identical, which implies, in particular, that the maximum valuations coincide ($\bar{v}(t) = \bar{v}^*(t)$ holds).

We analyze two different scenarios. In the first scenario one country has a comparative advantage of the Ricardian type in the $\mathcal{Q}$-industry so that its production dominates the world market. In the second scenario, productivity is identical across countries and intra-industry trade arises due to increasing returns to scale.

3.1   Ricardian Comparative Advantage

Consider an industry producing the $\mathcal{Q}$-type good described above. Nothing of the following analysis changes in presence of a larger number of $\mathcal{Q}$-type industries, which may differ in costs and maximum valuations v_{max}. Potential trade imbalances between the aggregate of these industries is balanced by costless trade in the homogeneous good $\mathcal{A}$, whose consumption levels are assumed to be high enough to do so. Without loss of generality we assume that Home has a absolute cost

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[9] This does not, of course, mean that each single firm can raise its profit by decreasing its productivity. The firm's profits would, instead, rise under a drop in productivity that affects all firms uniformly, given constant spacing.

advantage in the $\mathcal{Q}$-sector we analyze in the following. In particular, we assume that

$$\varphi\tau < \varphi^*$$

holds. This comparative advantage is strong enough that Home firms cover world supply of the $\mathcal{Q}$-good: Home's cost advantage, net of transport costs, eliminates Foreign competitors in both markets. At each point in time, an operating firm makes profits in Home and in Foreign. We denote the profits that accrue in the domestic market with π_d, while those from the foreign market are π_f.

By definition of the iceberg trade cost, for each unit of the $\mathcal{Q}$-type good with quality level q_m that arrives in Foreign, τ units must leave the mth firm's gates. Trade costs are thus formally equivalent to an increase in production costs by the factor τ, which affects sales and operating profits in the export market. By Lemma 2 (ii) operating profits in Foreign coincide with those in Home, but are scaled up by the factor τ, and sold in Foreign with a time-lag Δ_τ that induces $\bar{v}(t + \Delta_\tau) = \tau\bar{v}(t)$. By (20) the time lag is $\Delta_\tau = a^{-1}\ln(\tau)$. Consumption levels in Foreign thus lag by Δ_τ and profits in the export market are, consequently, discounted by the factor $\exp(-\rho\Delta) = \tau^{-\rho/a}$. Together, this implies

$$\tau^{1-\rho/a}\pi_f(t)/L^* = \pi_d(t)/L.$$

Thus, when adopting the notation from (23) while dropping the arguments γ_k, aggregate profits from both markets are

$$\begin{aligned}
\bar{\Pi} &= \int_t^\infty e^{-\rho(t'-t)}\left[\pi_d(\bar{v}(t')) + \pi_f(\bar{v}(t'))\right]dt' \\
&= \int_t^\infty e^{-\rho(t'-t)}\pi_d(\bar{v}(t'))\left[1 + \tau^{1-\rho/a}L^*/L\right]dt' + \int_t^{t+\Delta_\tau} e^{-\rho(t'-t)}\pi_f(\bar{v}(t'))dt'
\end{aligned} \tag{30}$$

Turning to the entry game, we continue to assume, in line with condition (21), that firms cannot setup their plant before they make a marginal unit of revenue – be it in the domestic or in the export market. This amounts to assuming that at least one of the following conditions must be satisfied at entry date t_m: either $\bar{v}(t)q_m - c_m \geq \bar{v}(t)q_{m-1} - p_{m-1}$ or $\bar{v}^*(t)q_m - \tau c_m \geq \bar{v}^*(t)q_{m-1} - p^*_{m-1}$, where p_{m-1} (p^*_{m-1}) are prices in Home (Foreign) before the mth firm's entry. By the assumption $\bar{v}(t) = \bar{v}^*(t)$ and Lemma 2 (ii) the condition from the foreign market is

$$\tau\bar{v}(t - \Delta_\tau)(q_m - q_{m-1}) \geq \tau c_m - \tau p_{m-1}$$

which is satisfied whenever the according constraint (21) for Home's domestic market holds. (This is quick to see when reading it as a constraint on the entry date t_m). Hence (21) for the domestic market remains the relevant constraint in the open economy.

