

Proceedings of IMECE2005
2005 ASME International Mechanical Engineering Congress and Exposition
November 5-11, 2005, Orlando, Florida USA

IMECE2005-81653

SYSTEM HIERARCHY OPTIMIZATION METHOD FOR MEMS APPLICATIONS

Nan Liu
Design and Manufacturing Institute
Mechanical Engineering Department
Stevens Institute of Technology
Castle Point on Hudson
Hoboken, NJ 07030
nliu1@stevens.edu

Souran Manoochehri
Design and Manufacturing Institute
Mechanical Engineering Department
Stevens Institute of Technology
Castle Point on Hudson
Hoboken, NJ 07030
smanooch@stevens.edu

ABSTRACT

A new design optimization methodology for Micro-Electro-Mechanical Systems (MEMS) application is presented. The optimization approach considers minimization of several uncertainty factors on the overall system performance while satisfying target requirements specified in the form of constraints on micro-fabrication processes and materials system. The design process is modeled as a multi-level hierarchical optimal design problem. The design problem is decomposed into two analysis systems; uncertainty effects analysis and performance sensitivity analysis. Each analysis system can be partitioned into several subsystems according to the different functions they perform. The entire problem has been treated as a multi-disciplinary design optimization (MDO) for maximum robustness and performance achievement. In this study, the analysis results are provided as optimized device geometry parameters for the example of the selected micro accelerometer device.

INTRODUCTION

Micro Electro-Mechanical System (MEMS) is an area of technology of rapidly increasing economic importance. Commercial devices include accelerometers, micro resonator, pressure sensors, ink jet printer heads, digital mirror arrays for projectors and atomic force microscopes. Most current research works in MEMS are focused on experimental studies related to fabrication methods and material systems. These experimental studies are expensive and time consuming. There are many uncertainty factors that exist in the fabrication processes. With the present micro-fabrication techniques, fabrication uncertainty in manufactured MEMS devices are inevitable and result in the error in device geometry parameters and material properties such as, young's modulus. The performance and function of the system, especially important mechanical responses such as resonant frequency, actuation force, and output sensitivity, will be affected severely due to variability

introduced through these uncertainty factors [1]. As the current fabrication methods for MEMS are still under developing, and micro level devices have the relatively large error-to-size ratio compared to the macro-scaled mechanical structures, the effect of fabrication uncertainty errors is a critical factor to be considered in MEMS device design and fabrication. Another goal for MEMS design is to make the performance of device as optimum as we can. Usually these optimization steps are based on the experiences of the designer, who analyzes the system performance and modifies individual system parameters. This complicated task requires an engineer who is not only familiar with the system uncertainty optimization but also with MEMS device modeling and fabrication knowledge. A more efficient approach to find the best values of parameters to satisfy the performance requirements with the lowest uncertainty effects on the system is still highly desirable.

Using optimization methods to make system response less sensitive to uncertainties becomes an important concept to assure reliable performances and improve yield rate in MEMS mass production. From this point of view, a design technique including a robust system optimization stage should be part of the early stage design of MEMS devices. A number of robust design methods for mechanical systems have been developed from the initial Taguchi's "parameter design method" [2] to recent nonlinear programming methods [3]. Belegundu and Chandrupatla [4] proved that the reduction in sensitivity implies reduction in probability of failure, if an uncertain variable is considered to be a random variable described by a probability distribution. In a method based on Taguchi's robust design [5], the concept of signal-to-noise ratio is presented as robustness criteria that provides for a fixed built-in trade off between performance and robustness factors. Probabilities of failure are determined and minimized, by assuming that design parameters are random variables with some distribution. Yu and Lan [6] developed the structure analysis and system

modeling of piezoelectric micro-accelerometer and conducted the design optimization and robustness analysis using Taguchi's method to reduce the sensitivity of the sensor response to the dimensional errors and variations of material properties. Mawardi and Pitchumani [7] introduced a methodology for robust design analysis of micro resonator applications by considering uncertainties in parameters governing the resonant frequency and the trans-conductance values. In another study [8], an approach to behavioral modeling of micro electro-mechanical systems (MEMS) is presented emphasizing robust design that minimizes the effects of device parametric variability on overall performance and performance sensitivity analysis. Liu et al. [9] presented sensitivity analysis by considering manufacturing-induced variability in the width of the resonator. Du and Chen [10] introduced an efficient uncertainty analysis method for multidisciplinary robust design. Ongkodjojo and Tay [11] introduced a global optimization design for micro electro-mechanical systems based on simulated annealing and applied this method to the modeling of a vibrating micro gyroscope. Han and Kwak [12] proposed a formulation to improve robustness of the objective function by minimizing a gradient index, defined as a function of gradients of performance functions with respect to uncertain variables by using the DOT (Design Optimization Tools) (VR&D 1995) and ABAQUS FEM analysis tools. Several software packages are also developed for the simulation and modeling of the MEMS design such as, Coventor and Intellisense.

The MEMS design as a multidisciplinary system optimization problem is a main focus of this paper. Multidisciplinary Design Optimization (MDO) is a body of methods and techniques for performing the optimization so as to balance the design considerations at the system and detailed levels. This approach is widely used for the large-scale mechanical designs, such as vehicles, airplanes, manufacturing machineries, etc. MDO applications in MEMS design will be a new promising area. Several MDO gradient based methods exist, including: All-in-One (A-i-O) method [13], Individual Discipline Feasible (IDF) method [13], Concurrent Subspace Optimization-Neural Network (CSSO-NN) [14], Collaborative Optimization (CO) method [15], and Bi-Level Integrated System Synthesis (BLISS) method [16]. With BLISS, the general system optimization problem is decomposed into a set of local optimizations dealing with a large number of detailed local design variables and a system level optimization dealing with global linking variables. Sensitivity analysis will be performed before each optimization routine to find the right search direction fast. Analytical Target Cascading (ATC) [17] as a classical MDO method introduced the concept that linking variables are transferred to lower level problems as targets after solving the top-level problem, and some of the top-level optimization variables are also transferred from the lower level as response targets.

