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Research Article

External Periodic Force Control of a Single-Degree-of-Freedom Vibroimpact System

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A single-degree-of-freedom mechanical model of vibro-impact system is established. Bifurcation and chaos in the system are revealed with the time history diagram, phase trajectory map, and Poincaré map. According to the bifurcation and chaos of the actual vibro-impact system, the paper puts forward external periodic force control strategy. The method of controlling chaos by external periodic force feedback controller is developed to guide chaotic motions towards regular motions. The stability of the control system is also analyzed especially by theory. By selecting appropriate feedback coefficients, the unstable periodic orbits of the original chaotic orbit can be stabilized to the stable periodic orbits. The effectiveness of this control method is verified by numerical simulation.

1. Introduction

In the field of nonlinear, along with people understanding the nature of chaos, how to control chaos and chaos synchronization has been a hot topic studied by researchers. In the 1990s, Ott et al. proposed the OGY chaos control method [1]. Scholars put forward a lot of modified control methods [2, 3] based on the OGY method. Although small perturbation of the system parameters for the control of chaos has been confirmed, the method requires detailed information about the target trajectory and brings a lot of inconvenience to the practical application. Therefore the domestic and foreign researchers put the traditional control theory and chaotic motion characteristics used in chaos control and present a lot of chaos control method, such as delayed feedback control [4, 5], periodic parameter perturbation control [6], continuous feedback control [7], pulse feedback control [8], and adaptive control [9], and so forth. There are also examples of control on the vibroimpact system [10, 11].

Vibroimpact system as a typical nonsmooth dynamical system generally exists in practical engineering. Because of the frequent collision, the system has strong nonlinearity and discontinuities compared with a smooth nonlinear system, presents more complex nonlinear phenomena, and causes hazards on the safe operation of the system. Because of the collision interface differential discontinuities, the original method applied to continuous system can not be used for such system.

This paper puts forward a sine periodic force feedback controller based on the periodic external force feedback control strategy and analyzes the stability of control theory. When selecting the appropriate feedback coefficients, the chaotic orbits can be controlled onto the stable periodic orbits. A single-degree-of-freedom vibroimpact system is transformed into a form of Poincaré map for numerical simulation. The results of numerical simulation show that the method is effective in practical engineering, so it has certain practical significance.

2. Mechanics Model of the Vibroimpact System

Figure 1 shows a single-degree-of-freedom mechanical model of vibroimpact system. Oscillator M is connected to the left side support by the spring with stiffness K and the damper with damping C. In the harmonic excitation $F \sin(\Omega T + \theta)$, motion of the oscillator is in the horizontal direction. X is

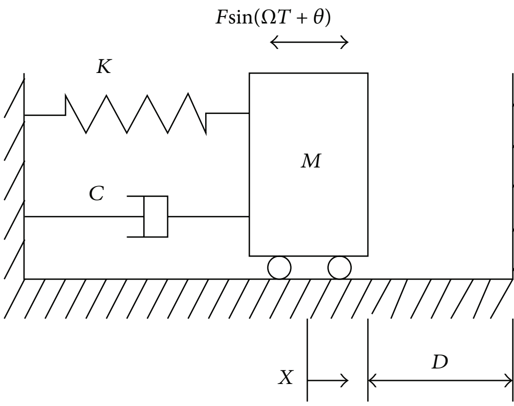


Figure 1: Mechanics model of a single-degree-of-freedom vibroimpact system.

the displacement of motion. When the oscillator is in the equilibrium position, the gap between the rigid constraints on the right side is D. Considering the collision as the rigid collision and R as the coefficient of restitution, the differential equation of motion of system is

$$M\ddot{X} + C\dot{X} + KX = F\sin(\Omega T + \theta) \quad (X < B). \tag{1}$$

The shock equation of system is

$$\dot{X}_{+} = -R\dot{X}_{-} \quad (X = B), \tag{2}$$

where $\dot{X}_{-}$ and $\dot{X}_{+}$ represent the impacting mass velocities of approach and departure at the instant of impacting, respectively.