Comparing (23) and (30) we observe that the discounted flow of operating profits in the open economy is equivalent to that of a firm operating in an economy with population size $L + \tau^{1-\rho/a}L^*$ plus the profits that accrue from sales in the export market $(\pi_f(\bar{v}(t')))$ at times $t' < t_1 + \Delta_\tau$. Applying now Lemma 4 to a virtual closed economy with population size $L + \tau^{1-\rho/a}L^*$, we know that the constraint (21) binds provided that $\phi < \bar{\phi}$ holds. Letting t_1^v be the first firm's entry date of this virtual economy, we can thus infer that firms in Home that enter at date t_1^v make no profits in Foreign before date $t' < t_1^v + \Delta_\tau$.[10] In this case we have

$$\bar{\Pi} = \left[1 + \tau^{1-\rho/a}L^*/L\right] \int_t^\infty e^{-\rho(t'-t)}\pi_d(t')\,dt'.$$

Consequently, whenever the constraint (21) binds, the open economy with population sizes L and L^* and trade costs τ sustains the same equilibrium as the virtual closed economy of population size $L + \tau^{1-\rho/a}L^*$, and all implications from Lemma 4 apply.[11]

Notice that, if the growth rate of valuation is larger than the market discount rate $(a > \rho)$ the exponent on the trade costs τ is positive. In this case we can tell the following story. Initially, Home is a closed economy due to prohibitive trade costs and spacing in its $\mathcal{Q}$-type industry is determined accordingly. One day, trade costs drop to moderate levels that induce complete specialization but are still positive. In the long run, a new market structure establishes, which is characterized by a smaller spacing parameter $\bar{\gamma}$, since by the increase in market size induces more frequent firm entry with a tighter relative quality spacing (Lemma 4). The larger the market size of the trade partner, the more pronounced is this increase in the toughness of competition. Interestingly, however, further reductions of τ tend to increase the long run equilibrium spacing. The key effect is the one discussed in the closed economy already: trade costs act like an additional production cost specific to the export market. These additional cost effect all firms equally and thus, at constant spacing and markups, increase each firms profits, thus inducing a denser relative quality spacing. Conversely, a reduction of these trade costs cut into firms' profits, which entering firms compensate by less frequent entry and wider relative spacing.

Having analyzed an industry whose world supply is dominated by one country, we next turn

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[10] Notice that $\bar{v}(t_1^v)(q_m - q_{m-1}) = c_m - p_{m-1}$ implies $\tau\bar{v}(t_1^v)(q_m - q_{m-1}) = \tau c_m - \tau p_{m-1}$ and hence $\bar{v}(t_1')(q_m - q_{m-1}) < \tau c_m - p_{m-1}^*$ for $t' < t_1^v + \Delta$

[11] A slight adaptation of the proof of Proposition 3 can show that the equilibrium is unique.

to the case of intra industry trade.

3.2 Intra Industry Trade

We consider next competition between domestic and foreign firms within a given industry, *i.e.*, we consider the case of intra industry trade. In particular, we aim to analyze the effects of international competition on prices and markup when foreign firms sell into domestic markets and domestic firms sell into foreign markets within the same $\mathcal{Q}$-type industry. To this end, we restrict the analysis to the case of countries with equal population sizes ($L = L^*$) and identical technologies ($\phi = \phi^*$ and $\varphi = \varphi^*$).

Contributions: prove a regularity concerning endogenous quality supply is models of quality competition in the spitit of [?] as used in [?]

3.2.1 Exogenous Spacing

In a first step, we return to exogenous spacing and analyze the two country world with equal spacing (11) but locations of firms alternate with index n. More precisely, if firm n is located in Home (Foreign) then firm $n-1$ is assumed to be located in Foreign (Home). As in subsection 2.1.4, we assume that firm quality is ascending with index and $n = 0$ marks the highest quality firm. Effective production costs of sales to either of the markets are φq_n^{θ} if q_n is produced domestically and $\tau \varphi q_n^{\theta}$ otherwise. In this case, we can, just as in the closed economy, derive the equilibrium prices explicitly.

Proposition 4 *Assume that (11) holds and firm locations alternate in n. Then, for any of the two countries consumer prices are*

$$p_n = \left(A \left(\lambda / \gamma^{\theta} \right)^n + \alpha_n \right) c_n \tag{31}$$

where

$$\alpha_n = \begin{cases} \left(2 + \frac{\gamma^{\theta} + \gamma^{1-\theta}}{\gamma + 1} \tau \right) \left(4 - \left(\frac{\gamma^{\theta} + \gamma^{1-\theta}}{\gamma + 1} \right)^2 \right)^{-1} & \text{if } q_n \text{ is produced domestically} \\ \left(2\tau + \frac{\gamma^{\theta} + \gamma^{1-\theta}}{\gamma + 1} \right) \left(4 - \left(\frac{\gamma^{\theta} + \gamma^{1-\theta}}{\gamma + 1} \right)^2 \right)^{-1} & \text{else} \end{cases} \tag{32}$$

$$A = \frac{\lambda}{2\lambda - 1} \left(1 - \alpha_0 2 + \alpha_{-1} \gamma^{-\theta} + \frac{\gamma - 1}{\gamma} \frac{q_0 \bar{v}(t)}{c_0} \right) \tag{33}$$

and λ is from (14).

