OPTIMAL REGULATOR FOR LINEAR SYSTEMS WITH TIME DELAY IN CONTROL INPUT

MICHAEL BASIN   JESUS RODRIGUEZ-GONZALEZ

Department of Physical and Mathematical Sciences
Autonomous University of Nuevo Leon
Apdo postal 144-F, C.P. 66450, San Nicolas de los Garza
Nuevo Leon, Mexico
Phone: +52 81 83294030   Fax: +52 81 83522954
E-mail: mbasin@fcfm.uanl.mx   jgrg17@yahoo.com.mx

RODOLFO MARTINEZ-ZUNIGA
Department of Mechanical and Electrical Engineering
Autonomous University of Coahuila
Calle Barranquilla, S/N, Col.Guadalupe
Apdo postal 189, C.P. 25750, Monclova
Coahuila, Mexico
Phone: +52 86 66353846   Fax: +52 86 66351564
E-mail: rodolfomart62@hotmail.com

Abstract. This paper presents the optimal regulator for a linear system with time delay in control input and a quadratic criterion. The optimal regulator equations are obtained using the duality principle, which is applied to the optimal filter for linear systems with time delay in observations, and then proved using the maximum principle. Performance of the obtained optimal regulator is verified in the illustrative example against the best linear regulator available for linear systems without delays. Simulation graphs and comparison tables demonstrating better performance of the obtained optimal regulator are included.

Keywords: Linear Time-Delay System; Optimal Control; Filtering

Category of the paper: Regular

1 Introduction

Although the optimal control (regulator) problem for linear system states was solved, as well as the filtering one, in 1960s [1, 2], the optimal control problem for linear systems with delays is still open, depending on the delay type, specific system equations, criterion, etc. Such complete reference books in the area as [3, 4, 5, 6, 7] note, discussing the maximum principle [8] or the dynamic programming method [9] for systems with delays, that finding a particular explicit form of the optimal control function might still remain difficult. A particular form of the criterion must be also taken into account: the studies mostly focused on the time-optimal criterion (see the paper [10] for linear systems) or the quadratic one [11, 12, 13]. Virtually all studies of the optimal control in time-delay systems are related to systems with delays in the state (see, for example, [14]), although the case of delays in control input is no less challenging, if the control function should be causal, i.e., does not depend on the future values of the state. A considerable bibliography existing for the robust control problem for time delay systems (such as [15, 16]) is not discussed here.

This paper concentrates on the solution of the optimal control problem for a linear system with delay in control input and a quadratic criterion, which is based on the duality principle in a closed-form situation [17] applied to the optimal filter for linear systems with delay in observations obtained in [18]. Taking into account that the optimal control problem can be solved in the linear case applying the duality principle to the solution of the optimal filtering problem [1], this paper exploits the same idea for designing the optimal control in a linear system with time delay in control input, using the optimal filter for linear systems with delay in observations. In doing so, the optimal regulator gain matrix is constructed as dual transpose to the optimal filter gain one and the optimal regulator gain equation is obtained as dual to the variance equation in the optimal filter. The results obtained by virtue of the duality principle can be rigorously verified through the general equations of the maximum principle [19, 8] or the dynamic programming method [20, 9] applied to a specific time-delay case, although the physical duality seems obvious: if the optimal filter exists in a closed form, the optimal closed-form regulator should also exist, and vice versa [17]. In this paper, the obtained results are proved using the maximum principle [19, 8]. It should be noted, however, that application of the maximum principle to the present case gives one only a system of state and co-state equations and does not provide the explicit form of the optimal control or co-state vector. So, the duality principle approach actually provides one with the explicit form of the optimal control and co-state vector, which should be then substituted into the equations given by the rigorous optimality tools and thereby verified.

Finally, performance of the obtained optimal control for a linear system with time delay in control input and a quadratic criterion is verified in the illustrative example against the best linear regulator available for linear systems without delays. The simulation results show a definitive advantage of the obtained optimal regulator in both the criterion value and the value of the controlled variable.

The paper is organized as follows. Section 2 states the optimal control problem for a linear system with time delay in control input and describes the duality principle for a closed-form situation [17]. For reference purposes, the optimal filtering equations for a linear state and linear observations with delay [18] are briefly reminded in Section 3. The optimal control problem for a linear system with time delay in control input is solved in Section 4, based on application of the duality principle to the optimal filter of the preceding section. The proof of the obtained results, based on the maximum principle [19, 8], is given in Appendix. Section 5 presents an example illustrating the quality of control provided by the obtained optimal regulator for linear systems with time delay in control input against the best linear regulator available for systems without delays. Simulation graphs and comparison tables demonstrating better performance of the obtained optimal regulator are included.