After the dimensionless transformation, when $x < d$, the differential equations of motion of the system between two collisions are as follows:

$$\ddot{x} + 2\zeta\dot{x} + x = \sin(\omega t + \theta). \tag{3}$$

In which, the nondimensional quantities are

$$x = \frac{XK}{F}, \qquad \zeta = \frac{C}{2\sqrt{MK}}, \qquad \omega = \Omega\sqrt{\frac{M}{K}}, \qquad t = T\sqrt{\frac{K}{M}}, \qquad d = \frac{DK}{F}. \tag{4}$$

When $x = d$, shock equation of the system at collision transient is given by

$$\dot{x}_{+} = -R\dot{x}_{-}, \tag{5}$$

where $\dot{x}_{-}$ and $\dot{x}_{+}$ represent the impacting mass velocities of approach and departure at the instant of impacting, respectively.

By (1), the general solution of the system between two collisions between is

$$\begin{aligned} x = {} & e^{-\zeta t}\left(a\cos\omega_d t + b\sin\omega_d t\right) + A\sin(\omega t + \theta) \\ & + B\cos(\omega t + \theta), \end{aligned} \tag{6}$$

where $\omega_d = \sqrt{1-\zeta^2}$. A and B are amplitude constants. By the initial conditions of the system: $x(t_0) = x_0$ and $\dot{x}(t_0) = \dot{x}_0$, let $t_0 = 0$; the integral constants A and B can determined

$$\begin{aligned} a &= x_0 - A\sin\theta - B\cos\theta, \\ b &= \frac{\left(\dot{x}_0 + \zeta x_0 + (B\omega - A\zeta)\sin\theta - (A\omega + B\zeta)\cos\theta\right)}{\omega_d}, \\ A &= \frac{1-\omega^2}{\left(1-\omega^2\right)^2 + (2\zeta\omega)^2}, \\ B &= \frac{-2\zeta\omega}{\left(1-\omega^2\right)^2 + (2\zeta\omega)^2}. \end{aligned} \tag{7}$$

Periodic motion of the system under certain parameter conditions can be expressed as $n-p$, n represents a force cycle number, and p represents the number of collisions. Considering the $n-1$ periodic motion, the collision instantaneous dimensionless time $t = 0$, and next collision time $t_z = 2n\pi/\omega$, then boundary conditions of the $n-1$ periodic motion are $x(0) = x(t_z) = d$ and $\dot{x}(0) = -Rx(t_z)$. With (6) applying to the boundary conditions, the existence conditions of periodic motion system are

$$\left|\frac{-H\tan\overline{\theta} - \sqrt{1+\tan^2\overline{\theta} - H^2}}{1+\tan^2\overline{\theta}}\right| \le 1, \tag{8}$$

where $\overline{\theta} = \arctan(-H_2/H_1)$, $c = \cos\omega_d t_z$, $e = e^{-\zeta t_z}$, $s = \sin\omega_d t_z$,

$$\begin{aligned} H = {} & \frac{d}{H_1}\left(\left(\zeta + \mathrm{Re}\left(\omega_d s + \zeta c\right)\right)es \right. \\ & \left. + \left(\omega_d + \mathrm{Re}\left(\omega_d c - \zeta s\right)\right)(ec-1)\right), \\ H_1 = {} & A\left(\zeta + \mathrm{Re}\left(\omega_d s + \zeta c\right)\right) - B\omega(1+R)es \\ & + A\left(\omega_d + \mathrm{Re}\left(\omega_d c - \zeta s\right)\right)(ec-1), \\ H_2 = {} & B\left(\zeta + \mathrm{Re}\left(\omega_d s + \zeta c\right)\right) - A\omega(1+R)es \\ & + B\left(\omega_d + \mathrm{Re}\left(\omega_d c - \zeta s\right)\right)(ec-1). \end{aligned} \tag{9}$$

At the same time, theory fixed point of the $n-1$ periodic motion is given by

$$\begin{aligned} \theta_d &= \arccos\left(\frac{-H\tan\overline{\theta} - \sqrt{1+\tan^2\overline{\theta} - H^2}}{1+\tan^2\overline{\theta}}\right), \qquad x_d = d, \\ \dot{x}_d &= \frac{\left(d - A\sin\theta_d - B\cos\theta_d\right)\left((1-ec)\omega_d - es\zeta\right)}{es} \\ & \quad + A\omega\cos\theta_d - B\omega\sin\theta_d. \end{aligned} \tag{10}$$