Proof. Setting $p_n = u_n + \alpha c_n$ if firm n is located in the domestic market and $p_n = u_n + \alpha^* c_n$ if not, the system (8) for the consumer prices in Home is

$$2\left[u_n + \alpha c_n\right] = c_n + \frac{1}{\gamma+1}\left[u_{n+1} + \alpha^* c_{n+1}\right] + \frac{\gamma}{\gamma+1}\left[u_{n-1} + \alpha^* c_{n-1}\right] \qquad n < 0 \text{ domestic}$$

$$2\left[u_n + \alpha^* c_n\right] = \tau c_n + \frac{1}{\gamma+1}\left[u_{n+1} + \alpha c_{n+1}\right] + \frac{\gamma}{\gamma+1}\left[u_{n-1} + \alpha c_{n-1}\right] \qquad n < 0 \text{ foreign}$$

The contributions multiplied by c_n vanish iff

$$2\alpha = 1 + \frac{\gamma^\theta}{\gamma+1}\alpha^* + \frac{\gamma^{1-\theta}}{\gamma+1}\alpha^* \qquad n < 0 \text{ domestic}$$

$$2\alpha^* = \tau + \frac{\gamma^\theta}{\gamma+1}\alpha + \frac{\gamma^{1-\theta}}{\gamma+1}\alpha \qquad n < 0 \text{ foreign}$$

Solving for α and α^* leads to (32). The remaining problem is

$$2u_n = \frac{1}{\gamma+1}u_{n+1} + \frac{\gamma}{\gamma+1}u_{n-1} \qquad n < 0$$

with the general solution

$$u_n = A\lambda^n + B\mu^n$$

and $\lambda = \gamma + 1 + \sqrt{\gamma^2 + \gamma + 1}$ and $\mu = \gamma + 1 - \sqrt{\gamma^2 + \gamma + 1}$. Transversality $\lim_{n\to-\infty} p_n = 0$ and $\mu < 1$ imply $B = 0$. (8) for $n = 0$ leads to

$$2A + 2\alpha_0 c_0 = c_0 + \frac{\gamma-1}{\gamma}q_0 v_{\max} + A/\lambda + \alpha_{-1}\gamma^{-\theta}c_0$$

proving (33). ∎

Notice that in the limit of vanishing trade costs ($\tau \to 1$) the local prices (31) collapse to those of the closed economy (12). In this case, the location of firms is, obviously, irrelevant.

When considering positive trade costs ($\tau > 1$) we focus on relative small relative spacing so that $\lambda > \gamma^\theta$ holds, which is, by (18), equivalent to the assumption that

$$\vartheta \equiv \frac{\gamma^\theta + \gamma^{1-\theta}}{\gamma+1} < 2$$

holds. Further, to facilitate notation, we denote the values defined in (32) with α_d if n is located in Home and α_f if n is located in Foreign. Similarly, we label A from (33) with A_d whenever $\alpha_0 = \alpha_d$ (highest quality firm is located in the domestic market) and A_f otherwise.

Now observe that $\tau > 1$ implies $\alpha_f > \alpha_d$ and that both α_d and α_f, are larger than α_{CE} of the closed economy (13), since

$$\alpha_f > \alpha_d > \frac{2+\vartheta}{4-\vartheta^2} = \frac{1}{2-\vartheta} = \alpha_{CE}.$$

Moreover, the inequality $\alpha_f > \alpha_d$ implies that, for a given firm n, the A from (33) is larger if the highest quality firm ($n = 0$) is located in the domestic market instead of the foreign one ($A_d > A_f$).

Firms do not sell to all markets under all circumstances. Local firms sell into the export market whenever their markups are weakly positive:

$$A_f \left(\lambda / \gamma^\theta \right)^n + \alpha_f - \tau \geq 0. \tag{34}$$

By $\lambda > \gamma^\theta$ this condition is satisfied for all $n \leq 0$ if it holds for $n = 0$ as well as for $n \to -\infty$, *i.e.* whenever conditions

$$A_f + \alpha_f - \tau \geq 0 \qquad \text{and} \qquad \alpha_f - \tau \geq 0$$

are satisfied. It is quick to check that these conditions are more restrictive than the according conditions for positive sales of domestic firms.