A methodology for system conceptual modeling and design of MEMS should provide valuable suggestions about the key MEMS parameters before fabrication. The primary contribution of the paper is looking at the analysis and design of MEMS device as a system level multi-objective optimization decision making problem, which efficiently satisfies the target performance requirements with decreasing uncertainties effects

on the system performance. A new objective formulation is introduced by considering not only satisfaction of target requirements for main system function parameters but also minimizing the effects on the system caused by maximum uncertainty values. Once this general method is developed, it will be widely applied to any complex MEMS devices for early stage robust design process.

The following sections present a general description of the steps involved in the new method proposed, followed by an application example of a micro accelerometer modeling and optimization design. The design problem is solved by considering two important performance requirements namely, resonance frequency and system performance sensitivity with design constraints. MEMS design optimization process can be decomposed into three steps: sensitivity analysis, performance design and uncertainty analysis. The DecisionPro (Vanguard Software Corporation) is used to build the system hierarchy tree. A Matlab programming is developed for finding the best values of design variables considering the maximum uncertainty variances. The results are tested by Monte Carlo simulation [18] to achieve the desired resonant frequency.

NOMENCLATURE

| | |
|----------|---|
| l | Rang of Beam Length |
| b | Rang of Beam Width |
| h | Rang of Beam Thickness |
| l_{Mt} | Rang of Length of mass |
| h_M | Rang of Thickness of mass |
| w_n | Resonance frequency |
| K | Stiffness of beam |
| E | Young's modulus |
| I | Moment of inertia of the rectangular beam |
| SPS | System Performance Sensitivity |
| S | Output sensitivity |
| l_p | Length of PZT film |
| d_{31} | Piezoelectric constant |
| C | Electric capacity constant |
| M | Mass's weight |
| ρ | Mass's density |
| PF | Performance Design Objective |
| UD | Uncertainty Design Objective |
| T | Final Objective |

SYSTEM OPTIMIZATION METHOD

2.1. Procedure

The methodology presented in this paper considers performance issues at the sub-system level and process uncertainties at the system level. This process is shown

diagrammatically in Figure 1. In this study, several design variables are selected related to the device geometry. In addition, limitations from the current fabrication processes are given as constraints imposed on the designs.

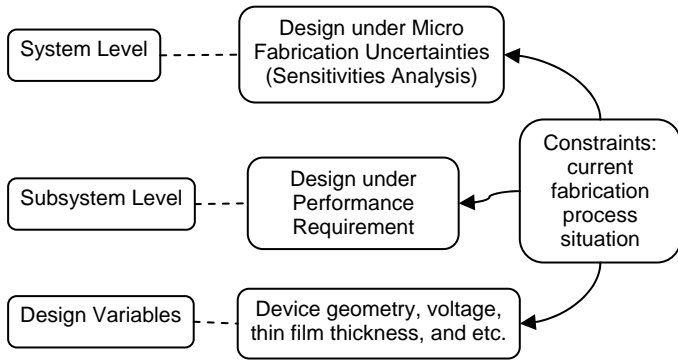


Fig. 1: System decomposition of MEMS optimization design

The steps involved in this approach are presented in Figure 2. First of all, the design problem is analyzed. Critical function parameters for the system are decided and considered for future analysis steps. Every system has its own critical function parameters. For example, in the resonators system, the important mechanical design property is the resonant natural frequency, which is crucial for the performance of the device in which the resonators serve as components; and important electrical functional property is the trans-conductance, which relates to the magnitude of the amplification gain that the resonators, as electronic devices, are capable of delivering [7]. Another example is micro accelerometer; the resonance frequency of structure is an important system function. Also another critical function is the system performance sensitivity (*SPS*) used for the evaluation of the system performance, which is presented by the ratio of the system output over input. When the *SPS* is as large as possible, meaning a big output signal with a small input signal, the performance of system is the best. Subsequent task in the design process is mathematical problem formulation and modeling. According to mechanical, material, electric and physics knowledge, critical system function parameters are formulated as functions of all the design variables, such as device geometries and material properties. Sensitivity analysis for all the variables is performed. The importance of each variable for each function is decided accordingly. The more important variables will be first optimized to satisfy the performance function parameters target by assuming other variable values according to the prior knowledge from experiments. This idea is especially for the problems with a large number of variables. Understanding of the MEMS fabrication process is also another important factor for the selection of optimized design variables with uncertainties. For example, beam width and length, hole's diameter, and plate thickness are design variables and can be taken as uncertain variables at the same time because fabrication errors will affect these variables. Some material properties can also be considered as uncertain variables. Those uncertain design variables only decided by sensitivity analysis are considered in the presented example. Objective function is developed according to the method presented in this paper, which includes performance design and uncertainty

consideration. The constraints including the range of each design variables will be decided based on the limitations of the current fabrication methods. The maximum uncertainty values for each design variables are assumed according to fabrication experiments and experience in this field. The proposed optimal design method utilizes conventional deterministic optimization methods to enhance both performance and robustness of MEMS structures.