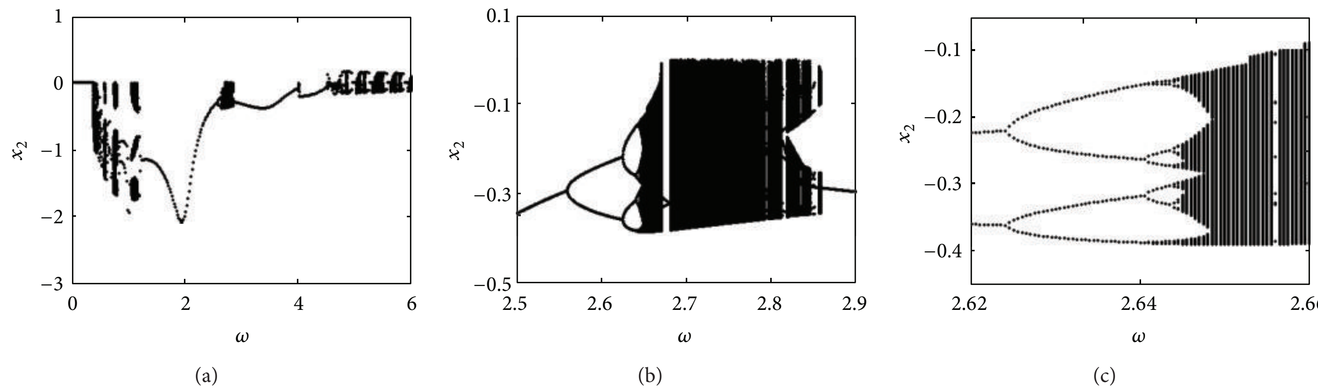


Figure 2: Bifurcation diagram of the system.

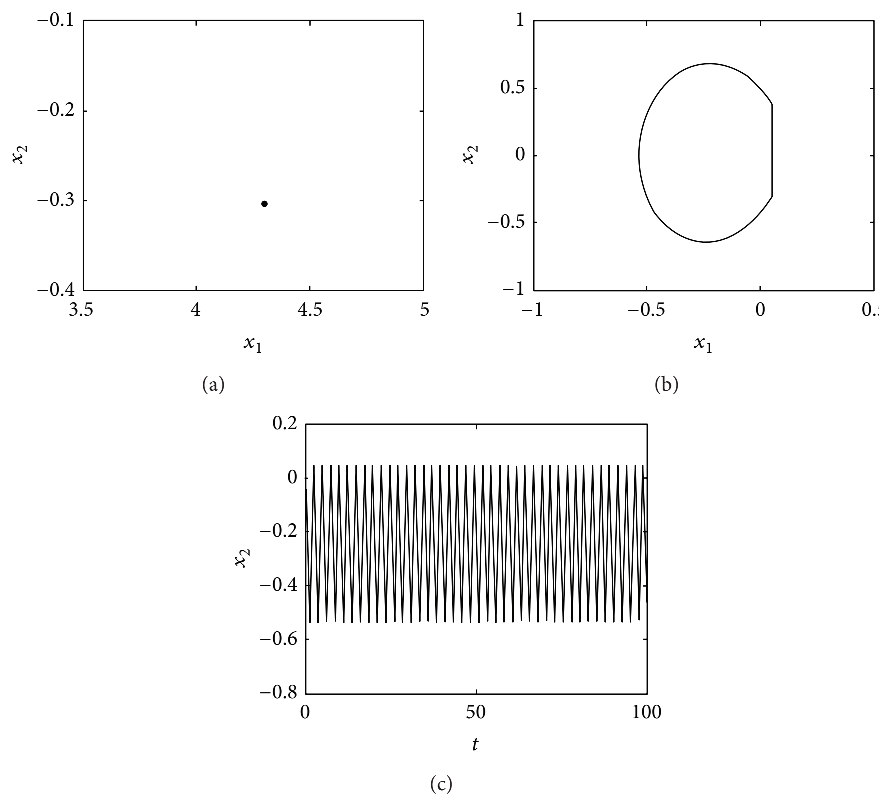


Figure 3: Poincaré map, phase portrait, and time course diagram of the system with $\omega = 2.55$.