If these conditions hold, the total operating profits – the sum of operating profits in both markets – can be calculated by using the explicit prices from (31) and the generic expression for the operating profits (10). For firms located in Home, this leads to

$$\pi_{n,tot} = \begin{cases} \dfrac{c_0^2}{q_0} \dfrac{L}{\bar{v}(\tau)} \dfrac{\bar{\gamma}}{\bar{\gamma} - 1} \left[(A + \alpha_d - 1)^2 + \left(A^* + \alpha_f^* - \tau \right)^2 \right] & n = 0 \\[4mm] \dfrac{c_n^2}{q_n} \dfrac{L}{\bar{v}(\tau)} \dfrac{\bar{\gamma} + 1}{\bar{\gamma} - 1} \left[\left(A \left(\lambda / \bar{\gamma}^\theta \right)^n + \alpha_d - 1 \right)^2 + \left(A^* \left(\lambda / \bar{\gamma}^\theta \right)^n + \alpha_f^* - \tau \right)^2 \right] & n < 0 \end{cases} \tag{35}$$

From these expression we read that profits from the domestic market are, not surprisingly, bigger that those from foreign markets. In particular, for the highest quality firm we compute with (32) and (33)

$$A_d + \alpha_d - 1 - \left(A_f + \alpha_f^* - \tau \right) = \frac{\tau - 1}{2\lambda - 1} \left[\left(\lambda \gamma^{-\theta} + 1 \right) \frac{1}{2 + \vartheta} + (2\lambda - 1) \right] > 0$$

For $n < 0$ the equivalent result holds since the difference $\left(A \left(\lambda / \gamma^\theta \right)^n + \alpha_d - 1 \right) - \left(A^* \left(\lambda / \gamma^\theta \right)^n + \alpha_f^* - \tau \right)$ is smallest if $A = A_f$ and $A^* = A_d$ (remember $A_f < A_d$) and, moreover, increases in n so that we need to consider the case $n = -1$ only, for which we compute

$$A_f \left(\tfrac{\lambda}{\gamma^\theta} \right)^{-1} + \alpha_d - 1 - \left(A_d \left(\tfrac{\lambda}{\gamma^\theta} \right)^{-1} + \alpha_f^* - \tau \right) = \frac{\tau - 1}{2\lambda - 1} \left[\frac{-2 \left(\gamma^\theta + \lambda \right)}{2 + \vartheta} + (2\lambda - 1) \right] > 0$$

This expression is positive for small γ as $\lambda > \gamma^\theta$ and $\gamma^\theta + \gamma^{1-\theta} > \gamma + 1$ implies that the term in square brackets is positive since $-\tfrac{2}{3} \left(\gamma^\theta + \lambda \right) + (2\lambda - 1) > -\tfrac{4}{3}\lambda + (2\lambda - 1) = \tfrac{2}{3}\lambda - 1 > 0$, where we used $\lambda > 3$ from (14) for the last inequality.

3.2.2 Endogenous Spacing Under Trade (Incomplete)

The results derived in the previous sections allow some preliminary thoughts about the effects of free entry under intra industry trade. Clearly, when moving from prohibitive to negligible trade costs, the long run Equal Relative Spacing Equilibrium exhibits denser relative spacing since the increase in total market size induces, just as in the closed economy, a more frequent entry at denser relative spacing. In this sense, consumers enjoy a real pro-competitive effect from trade integration as they can choose from a more ample quality supply and are being charged lower markups at the same time.[12]

Further, considering continuity of equilibrium variables in trade costs, we expect that the world economy behaves reasonably close to the integrated economy whenever trade costs are very small. This involves, in particular, the constraint (21) requiring marginally positive revenues, which can thus be assumed to be bind in at least one of the two markets (either the export or the domestic one) for $\phi > 0$ and $\tau > 1$ small enough. In case that constraint (21) binds we observe that the assumption of alternating firm locations made in the previous subsection is the natural outcome. In particular, if the highest quality firm is located in Foreign,we can check with (31) that the price for its good is larger in Home than in Foreign (use (33) and (32) to observe that $A_f + \alpha_f > A_d + \alpha_d$). This implies that the constraint (21) allows firms in Home to enter the market before Foreign firms can do so. Together with the observation that the share of profits from the domestic market to total profits (28) exceeds the share from the export market, we conjecture that equilibrium firm location does, in fact, oscillate between both countries.[13]