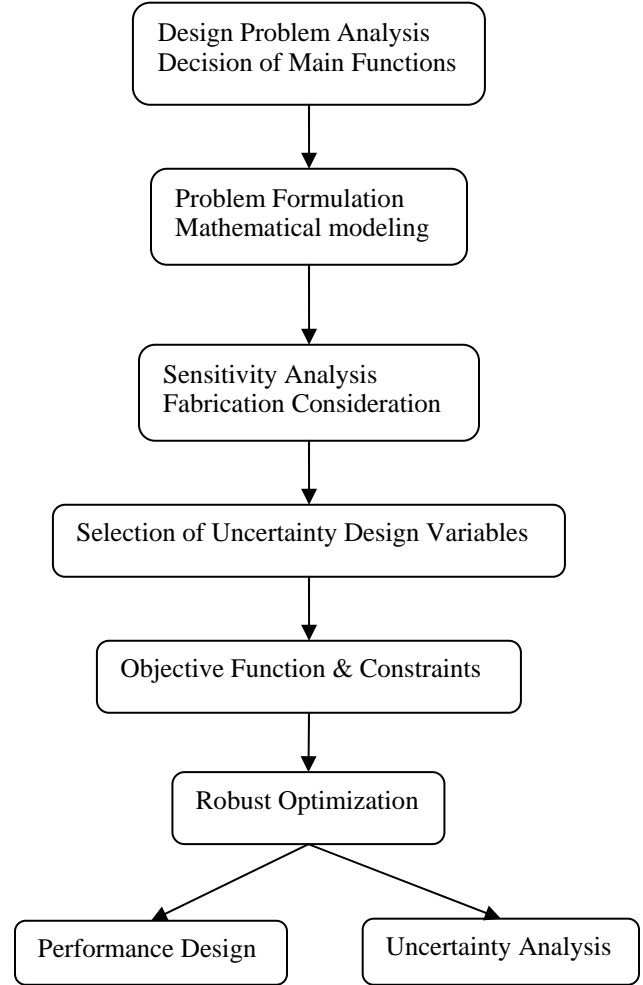


Fig. 2: Proposed optimal design method procedure

2.2. Formulation

1. Sensitivity Analysis

Assume the critical function parameters f_i and constraints related to this function for the system can be expressed as following:

$$f_i = f_i(x_1, x_2, x_3 \cdots x_i); i = 1, 2, \dots, n$$

$$\text{Subject to } g_j(x_1 \cdots x_i) \leq 0 \quad j = 1, 2, \dots, m \quad (1)$$

$$x_1^L \leq x_1 \leq x_1^U \cdots x_i^L \leq x_i \leq x_i^U$$

where $x_1, x_2, x_3 \dots x_i$ are all the system design variables, such as device geometry parameters and properties such as the Young's modulus. Values x_i^L and x_i^U are the lower and upper bounds of the design variables. Sensitivity analysis for each variable is defined by computing the ratio of $\frac{\partial f_i}{\partial x_i}$. Critical uncertain variables are then chosen according to the magnitude of the sensitivity values.

2. Performance Design

The proposed performance design should make all the main function parameters for the system to achieve the target values while the system performance sensitivity is maximized meaning the system has big output signals. Assume f_i^* is the target value of each main function parameter f_i , the problem can then be formulated as:

$$\text{Minimize: } \prod_{i=1}^n |f_i - f_i^*| \quad (2)$$

Maximize: system performance sensitivity ratio

$$SPS = \frac{\text{Output}}{\text{Input}} \quad (3)$$

By normalize $|f_i - f_i^*|$ as $\left|1 - \frac{f_i}{f_i^*}\right|$, the new formulation

of performance design objective function can be introduced as:

$$\text{Minimize: } PF = \frac{\prod_{i=1}^n \left|1 - \frac{f_i}{f_i^*}\right|}{SPS} \quad (4)$$

Subject to $g_j(x_1 \dots x_i) \leq 0 \quad j = 1, 2, \dots, m$

$$x_1^L \leq x_1 \leq x_1^U \dots x_i^L \leq x_i \leq x_i^U$$

3. Uncertainty Effects Design

The maximum uncertainty value for each variable caused by fabrication or material processes can be assumed as Δx_i . So variance of performance caused by the uncertainty can be formulated as:

$$\begin{aligned} \Delta f_i &= f_i(x_i + \Delta x_i) - f_i(x_i - \Delta x_i) \\ \Delta SPS &= SPS(x_i + \Delta x_i) - SPS(x_i - \Delta x_i) \end{aligned} \quad (5)$$

The goal for uncertainty effects design is to find the values of the design variables with the minimum performance variance. This means that if the device is designed using the optimum design variables' values, uncertainty will have minimum effects on its performance. By assuming there is only one main function parameter f_1 , uncertainty objective function can be formulated accordingly. w_1 and w_2 are used for

controlling trade offs of uncertainty effects between main function parameter and system performance sensitivity. The formulation can be introduced as following.

$$\text{Minimize: } UD = w_1 \cdot \Delta f_1 + w_2 \cdot \Delta SPS$$

$$\text{Subject to } g_j(x_1 \dots x_i) \leq 0 \quad j = 1, 2, \dots, m \quad (6)$$

$$x_1^L \leq x_1 \leq x_1^U \dots x_i^L \leq x_i \leq x_i^U$$

4. Objective Function

The objective function is defined by the combination of the performance design and uncertainty effects design as shown below. w_3 and w_4 are the weight factors for considering the trade offs of performance design and uncertainty design for the whole system. It can be stated as:

$$\text{Minimize: } T = w_3 \cdot PF + w_4 \cdot UD$$

$$\text{Subject to } g_j(x_1 \dots x_i) \leq 0 \quad j = 1, 2, \dots, m \quad (7)$$

$$x_1^L \leq x_1 \leq x_1^U \dots x_i^L \leq x_i \leq x_i^U$$

MEMS APPLICATION

Micro Four Cantilever Accelerometer

Accelerometers have been used in many fields, including for activation of automotive safety systems (airbags, electronic suspension), for machine and vibration monitoring, and in biomedical applications for activity monitoring, because of their low cost, small size, and broad frequency response [19]. Three sensing mechanisms, piezoresistive, capacitive, and piezoelectric are most commonly utilized for MEMS accelerometers; each one has limitations and advantages. A piezoelectric micro accelerometer presented here which consists of a centered seismic mass suspended by four symmetric cantilever beams proposed by Yu and Lan [5] for automotive airbag applications is shown in Figure 3. When acceleration is acted on the mass, the upper electrode, the PZT thin film and the lower electrode on the beam will develop the voltage.