Define the following section: $\sigma = \{(\theta, x, \dot{x}) \in R^2 \times S_1 | x = d\}$. In the paper, we choose the section σ to establish the Poincaré map

$$\theta_{k+1} = \omega t + \theta_k, \tag{11}$$

$$x_{k+1} = e^{-\zeta t}\left(\hat{a}\cos\omega_d t + \hat{b}\sin\omega_d t\right) + A\sin\left(\omega t + \theta_k\right) + B\cos\left(\omega t + \theta_k\right), \tag{12}$$

$$\dot{x}_{k+1} = -R\Big(e^{-\zeta t}\left(\left(\hat{b}\omega_d - \hat{a}\zeta\right)\cos\omega_d t - \left(\hat{b}\zeta + \hat{a}\omega_d\right)\sin\omega_d t\right) + A\omega\cos\left(\omega t + \theta_k\right) - B\omega\sin\left(\omega t + \theta_k\right)\Big), \tag{13}$$

where $\hat{a} = x_k - A\sin\theta_k - B\cos\theta_k$, and $\hat{b} = (\dot{x}_k + \zeta x_k + (B\omega - A\zeta)\sin\theta_k - (A\omega + B\zeta)\cos\theta_k)/\omega_d$. In which, initial iteration value of the Poincaré map of the $n-1$ periodic motion is

$$\theta_0 = \theta_d, \qquad x_{k+1} = x_k = x_0 = x_d = d, \qquad \dot{x}_0 = \dot{x}_d. \tag{14}$$

3. Chaos and Bifurcation

The single-degree-of-freedom mechanical model of vibroimpact system, with system parameters $\zeta = 0.01$, $d = 0.05$, and $R = 0.8$, has been chosen to be analyzed. The system parameter ω is taken as the bifurcation parameter. The global bifurcation diagram can be obtained with the ω changing

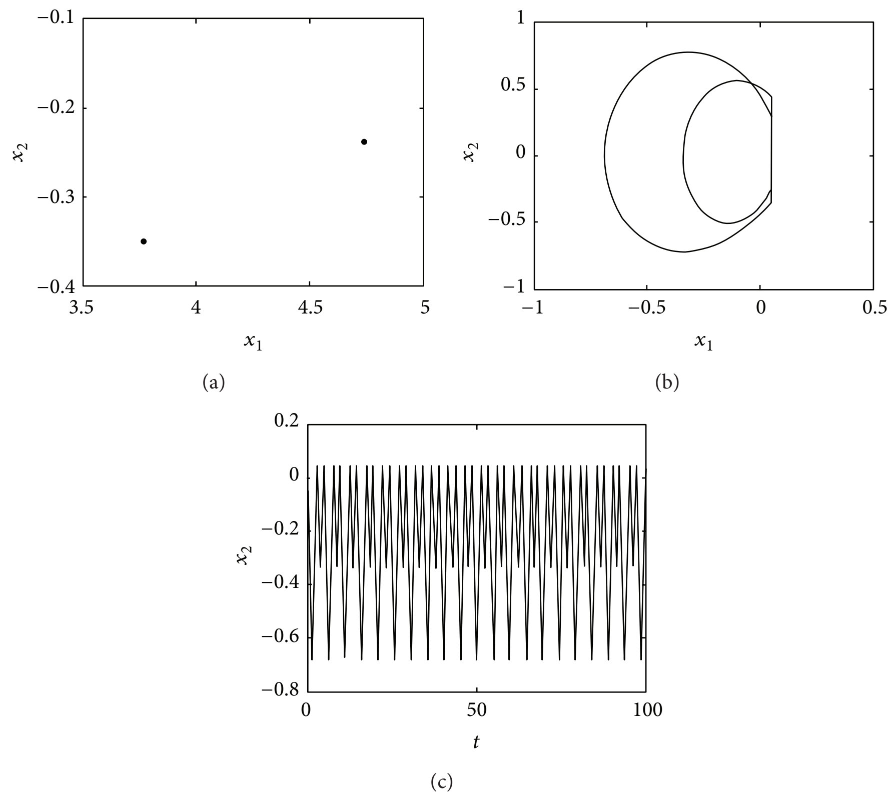


FIGURE 4: Poincaré map, phase portrait, and time course diagram of the system with $\omega = 2.6$.