Regarding the comparative statics with respect to trade costs, intuition gained in the previous sections offers some basic insights already. Most importantly, an increase in τ generates an obvious "competition effect": penetration of export market becomes less easy, which implies that each firm tends to loose market shares in the export market while it gains market shares in the domestic market. Focussing on the common term of profits (35)

$$(\alpha_d - 1)^2 + (\alpha_f^* - \tau)^2 \tag{36}$$

it is quick to check that around zero tariffs, the losses dominate. While profits from the domestic market (associated with the first term) increase in τ, the profits from the export market (the

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[12] In this sense, our model differs from those based on Meltiz (2003) that exhibit no scale effects and therefore no pro-competitive effects due to full trade integration.

[13] These considerations neglect the impact of off-equilibrium deviations on the location choices of future firms and need to be cast in a rigorous proof.

second term) decrease faster. The sum of both therefore decreases in τ. Intuitively, ... For larger τ, however, the contribution of foreign profits becomes less important and its percentage decrease is outweighed by the increase in the profits from the domestic market. The effect of increases in τ on A and A^* in total profits (35) is less obvious but may in fact sum to a negative total effect.

Against the background of the possibly ambiguous effects of trade costs on the equilibrium spacing, we turn to a numerical evaluation of τ on $\bar{\gamma}$ of the ERSE. We do so assuming that the constraint (21) binds, wherefore we focus on small spacing parameters $\bar{\gamma}$ (hence we choose ϕ to be small). Since $A_f \to 0$ and $\alpha_f \to 1$ at $\bar{\gamma} \to 1$, the condition (34) shows that for small $\bar{\gamma} > 1$ trade flows are positive only under small transport costs $\tau > 1$. Notice that, whenever the constraint (21) binds for the Home market, the constraint (34) is violated for $n = 0$ so that the highest productivity firm does not sell into the export market. Taking account for these cases, we therefore consider parametrizations for which (34) holds for all $n \leq -1$ only. Throughout the exercise, we make sure that $\alpha_f - \tau \geq 0$ is satisfied so that all low quality firms sell into the foreign market.

Figure 2 shows that the equilibrium spacing tends to increase in trade costs for a wide parameter range: since increasing profits tend to curb profits, they force firms to choose a wider spacing, which grants the profits needed to cover setup costs. Not surprisingly, and just as in the closed economy, the top panel illustrates that the spacing decreases when the market size, associated with population L, increases. Similarly, the bottom panel shows that a faster increase of setup cost in quality (larger θ) forces firms to a tighter relative spacing.

Figure 3 finally shows that an increase in the discount rate ρ tends to increase the relative spacing: higher discounting implies less net present value to cover setup cost, which must be compensated by larger spacing. More interestingly, however, the same figure shows that the relative spacing may actually decrease in the trade costs τ. As stated above, for larger τ the negative impact of an incremental increase in τ increases profits earned domestically but depresses those from the export market. Since, at high τ the latter are not substantial to start with, the gains from domestic market dominate, so that, under free entry, firms react by entering production earlier and under denser relative spacing. It is noteworthy that this last effect is stronger for less heavy discounting. This effect of variations in the discount rate underlines that the positive effect of trade costs on profits works through the term (36), which dominates for firms far away from the quality leader ($n \to \infty$) and hence for periods long after entry. If, as under light discounting, these profits contribute substantially to the net present value of entry, the aforementioned effect

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is strong. Conversely, under heavy discounting, it is feeble at best.

4   Conclusion

The main contribution of our work is to extend the insights brought forward by Aved Shaked and John Sutton to many firm environments, and to, therefore, highlight how the ideas of Hotelling (1929) or Lancaster (1966) work out in the quality space.

Hotelling's classic 'location' paradigm is widely used to reflect generic product characteristics.[14] The well-studied formalism of the spacing model, however, does not apply to competition in quality. By its very definition, quality requires that individuals agree on the ranking of varieties and, in particular, their individually preferred "ideal variety". When it comes to vertical differentiation – or differentiation in quality – only the higher price tag of the universally preferred higher quality goods makes different consumers buy distinct qualities.[15]

Conceptually, the fundamental departure of our model of spatial competition in quality from the existing literature is the absence of the "transportation cost" and the associated one-to-one ordering of tastes and characteristics. In the Hotelling model, if we consider only consumers with valuations between two firms A and B, if consumer 1 buys from firm A and consumer 2 is closer to firm A than consumer 1, then consumer 2 necessarily buys from firm A. In particular, firm A is always more likely to sell to consumer 2 than to consumer 1, independently of the market environment. In our model, such a canonical matching of firm characteristic and consumer valuation does not exist: for example, if costs increase for all firms by 10%, a firm can end up selling to an entirely different set of consumers than it did before the cost shock.