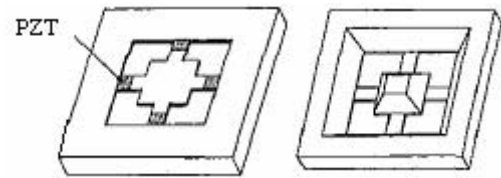


Fig. 3: Mechanical structure of the micro four cantilever accelerometer

Modeling

All the mechanical models are developed based on assumptions, such as beam's weight is negligible compare to mass; mass and rim are rigid; beam's deflections observe linear elasticity and Hooke's law; PZT film and electrodes are much thinner than beam without effect on the stiffness. Device geometry variables are shown in Figure 4, where l_{Mt} and

h_M are mass' geometries, and l is the beam length, b is the beam width, l_p is the PZT film length, and h is the beam thickness.

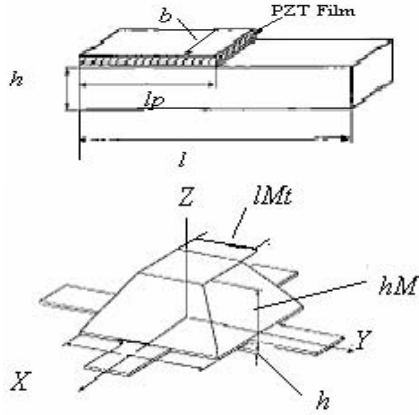


Fig. 4: Details of the beam suspension and the pyramid shaped seismic mass

The critical function parameter for this system is the resonance frequency defined by $w_n = \sqrt{\frac{K}{M}}$, where the stiffness of beam

$K = \frac{48EI}{l^3}$ (E is Young's modulus); $I = \frac{bh^3}{12}$ (I is the moment of inertia of the rectangular beam). After substitution, w_n can be written as Eq. (8).

$$w_n = \sqrt{\frac{48EI}{Ml^3}} = \sqrt{\frac{4Ebh^3}{Ml^3}} \quad (8)$$

System Performance Sensitivity is evaluated using output sensitivity term S presented in Eq. (9)

$$S = \frac{3Md_{31}l_p \left(1 - \frac{l_p}{l}\right)}{h^2C} \quad (9)$$

where $l_p = \frac{l}{2}$. Substitute l_p into Eq. (9). S is finally defined as,

$$S = \frac{3Md_{31}l^2}{4h^2C} \quad (10)$$

where d_{31} is piezoelectric constant; C is electric capacity constant and M is the mass's weight, which can be calculated by,

$$M = \rho \cdot V = \frac{\sqrt{2} \cdot \rho}{6} \left[l_{Mt}^3 - (l_{Mt} - \sqrt{2} \cdot h_M)^3 \right] + \rho \cdot l_{Mt}^2 \cdot h \quad (11)$$

where ρ is the mass's density.

Sensitivity analysis

The following first order differential equations show the variance of resonance frequency and SPS with respect to the change of all design variables (l_{Mt}, l, h, h_M, b). The results of the sensitivity of each variable here for the selected functions based on an assumed initial point are shown in Figures 5 and 6. The equations used for these calculations are given below.

The sensitivity of the thickness of mass h_M to the resonance frequency is presented by,

$$\begin{aligned} \frac{\partial w_n}{\partial h_M} &= \frac{\partial w_n}{\partial M} \frac{\partial M}{\partial h_M} \\ &= \left(-\frac{1}{2} \right) \sqrt{\frac{4Ebh^3}{M^3l^3}} \cdot \frac{\sqrt{2} \cdot \rho}{6} \cdot 3\sqrt{2} (l_{Mt} - \sqrt{2} \cdot h_M)^2 \end{aligned} \quad (12)$$

The sensitivity of the beam length l to the resonance frequency is given as,

$$\frac{\partial w_n}{\partial l} = \left(-\frac{3}{2} \sqrt{\frac{4Ebh^3}{Ml^5}} \right) \quad (13)$$

The sensitivity of the beam width b to the resonance frequency is presented by,

$$\frac{\partial w_n}{\partial b} = \frac{1}{2} \sqrt{\frac{4Eh^3}{Ml^3b}} \quad (14)$$

The sensitivity of the beam thickness h to the resonance frequency is given as,

$$\begin{aligned} \frac{\partial w_n}{\partial h} &= \frac{\partial w_n}{\partial h} + \frac{\partial w_n}{\partial M} \cdot \frac{\partial M}{\partial h} \\ &= \frac{3}{2} \sqrt{\frac{4Ebh^3}{Ml^3}} + \left(-\frac{1}{2} \right) \sqrt{\frac{4Ebh^3}{M^3l^3}} \rho \cdot l_{Mt}^2 \end{aligned} \quad (15)$$

The sensitivity of the length of mass l_{Mt} to the resonance frequency is presented by,

$$\frac{\partial w_n}{\partial l_{M_t}} = \frac{\partial w_n}{\partial M} \frac{\partial M}{\partial l_{M_t}} = \left(-\frac{1}{2} \right) \sqrt{\frac{4Eb\hbar^3}{M^3 l^3}} \cdot \frac{\sqrt{2} \cdot \rho}{2} \left[l_{M_t}^2 - (l_{M_t} - \sqrt{2} \cdot h_M)^2 \right] + 2\rho \cdot l_{M_t} \cdot h \quad (16)$$