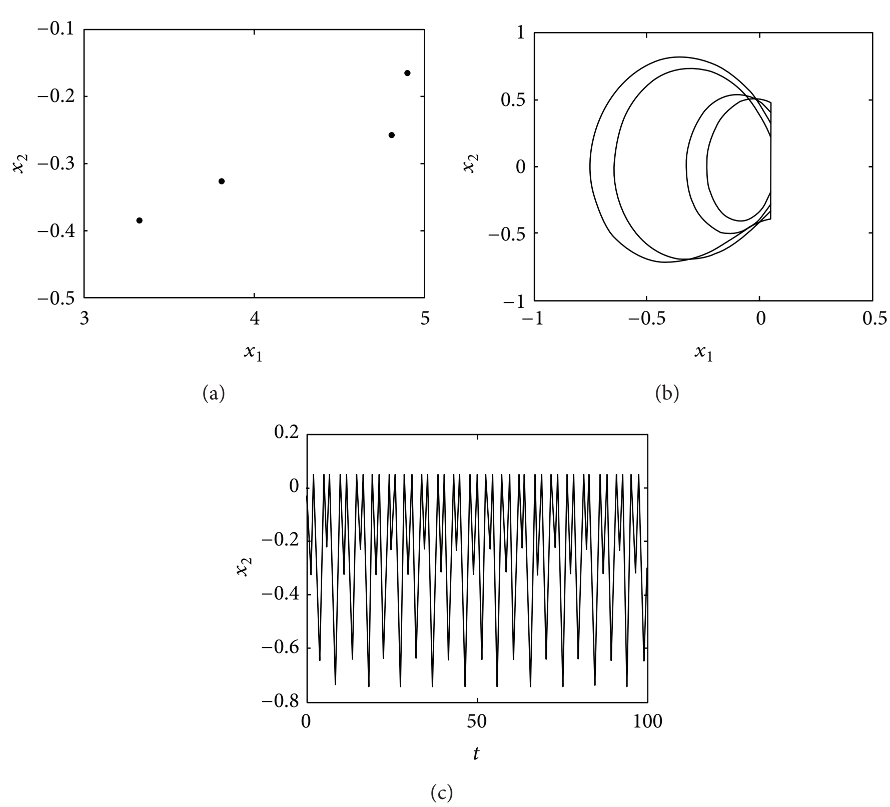


FIGURE 5: Poincaré map, phase portrait, and time course diagram of the system with $\omega = 2.635$.

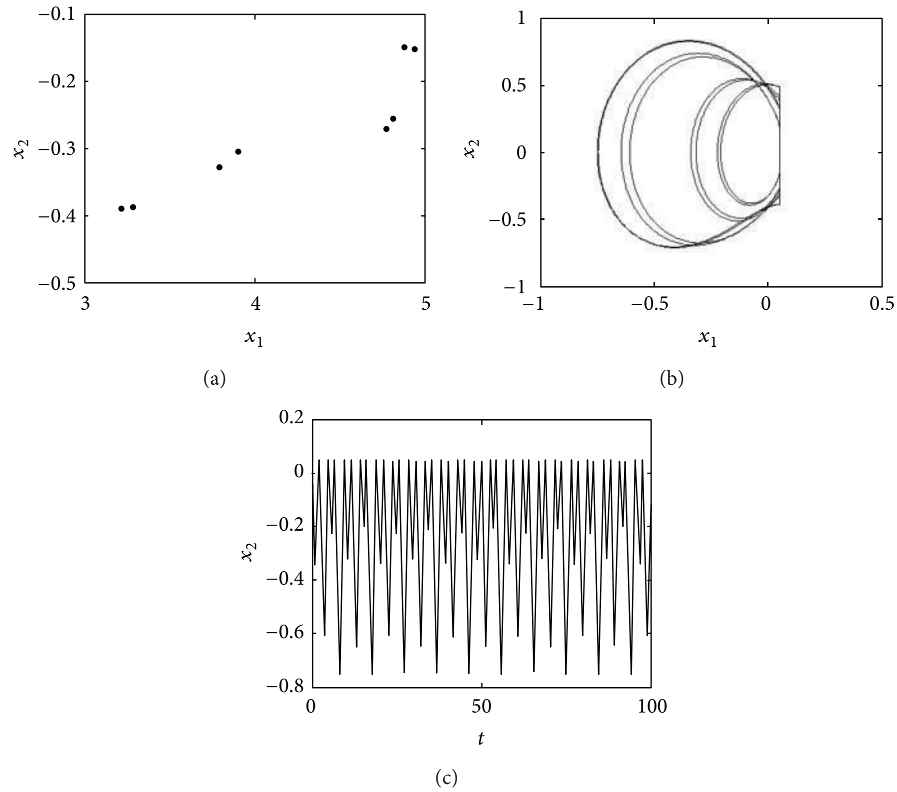


Figure 6: Poincaré map, phase portrait, and time course diagram of the system with $\omega = 2.42$.