Owing to the absence of the physical transportation cost, mark-ups are determined by the shape of the marginal cost schedule, that is, by how the marginal costs to produce goods of different qualities compare. In standard models of spatial competition, the transportation cost and the varying location of consumers directly determine a firm's market power. In our setup,

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[14] In particular, Lancaster (1966, 1975 and 1980) has coined the term "ideal variety" re-interpreting physical distance as the distance of the good's attribute from the consumer's taste.

[15] There is now ample empirical evidence that quality is an important dimension of product differentiation. At the consumer level, Bills and Klenow (2001) and Broda and Romalis (2009) document that wealthier consumers tend to buy a much larger fraction of high quality goods. At the producer level, Baldwin and Harrigan (2008), Crozet, Head, and Mayer (2007), Hallak and Sivadasan (2009), Johnson (2008), and Kugler and Verhoogen (2008) document that high quality producers tend to be more profitable in equilibrium and are more likely to export than low quality producers. Last, quality differentiation also explains aggregate trade flows: richer countries export goods with high unit value (Schott (2004) and Hummels and Klenow (2005)) and of high quality (Hallak and Schott (2008)), while richer countries also tend to consume high unit-value goods (Hallak (forthcoming)).

all consumers would like to improve the quality off their good, and it is the increasing steepness of the prices of such goods (i.e. the convexity of the marginal cost schedule) that prevents them from doings so. In turn, the firm's market power is thus determined by the interplay of the shape of the marginal cost schedule with respect to quality and the spacing of quality competition.

A firm's optimal price depends on its cost, on the extent of its market power, and also on the prices of the two adjacent competitors with next highest respectively next lowest quality. Even though each firm only responds to its direct competitors, all firms are indirectly connected along the chain of quality spacing. In equilibrium, the markup of any firm thus depends on the market power of all other firms and on the density of spacing on the quality schedule in the entire economy.

A   Appendix

Proof: Proposition 2.   Consider the location γ_1 of the first entrant, $m = 1$ and observe that $\gamma_1 = 1$ cannot be part of an equilibrium since Bertrand competition implied $\Pi_{t_1}(\gamma_1) = 0$ (regardless of t_1) in this case, thus violating the free entry condition. Hence, $\gamma = 1$ implies $\gamma_1 > \gamma$. At the same time, γ_1 must trivially be finite so that $\gamma_1 < \gamma$ holds for γ large enough. By continuity there is a $\gamma > 1$ so that $\gamma_1 = \gamma$, which we denote by $\bar{\gamma}$. Thus, under $\gamma = \bar{\gamma}$, the first firm chooses a quality level that extends the equal relative spacing rule (11) to its own quality level q_1.

Now assume $\gamma = \gamma_1 = \bar{\gamma}$ and call the spacing problem of the remaining additional firms ($m = 2, 3, ...$) the *residual spacing problem*. With the notation

$$\gamma'_m = \gamma_{m+1} \qquad (m \geq 1) \qquad q'_0 = \bar{\gamma}q_0 = q_1 \qquad \text{and} \qquad \bar{v}'(\tau) = \bar{v}(\tau)\bar{\gamma}^{\theta-1} \qquad \text{(A1)}$$

the residual spacing problem solves the corresponding system (24) - (26) above, where now all state and choice variables bear a prime (q', v', γ'). Apply Lemma 2 (i) to verify that

$$\pi_m\left(\bar{v}'(\tau), \tilde{\Gamma}'_{m,k}\hat{\gamma}'\Gamma'^{(*)}_{0,m-1}q'_0, \gamma'_n, \gamma'_{n-1}, ..., \gamma'_1, \bar{\gamma}\right) = \bar{\gamma}^\theta \pi_m\left(\bar{v}(\tau), \tilde{\Gamma}'_{m,k}\hat{\gamma}'\Gamma'^{(*)}_{0,m-1}q_0, \gamma'_n, \gamma'_{n-1}, ..., \gamma'_1, \bar{\gamma}\right)$$