The sensitivity of the beam thickness h to the *SPS* is given as,

$$\begin{aligned} \frac{\partial S}{\partial h} &= \frac{\partial S}{\partial h} + \frac{\partial S}{\partial M} \frac{\partial M}{\partial h} \\ &= -\frac{3Md_{31}l^2}{2h^3C} + \frac{3d_{31}l^2}{4h^2C} \rho \cdot l_{M_t}^2 \end{aligned} \quad (17)$$

The sensitivity of the length of mass l_{M_t} to the *SPS* is presented by,

$$\begin{aligned} \frac{\partial S}{\partial l_{M_t}} &= \frac{\partial S}{\partial M} \frac{\partial M}{\partial l_{M_t}} \\ &= \frac{3d_{31}l^2}{4h^2C} \cdot \left[\frac{\sqrt{2} \cdot \rho}{2} \left[l_{M_t}^2 - (l_{M_t} - \sqrt{2} \cdot h_M)^2 \right] + 2\rho \cdot l_{M_t} \cdot h \right] \end{aligned} \quad (18)$$

The sensitivity of the thickness of mass h_M to the *SPS* is given as,

$$\frac{\partial S}{\partial h_M} = \frac{\partial S}{\partial M} \frac{\partial M}{\partial h_M} = \frac{3d_{31}l^2}{4h^2C} \cdot \rho \cdot (l_{M_t} - \sqrt{2}h_M)^2 \quad (19)$$

The sensitivity of the beam length l to the *SPS* is presented by,

$$\frac{\partial S}{\partial l} = \frac{3Md_{31}l}{2h^2C} \quad (20)$$

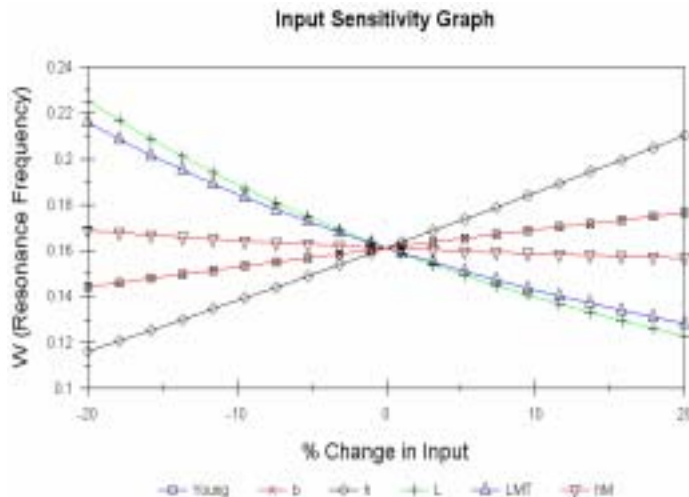


Fig. 5: Sensitivity analyses of design variables to resonance frequency

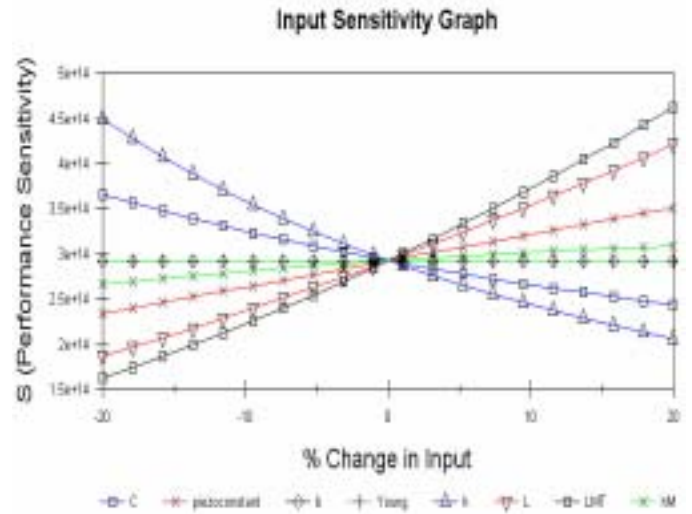


Fig. 6: Sensitivity analyses of design variables to system performance sensitivity

From the sensitivity analysis shown in Figures 5 and 6, it is obvious that h , l and l_{M_t} have more impact on the performance function than other design variables. So, these three design variables are chosen to be optimized in a later stage.

Performance Design under Uncertainty

Table 1 tabulates the uncertainties in each geometric parameter and in the Young's modulus quantified by their respective maximum expected variations. Table 2 shows the range of each design variables specified. As the beam width b and the thickness of mass h_M have comparably small effects on the system performance based on the results shown in Figures 5 and 6, they are assumed as constants $b = 180\mu m$, and $h_m = 250\mu m$. As described before, the system design is looked as a multi-objective optimization problem. Performance design objective here is to maximize system performance sensitivity (*SPS*) presented by S and minimize $|w_n - w_n^*|$, the difference between the value of w_n and its' target value w_n^* , subject to a set of fabrication device geometry constraints as shown in Table 2. According to this formulation, performance design objective function *PF* for this purpose can be presented by Eq. (21).