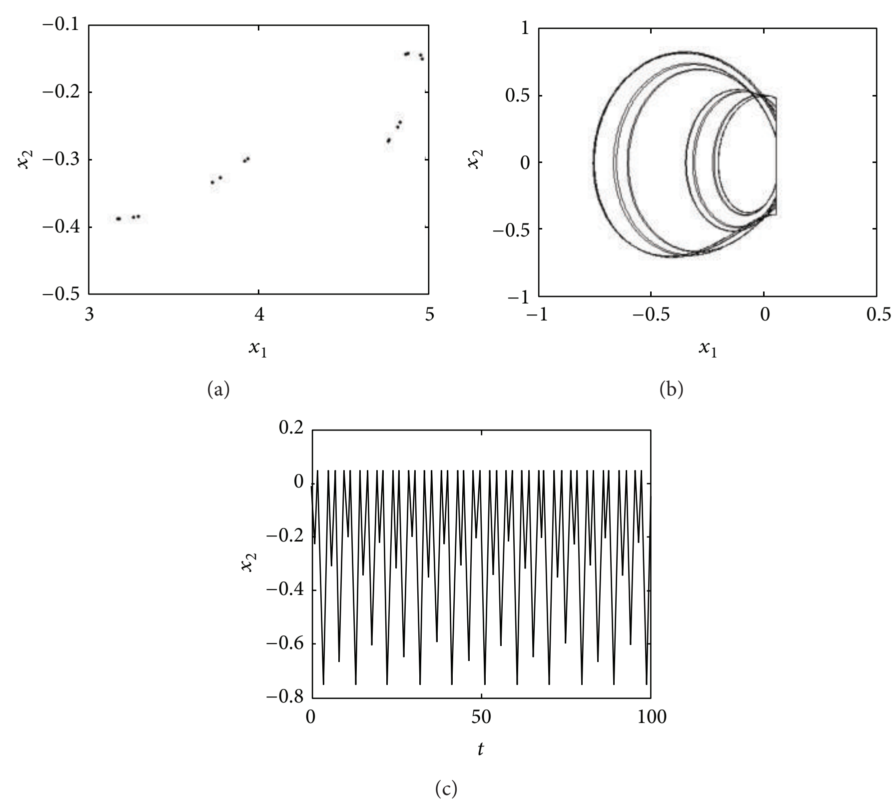


Figure 7: Poincaré map, phase portrait, and time course diagram of the system with $\omega = 2.644$.