and notice that the setup cost (3) satisfies $F(\Gamma'^{(*)}_{0,m-1}q'_0) = F(\Gamma'^{(*)}_{0,m-1}\bar{\gamma}q_0) = \gamma^\theta F(\Gamma'^{(*)}_{0,m-1}q_0)$. Hence, the term γ^θ factors out of the right hand side of (24) and of the term in square brackets in (25) and, moreover, leaves the respective constraints unaffected. Consequently, the solution of the residual spacing problem coincides with the original problem, implying $\gamma'_1 = \gamma_2 = \gamma_1 = \bar{\gamma}$. A simple induction argument completes the proof that $\gamma_m \equiv \bar{\gamma}$ for all $m \geq 1$.

Finally, the transformation (A1) shows either two consecutive entries occur at dates that satisfy $\bar{v}(t_m) = \bar{\gamma}^{\theta-1}\bar{v}(t_{m+1})$. With (20), this is $e^{a(t_{m+1}-t_m)} = \bar{\gamma}^{\theta-1}$ and proves the second statement. ∎

Proof: Lemma 3.   We consider the first firm's entry.

(i) First observe that net profits are zero in equilibrium. If the constraint (21) does not bind this claim holds by continuity of gross profits (23) in the first firm's entry date t_1 and $\Pi(\gamma_1, t_1) \to 0$ ($t_1 \to -\infty$). Thus, assume that the constraint (21) binds and firm 1 earns positive operating profits under strategy $\gamma_1 > 1$ and entry date $t_1 = a^{-1}\ln\left(\varphi\left(\gamma_1 q_0\right)^{\theta-1}/v_{\max,0}\right)$ (defined by (21)). A competitor firm can enter the market at $\tilde{t}_1 = t_1 - \varepsilon$ playing $\tilde{\gamma}_1 = \gamma_1 e^{-\varepsilon a(\theta-1)}$, thus satisfying (21). For $\varepsilon > 0$ small enough $\tilde{\gamma}_1 > 1$ holds and the competitors profits are, by continuity, positive, which contradicts the definition of t_1 by (25), proving that net profits are zero in equilibrium. – We can now easily rule out the existence of a sequence of $\phi_k \to 0$ ($k \to \infty$) and associated $\bar{\gamma}_k$ so

that $\lim_{k\to\infty} \bar{\gamma}_k = \bar{\gamma}_\infty > 1$. If there were such a sequence, their limit would generate positive net profits, which contradicts free entry.

(ii) By Bertrand competition we have $\gamma = 1 \Rightarrow \Pi = 0$. Together with part (i) of the Lemma this implies that $\phi \to 0$ is equivalent to $\gamma \to 1$. Assume that statement (ii) does not hold, i.e. there is a sequence $\{\gamma_k\}_{k\geq 0} > 1$ with $\gamma_k \to 1$ with entry dates $t_1^{(k)}$ and operating profits satisfying $\pi(t_1^{(k)}) > 0$. We restrict the following computations to the thus defined set of ERSEs, dropping indices k.

Now observe that the entry date determined by (25) implies that at t_1 profits from (23) satisfy condition $0 \leq \partial\Pi(t,\gamma)/\partial t$, which we rewrite with $\partial\Pi(t,\gamma)/\partial t = -\pi(t) + \rho\Pi(t,\gamma)$ as

$$\frac{1}{\rho} \leq \frac{\Pi(t_1,\gamma)}{\pi(t_1)}. \tag{A1}$$

With the operating profits of additional entrants m (28) compute for $m = 1$

$$\left((\gamma - 1)\frac{q_1}{c_1^2}\frac{v_{\max,t_1}}{L}\right)\Pi(t_1,\gamma) = \int_{t_1}^{t_1+\Delta} e^{-(\rho+a)(\tau-t_0)}\left(A\left(\tau\right) + \alpha - 1\right)^2 d\tau + ...$$

$$... + \sum_{m\geq 1}(\gamma + 1)\int_{t_1+m\Delta}^{t_1+(m+1)\Delta} e^{-(\rho+a)(\tau-t_0)}\left(A\left(\tau\right)\left(\frac{\gamma^\theta}{\lambda}\right)^m + \alpha - 1\right)^2 d\tau$$