$$\begin{aligned} \text{Minimize: } PF &= \frac{\left| 1 - \frac{w_n}{w_n^*} \right|}{S} \\ \text{Subject to } g_j(x_1 \cdots x_i) &\leq 0 \quad j = 1, 2, \dots, m \\ x_1^L &\leq x_1 \leq x_1^U \cdots x_i^L \leq x_i \leq x_i^U \end{aligned} \quad (21)$$

| UNCERTAINTY PARAMETERS | UNITS | MAXIMUM VARIANCE |
|------------------------|-----------|-----------------------|
| Beam Width | <i>um</i> | $\Delta b = 1.0$ |
| Beam Length | <i>um</i> | $\Delta l = 1.0$ |
| Beam Thickness | <i>um</i> | $\Delta h = 0.8$ |
| Length of mass | <i>um</i> | $\Delta l_{Mt} = 1.0$ |
| Thickness of mass | <i>um</i> | $\Delta h_M = 0.3$ |
| Young's Modulus | GPa | $\Delta E = 2.0$ |

Tab. 1: The associated maximum variance of uncertainty parameters.

| DESIGN PARAMETERS | SYMBOL | DESIGN INTERVAL |
|---------------------------|----------|----------------------------|
| Rang of Beam Length | <i>l</i> | $300 \leq l \leq 500$ |
| Rang of Beam Width | <i>b</i> | $180 \leq b \leq 220$ |
| Rang of Beam Thickness | <i>h</i> | $10 \leq h \leq 20$ |
| Rang of Length of mass | l_{Mt} | $700 \leq l_{Mt} \leq 900$ |
| Rang of Thickness of mass | h_M | $250 \leq h_M \leq 350$ |

Tab. 2: The associated range of design parameters values considered.

The variance of output parameters is defined as the difference between the maximum and minimum expected values due to the effects of the various input parameter uncertainty. The variance of mass weight is presented as,

$$\Delta M = \left\{ \frac{\sqrt{2} \cdot \rho}{6} \left[(l_{Mt} + \Delta l_{Mt})^3 - (l_{Mt} - \Delta l_{Mt})^3 - \sqrt{2} \cdot (h_M + \Delta h_M)^3 \right] + \rho \cdot (l_{Mt} + \Delta l_{Mt})^2 \cdot (h + \Delta h) \right\} - \left\{ \frac{\sqrt{2} \cdot \rho}{6} \left[(l_{Mt} - \Delta l_{Mt})^3 - (l_{Mt} + \Delta l_{Mt})^3 - \sqrt{2} \cdot (h_M - \Delta h_M)^3 \right] + \rho \cdot (l_{Mt} - \Delta l_{Mt})^2 \cdot (h - \Delta h) \right\} \quad (22)$$

The variance of system performance sensitivity is the difference between maximum and minimum values of *S* corresponding to the upper and lower bounds of the uncertainty parameters, which can be written as,

$$\Delta S = \frac{3(M + \Delta M)d_{31}(l + \Delta l)^2}{4(h - \Delta h)^2 C} - \frac{3(M - \Delta M)d_{31}(l - \Delta l)^2}{4(h + \Delta h)^2 C} \quad (23)$$

Similarly, the variance of resonance frequency can be expressed as,

$$\Delta w_n = \sqrt{\frac{4(E + \Delta E)(b + \Delta b)(h + \Delta h)^3}{(M - \Delta M)(l - \Delta l)^3}} - \sqrt{\frac{4(E - \Delta E)(b - \Delta b)(h - \Delta h)^3}{(M + \Delta M)(l + \Delta l)^3}} \quad (24)$$

If a reference frequency is defined as

$$\tilde{w}_n = \sqrt{\frac{4\Delta E \Delta b \Delta h^3}{\tilde{M} \Delta l^3}} \quad (25)$$

where the reference mass weight \tilde{M} can be given by,

$$\tilde{M} = \frac{\sqrt{2} \cdot \rho}{6} \left[\Delta l_{Mt}^3 - (\Delta l_{Mt} - \sqrt{2} \cdot \Delta h_M)^3 \right] + \rho \cdot \Delta l_{Mt}^2 \cdot \Delta h \quad (26)$$

Similarly, the reference system performance sensitivity can be expressed as,

$$\tilde{S} = \frac{3\tilde{M}d_{31}\Delta l^2}{4\Delta h^2 C} \quad (27)$$

Then, the normalized resonant frequency can be expressed as,

$$\Psi = \frac{\Delta w_n}{\tilde{w}_n} \quad (28)$$

Similarly, the normalized system performance sensitivity can be given as,

$$\Phi = \frac{\Delta S}{\tilde{S}} \quad (29)$$

Uncertainty design objective function can be represented by Eq. (30), and final objective function is given by Eq. (31). w_1, w_2, w_3, w_4 are weight factors to control trade offs between all the objective functions. w_1 and w_2 are used for controlling the importance of uncertainty effects on resonance frequency and system performance sensitivity. w_3 and w_4 are used for controlling the importance of considerations of performance design and uncertainty design for the whole design. Finding the best values to satisfy the system requirements based on the balance among all the factors has been achieved in this optimization problem. Table 3 shows the final design variables values and the results for this example. The flow chart for the optimal micro-accelerometer design process is shown in Figure 7. The optimization formulation for uncertainty design can be expressed as,

$$\begin{aligned} &\text{Minimize: } UD = w_1 \cdot \Psi + w_2 \cdot \Phi, \\ &\text{Subject to } g_j(x_1 \cdots x_i) \leq 0 \quad j = 1, 2, \dots, m \\ &x_1^L \leq x_1 \leq x_1^U \cdots x_i^L \leq x_i \leq x_i^U \end{aligned} \quad (30)$$