2 Optimal control problem for linear system with time delay in control input

Consider a linear system with time delay in control input

$$dx(t) = (a_0(t) + a_1(t)x(t))dt + B(t)u(t-h)dt, \tag{1}$$

with the initial condition $x(s) = \phi(s)$, $s \in [-h, 0]$, where $x(t) \in R^n$ is the system state, $u(t) \in R^m$ is the control variable, and $\phi(s)$ is a piecewise continuous function given in the interval $[-h, 0]$. Existence of the unique solution of the equation (1) is thus assured by the Caratheodori theorem (see, for example, [21]). The quadratic cost function to be minimized is defined as follows:

$$J = \frac{1}{2}[x(T) - x_1]^T \psi [x(T) - x_1] +$$

$$\frac{1}{2}\int_{t_0}^{T} u^T(s)R(s)u(s)ds + \frac{1}{2}\int_{t_0}^{T} x^T(s)L(s)x(s)ds, \tag{2}$$

where x_1 is a given vector, R is positive and ψ, L are nonnegative definite symmetric matrices, and $T > t_0$ is a certain time moment.

The optimal control problem is to find the control $u(t)$, $t \in [t_0, T]$, that minimizes the criterion J along with the trajectory $x^*(t)$, $t \in [t_0, T]$, generated upon substituting $u^*(t)$ into the state equation (1). To find the solution to this optimal control problem, the duality principle [1] can be used. For linear systems without delay, if the optimal control exists in the optimal control problem for a linear system with the quadratic cost function J, the optimal filter exists for the dual linear system with Gaussian disturbances and can be found from the optimal control problem solution, using simple algebraic transformations (duality between the gain matrices and between the gain matrix and variance equations), and vice versa (see [1]). Taking into account the physical duality of the filtering and control problems, the last conjecture should be valid for all cases where the optimal control (or, vice versa, the optimal filter) exists in a closed finite-dimensional form [17]. This proposition is now applied to the optimal filtering problem for linear system states over observations with delay, which is dual to the stated optimal control problem (1),(2), and where the optimal filter has already been obtained (see [18]).

3 Optimal filter for linear state equation and linear observations with delay

In this section, the optimal filtering equations for a linear state equation over linear observations with delay (obtained in [18]) are briefly reminded for reference purposes. Let the unobservable random process $x(t)$ be described by an ordinary differential equation for the dynamic system state

$$dx(t) = (a_0(t) + a(t)x(t))dt + b(t)dW_1(t), \quad x(t_0) = x_0, \tag{3}$$

and a delay-differential equation be given for the observation process:

$$dy(t) = (A_0(t) + A(t)x(t-h))dt + F(t)dW_2(t), \tag{4}$$

where $x(t) \in R^n$ is the state vector, $y(t) \in R^m$ is the observation process, the initial condition $x_0 \in R^n$ is a Gaussian vector such that x_0, $W_1(t)$, $W_2(t)$ are independent. The observation process $y(t)$ depends on the delayed state $x(t-h)$, where h is a fixed delay shift, which assumes that collection of information on the system state for the observation purposes is possible only after a certain time h. The vector-valued function $a_0(s)$ describes the effect of system inputs (controls and disturbances). It is assumed that $A(t)$ is a nonzero matrix and $F(t)F^T(t)$ is a positive definite matrix. All coefficients in (3)–(4) are deterministic functions of appropriate dimensions.

The estimation problem is to find the estimate of the system state $x(t)$ based on the observation process $Y(t) = \{y(s), 0 \le s \le t\}$, which minimizes the Euclidean 2-norm

$$J = E[(x(t) - \hat{x}(t))^T (x(t) - \hat{x}(t))]$$

at each time moment t. In other words, our objective is to find the conditional expectation

$$m(t) = \hat{x}(t) = E(x(t) \mid F_t^Y).$$

As usual, the matrix function

$$P(t) = E[(x(t) - m(t))(x(t) - m(t))^T \mid F_t^Y]$$

is the estimate variance.