With $\Delta = (\theta - 1)a^{-1}\ln(\bar{\gamma})$ from (27) and $A(\tau) = A(\tau + \Delta)$ (by (15) and (27)) this is

$$\left((\gamma - 1)\frac{q_1}{c_1^2}\frac{v_{\max,t_1}}{L}\right)\Pi(t_1,\gamma) = \int_0^\Delta e^{-(\rho+a)\tau}\left(A\left(\tau + t_1\right) + \alpha - 1\right)^2 d\tau + ...$$

$$... + (\gamma + 1)\sum_{m\geq 1}e^{-(\rho+a)\Delta m}\left(\frac{\gamma^\theta}{\lambda}\right)^{2m}\int_0^\Delta e^{-(\rho+a)\tau}\left[A\left(\tau + t_1\right)\right]^2 d\tau + ...$$

$$... + 2(\gamma + 1)\sum_{m\geq 1}e^{-(\rho+a)\Delta m}\left(\frac{\gamma^\theta}{\lambda}\right)^m\int_0^\Delta e^{-(\rho+a)\tau}A\left(\tau + t_1\right)(\alpha - 1) d\tau + ...$$

$$... + (\gamma + 1)\int_\Delta^\infty e^{-(\rho+a)\tau}(\alpha - 1)^2 d\tau$$

$$= \int_0^\Delta e^{-(\rho+a)\tau}\left(A\left(\tau + t_1\right) + \alpha - 1\right)^2 d\tau + ...$$

$$... + (\gamma + 1)\frac{\gamma^{-\frac{\rho+a}{a}(\theta-1)}\left(\frac{\gamma^\theta}{\lambda}\right)^2}{1 - \gamma^{-\frac{\rho+a}{a}(\theta-1)}\left(\frac{\gamma^\theta}{\lambda}\right)^2}\int_0^\Delta e^{-(\rho+a)\tau}\left[A\left(\tau + t_1\right)\right]^2 d\tau + ...$$

$$... + 2(\gamma + 1)\frac{\gamma^{-\frac{\rho+a}{a}(\theta-1)}\left(\frac{\gamma^\theta}{\lambda}\right)}{1 - \gamma^{-\frac{\rho+a}{a}(\theta-1)}\left(\frac{\gamma^\theta}{\lambda}\right)}\int_0^\Delta e^{-(\rho+a)\tau}(\alpha - 1)A\left(\tau + t_1\right) d\tau + ...$$

$$... + (\gamma + 1)\gamma^{-\frac{\rho+a}{a}(\theta-1)}(\alpha - 1)^2$$

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All expressions multiplying the integrals are uniformly bounded for $\gamma \geq 1$ small enough (as $\gamma^\theta / \lambda < 1$ holds uniformly for γ small). Moreover, the time interval Δ from (27) satisfies $\Delta = O(\gamma - 1)$. Hence, for the case $\lim_{\gamma \to 1} A(t_1) / (\alpha - 1) = \infty$ we conclude

$$\frac{\Pi(t_1, \gamma)}{\pi(t_1)} = \frac{\left((\gamma - 1) \frac{q_1}{c_1^2} \frac{v_{\max, t_1}}{L}\right) \Pi(t_1, \gamma)}{(A(t_1) + \alpha - 1)^2} \to 0.$$

which contradicts (A1). We are therefore left with $\lim_{\gamma \to 1} A / (\alpha - 1) = B < \infty$, in which case

$$\frac{\Pi(t_1, \gamma)}{\pi(t_1)} = \frac{\left((\gamma - 1) q_1 / c_1^2 \frac{v_{\max, t_1}}{L}\right) \Pi(t_1, \gamma)}{(A(t_1) + \alpha - 1)^2} \to \frac{2}{(B + 1)^2}.$$

For $\rho < 1/2$ this implies $B < 0$. Since, with (13), $\alpha \to 1$, $\partial \alpha / \partial \gamma \to 0$ and $\partial^2 \alpha / (\partial \gamma)^2 \to \theta (\theta - 1)$ as $(\gamma \to 1)$, the assumption $\lim_{\gamma \to 1} A / (\alpha - 1) = B < \infty$ implies $\partial A / \partial \gamma = 0$ and $\partial^2 A / (\partial \gamma)^2 < 0$. Now take the first partial derivative of A from (15) wrt γ, to find that $\partial A / \partial \gamma = 0$ implies

$$\frac{v_{\max, t_0} q_0}{c_0} \to \theta \qquad (\gamma \to 1)$$

Using this limit, the second partial derivative is

$$\frac{\partial^2 A}{(\partial \gamma)^2} \to 0 \qquad (\gamma \to 1)$$

This contradicts condition $\partial^2 A / (\partial \gamma)^2 < 0$ and proves the statement. ∎

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