The final optimization formulation considering both uncertainty design and performance design can be expressed as,

$$\begin{aligned} &\text{Minimize: } T = w_3 \cdot PF + w_4 \cdot UD \\ &\text{Subject to } g_j(x_1 \cdots x_i) \leq 0 \quad j = 1, 2, \dots, m \\ &x_1^L \leq x_1 \leq x_1^U \cdots x_i^L \leq x_i \leq x_i^U \end{aligned} \quad (31)$$

| DESIGN VARIABLES AND OBJECTIVE FUNCTIONS | RESULTS |
|--|----------|
| Resonance Frequency w_n | 27432 Hz |
| System Performance Sensitivity SPS | 1.23 |
| Performance Design Objective PF | 6.734E-4 |
| Uncertainty Design Objective UD | 1.21 |
| Final Objective T | 1.277E-2 |
| Rang of Beam Length l | 500um |
| Rang of Length of mass l_{Ml} | 800um |
| Rang of Beam Thickness h | 20um |
| Rang of Beam Width b | 180um |
| Rang of Thickness of mass h_M | 250um |

Tab. 3: Final Design Variable Values and Results.

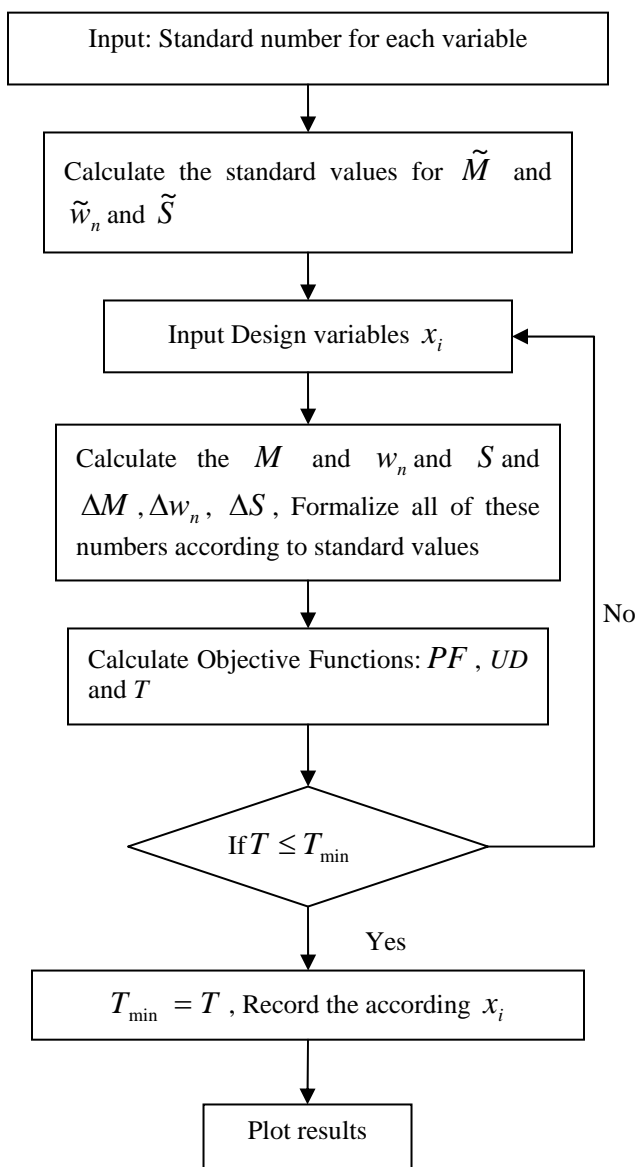


Fig. 7: Flow chart of optimization procedure

Monte Carlo Simulation

A Monte Carlo simulation is run with the histogram of a sample of 1000 estimates by assuming all the design variables to follow the normal distribution $N(\mu, \sigma)$. The mean values μ are chosen as the design values for all the parameters in Table 3. The values of σ are chosen as the maximum variance values for all the design variables as shown in Table 1. The uncertainty effects caused by the uncertainty design parameters variance at the final design variable values are tested. The corresponding frequency distributions of results of resonance frequency and system performance sensitivity are estimated in Figures 8 and 9.

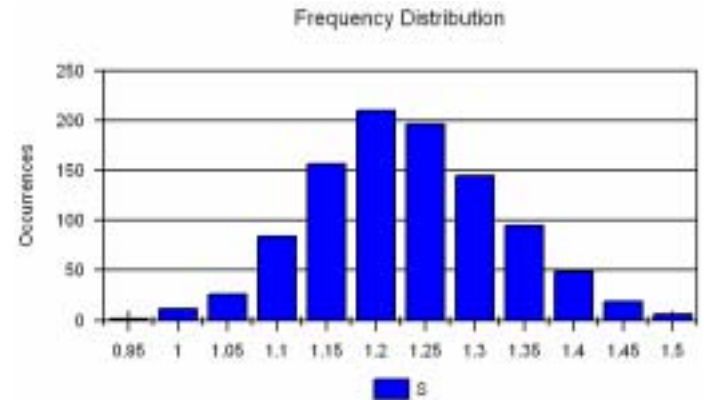


Fig. 8: Frequency distribution of system performance sensitivity

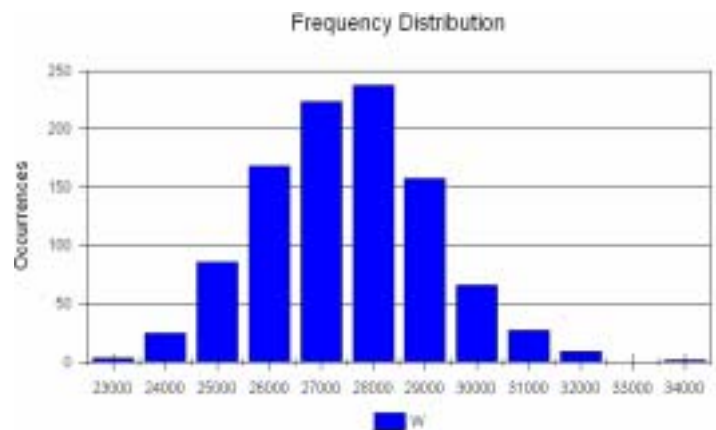


Fig. 9: Frequency distribution of resonance frequency

CONCLUSION

In this paper, a methodology for optimal MEMS design is presented to simultaneously optimize the effect of uncertainties in fabrication processes and the performance requirements of the device. A new formulation is discussed to consider the entire design activities as a multi-objective optimization problem. Uncertainty design is realized by minimizing the performance variance caused by the maximum uncertainty values possible. Performance design is considered by minimizing the variance between specified performance