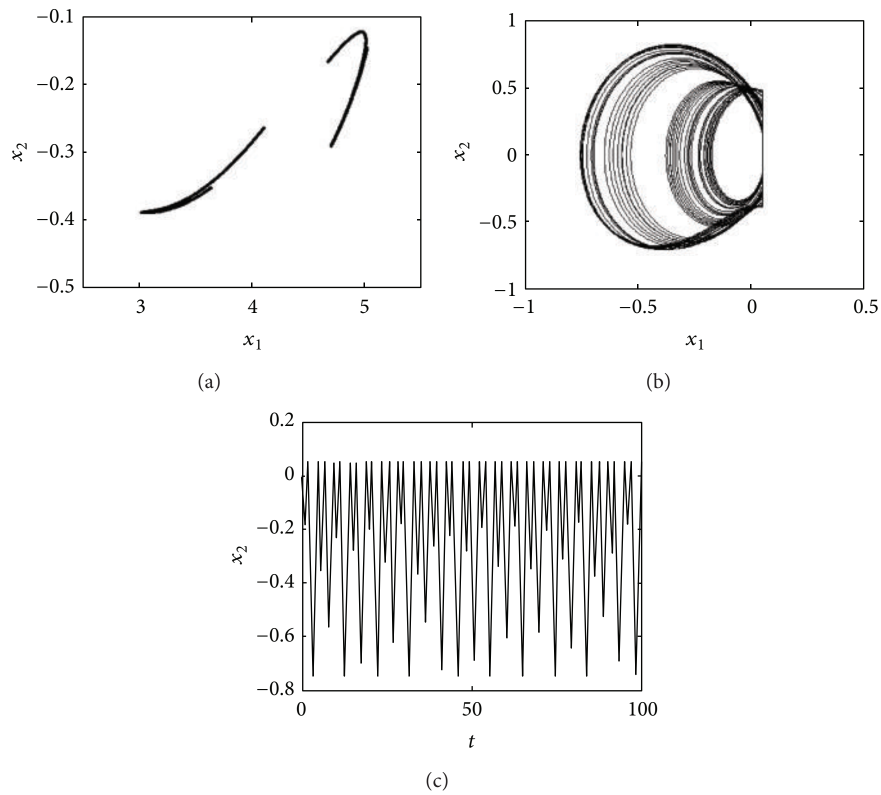


FIGURE 8: Poincaré map, phase portrait, and time course diagram of the system with $\omega = 2.52$.

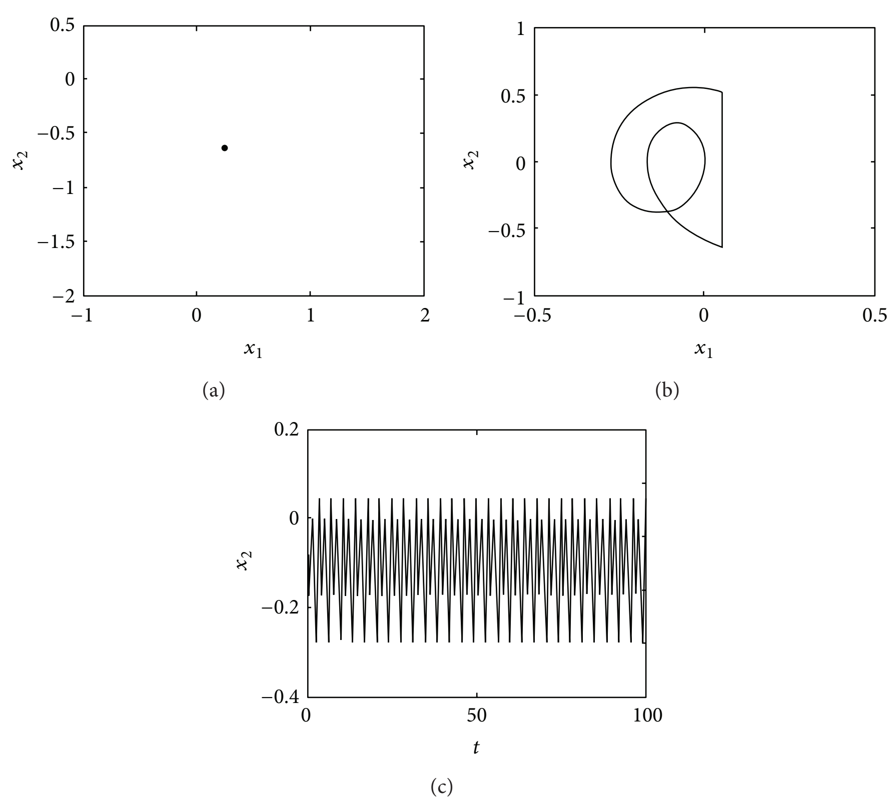
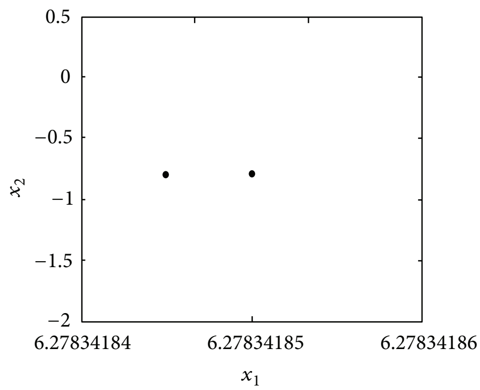
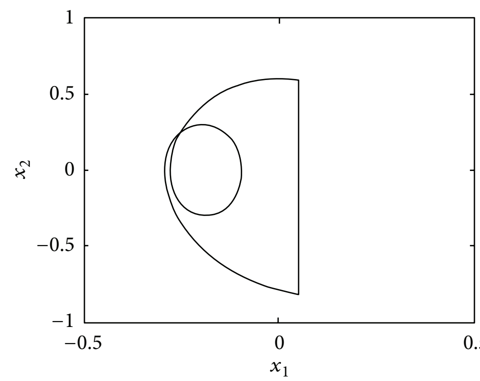