functional parameters and their target values while maximizing the system performance sensitivity. The optimization problem is solved by facilitating the trade offs between all the consideration factors. The micro-accelerometer is used as an example to demonstrate the application of this method. Geometric variables such as beam length, beam thickness, and mass length are considered as design variables in this study due to the reason that they have bigger sensitivity values with respect to the performance than other variables. For the micro accelerometer example, optimized values for the design variables and the objective function value are presented.

REFERENCES

- [1] Hong, Y. S., Lee, J. H., and Kim, S. H., 2000, "A laterally driven symmetric-micro-resonator for gyroscopic applications," *Journal of Micromechanics Microengineering*, **10**, pp. 452-458.
- [2] Phadke, M. S., 1989, *Quality Engineering using Robust Design*, Prentice Hall, Englewood Cliffs, New Jersey.
- [3] Ramakrishnan, B., and Rao, S.S., 1991, "A Robust Optimization Approach Using Taguchi's Loss Function for Solving Nonlinear Optimization Problems," *Advances in Design Automation*, ASME DE-3-1, pp.241-248.
- [4] Belegundu, A. D., and Chandrupatla, T. R., 1999, *Optimization Concepts and Applications in Engineering*, Prentice-Hall, Inc., Simon & Schuster / A Viacom Company, Upper Saddle River, New Jersey.
- [5] Chen, W., Allen, J. K., and Tsui, K. L., 1996, "A procedure for robust design: minimizing variations caused by noise factors and control factors," *ASME Journal of Mechanical Design*, **118**, pp. 478-485.
- [6] Yu, J. C., and Lan, C. B., 1999, "System Modeling and Robust Design of Micro-accelerometer using Piezoelectric Thin Film," *Proc., IEEE International Conference on Multi-sensor Fusion and Integration for Intelligent Systems*, Taipei, Taiwan, R.O.C., Vol. 1, pp. 99-104.
- [7] Mawardi, A., and Pitchumani, R., 2005, "Design of Microresonators under Uncertainty," *Journal of Micro-electromechanical Systems*, **14**, pp. 63-69.
- [8] Dewey, A., Ren, H., and Zhang, T. H., 2000, "Behavioral Modeling of Micro-electromechanical Systems (MEMS) with Statistical Performance-Variability Reduction and Sensitivity Analysis," *IEEE Transactions on Circuits and Systems-II: Analog and Digital Signal Processing*, **47**, pp. 105-113.
- [9] Liu, R., Pandey, B., and Turner, K., 2002, "MEMS resonators that are robust to process-induced feature width variation," *Journal of Micro electromechanical System*, **11**, pp. 505-511.
- [10] Ongkodjojo, A., and Tay, F. E H, 2002, "Global Optimization and Design for Micro-electromechanical Systems Devices based on Simulated Annealing," *Journal of Micromechanics and Microengineering*, **12**, pp. 878-897.
- [11] Du, X., and Chen, W., 2002, "Efficient Uncertainty Analysis Methods for Multidisciplinary Robust Design," *American Institute of Aeronautics and Astronautics Journal*, **40**, pp. 545-552.
- [12] Han, J. S., and Kwak, B. M., 2004, "Robust Optimization Using a Gradient Index: MEMS Applications," *Structure Multidisciplinary Optimization*, **27**, pp. 468-478.
- [13] Cramer, E. J., Dennis, J. E., Frank, P. D., Lewis, R. M., and Shubin, G. R., 1994, "Problem Formulation for Multidisciplinary Design Optimization," *SIAM Journal on Optimization*, **4**, pp. 754-776.
- [14] Sellar, R. S., Batill, S. M., and Renaud, J. E., 1996, "Concurrent Subspace Optimization using gradient-enhanced neural network approximations," *Technical Paper AIAA Paper 96-4019, 6th Symposium on Multidisciplinary Analysis and Optimization*, Bellevue, Washington, Vol. 1, pp 319-330.
- [15] Sobieski, I. P., and Kroo, I., 1998, "Collaborative Optimization using Response Surface Estimation," *36th AIAA Aerospace Sciences Meeting and Exhibit*, Reno, NV, Vol. 38, pp. 1931-1938.
- [16] Sobieszczanski-Sobieski, Agte, J., J., and Sandusky, R. Jr., 1998, "Bi-Level Integrated System Synthesis (BLISS)," *Technical Report NASA/TM-1998-208715*, NASA Langley Research Center, Hampton, Virginia.
- [17] Kim, H. M., Michelena, N. F., Papalambros, P. Y., and Jiang, T., 2003, "Target Cascading in Optimal System Design," *Journal of Mechanical Design*, **125**, pp. 1-7.
- [18] Dubi, A., 2000, *Monte Carlo Application in Systems Engineering*, John Wiley & Sons, Baffins Lane, Chichester, West Sussex, England.
- [19] Yazdi, N., Ayazi, F., and Najafi, K., 1998, "Micro-machined inertial sensors," *Proc. IEEE*, Vol. 86, pp. 1640-1659.