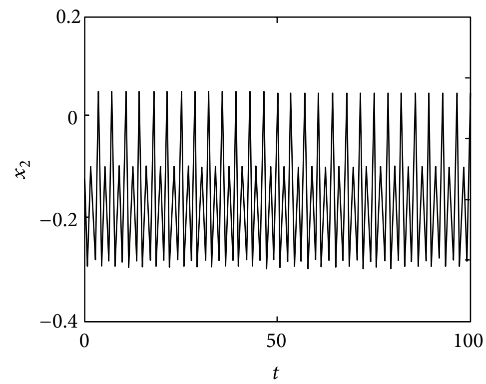
FIGURE 9: Poincaré map, phase portrait, and time course diagram of the controlled system with $\gamma = 0.05$ and $h = 4.6$.



(a)



(b)



(c)

FIGURE 10: Poincaré map, phase portrait, and time course diagram of the controlled system with $\gamma = 0.05$ and $h = 5.6$.

in the range of $[0, 6]$ as shown in Figure 2. As you can see from Figure 2, the system has stable $n - 1$ periodic motion in a certain range. But in different period of single touch movement, the system will produce double periodic bifurcation, from periodic motion to chaos in the process of transition. But with the increase in ω, the system will degenerate to the periodic motion. The Figures 2(b) and 2(c) are a partial enlargement of Figure 2(a). It can be seen that, when $\omega = 2.5585$, the vibroimpact system will be from periodic 1 motion to periodic 2 motion by bifurcation and when $\omega = 2.624$, the vibroimpact system will be from periodic 2 motion to periodic 4 motion by double periodic bifurcation. When $\omega = 2.641$, the vibroimpact system will be from periodic 4 motion to periodic 8 motion by double periodic bifurcation. When $\omega = 2.644$, the vibroimpact system will be from periodic 8 motion to periodic 16 motion by double periodic bifurcation. With the vibration frequency ω increasing further, the system leads to chaotic motion.

The excitation frequency ω takes 2.55, 2.6, 2.635, 2.642, 2.644, and 2.652. Poincaré map, phase portrait, and time course diagram of the system are shown in Figures 3, 4, 5, 6, 7, and 8. The system is, respectively, periodic 1 motion, periodic 2 motion, periodic 4 motion, periodic 8 motion, periodic 16 motion, and chaotic motion.

4. Chaos Control

The paper chooses the sine driving force for the periodic force excitation. Periodic force is easy to produce and control the external driving force in the actual project, so the sine driving force is used to suppress the bifurcation and chaotic motion of single-degree-of-freedom vibroimpact system. Based on the principle of parameter perturbation, periodic force excitation method can stabilize the chaotic motion by applying disturbance directly into the system. An unstable periodic motion of the chaotic system can produce resonance with external periodic force. The system can be from its unstable limit cycle to a stable limit cycle by resonating with external driving signal. So the chaos is controlled.

Periodic force excitation can be expressed as

$$f = h \sin(\gamma \omega \theta). \tag{15}$$

In the formula, h and γ are adjustable parameters.

As the previous analysis, when $\omega = 2.652$, the system is in chaotic motion. After the introduction of the external periodic force feedback controller, using the fourth-order Runge-Kutta method of numerical simulation, the Poincaré map, phase portrait, and time course diagram of the controlled system are obtained, as shown in Figures 9 and 10.

When $\gamma = 0.05$ and $h = 4.6$, the system is controlled to periodic 1 motion. When $\gamma = 0.05$ and $h = 5.6$, the system is controlled to periodic 2 motion. The simulation results show the effectiveness of the method. Because the method does not change the original system parameters, it can be applied to a 2-DOF and multiple-DOF vibroimpact mechanical system.

5. Conclusion

Based on a single-degree-of-freedom vibroimpact system as the research object, bifurcation and chaos have been researched with the system parameters changing. By adopting an external periodic force excitation method for suppressing its chaotic behavior, it delayed the occurrence of fault. Because this method does not change the original system parameters, it is easy to implement in engineering. This method is not limited to this kind of mechanical system with clearance collision and can also be used in other similar nonlinear system.

Acknowledgments

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