The solution to the stated problem is given by the following system of filtering equations, which is closed with respect to the introduced variables, $m(t)$ and $P(t)$:

$$dm(t) = (a_0(t) + a(t)m(t))dt + P(t)\exp\left(-\int_{t-h}^{t} a^T(s)ds\right)A^T(t)\times \tag{5}$$

$$\left(F(t)F^T(t)\right)^{-1}(dy(t) - (A_0(t) + A(t)m(t-h))dt).$$

$$dP(t) = (P(t)a^T(t) + a(t)P(t) + b(t)b^T(t) - \tag{6}$$

$$P(t)\exp\left(-\int_{t-h}^{t} a^T(s)ds\right)A^T(t)\left(F(t)F^T(t)\right)^{-1}\times$$

$$A(t)\exp\left(-\int_{t-h}^{t} a(s)ds\right)P(t))dt$$

The system of filtering equations (4) and (5) should be complemented with the initial conditions $m(t_0) = E[x(t_0) \mid F_{t_0}^Y]$ and $P(t_0) = E[(x(t_0) - m(t_0)(x(t_0) - m(t_0)^T \mid F_{t_0}^Y]$. As noted, this system is very similar to the conventional Kalman-Bucy filter, except the adjustments for delays in the estimate and variance equations, calculated due to the Cauchy formula for the linear state equation.

In the case of a constant matrix a in the state equation, the optimal filter takes the especially simple form ($\exp(-\int_{t-h}^{t} a^T ds) = \exp(-a^T h)$)

$$dm(t) = (a_0(t) + am(t))dt + P(t)\exp(-a^T h)A^T(t)\times \tag{7}$$

$$\left(F(t)F^T(t)\right)^{-1}(dy(t) - (A_0(t) + A(t)m(t-h))dt),$$

$$dP(t) = (P(t)a^T + aP(t) + b(t)b^T(t) - \tag{8}$$

$$P(t)\exp(-a^T h)A^T(t)\left(F(t)F^T(t)\right)^{-1}A(t)\exp(-ah)P(t))dt.$$

Thus, the equation (5) (or (7)) for the optimal estimate $m(t)$ and the equation (6) (or (8)) for its covariance matrix $P(t)$ form a closed system of filtering equations in the case of a linear state equation and linear observations with delay.

4 Optimal control problem solution

Let us return to the optimal control problem for the linear state (1) with time delay in linear control input and the cost function (2). This problem is dual to the filtering problem for the linear state (3) and linear observations with delay (4). Since the optimal filter gain matrix in (5) is equal to

$$K_f = P(t) \exp\,(-\int_{t-h}^{t} a^T(s)ds)A^T(t)(F(t)F^T(t))^{-1},$$

the gain matrix in the optimal control problem takes the form of its dual transpose

$$K_c = (R(t))^{-1}B^T(t) \exp\,(\int_{t-h}^{t} a^T(s)ds)Q(t),$$

and the optimal control law is given by

$$u_{opt}(t) = K_c x = (R(t))^{-1}B^T(t) \exp\,(\int_{t-h}^{t} a^T(s)ds)Q(t)x(t), \quad (9)$$

where the matrix function $Q(t)$ is the solution of the following equation dual to the variance equation (6)

$$dQ(t) = (-a^T(t)Q(t) - Q(t)a(t) + L(t) -$$
$$Q(t) \exp\,(\int_{t-h}^{t} a(s)ds)B(t)R^{-1}(t)\times$$
$$B^T(t) \exp\,(\int_{t-h}^{t} a^T(s)ds)Q(t))dt, \quad (10)$$

with the terminal condition $Q(T) = \psi$.

Upon substituting the optimal control (9) into the state equation (1), the optimally controlled state equation is obtained

$$dx(t) = (a_0(t) + a(t)x(t) + B(t)(R(t))^{-1}B^T(t)\times$$
$$\exp\,(\int_{t-h}^{t} a^T(s)ds)Q(t)x(t)dt, \quad x(t_0) = x_0,$$

The results obtained in this section by virtue of the duality principle are proved (the proof is given in Appendix) using the general equations of the Pontryagin maximum principle [19, 8]. (Bellman dynamic programming [20, 9] could serve as an alternative verifying approach). It should be noted, however, that application of the maximum principle to the present case gives one only a system of state and co-state equations and does not provide the explicit form of the optimal control or co-state vector. So, the duality principle approach actually provides one with the explicit form of the optimal control and co-state vector, which should be then substituted into the equations given by the rigorous optimality tools and thereby verified.

5 Example

This section presents an example of designing the optimal regulator for a system (1) with a criterion (2), using the scheme (9)–(10), and comparing it to the regulator where the matrix Q is selected as in the optimal linear regulator for a system without delays.

Let us start with a scalar linear system

$$\dot{x}(t) = x(t) + u(t - 0.1), \tag{11}$$

with the initial conditions $x(s) = 0$ for $s \in [-0.1, 0)$ and $x(0) = 1$. The optimal control problem is to find the control $u(t)$, $t \in [0, T]$, $T = 0.25$, that minimizes the criterion

$$J = \frac{1}{2}[x(T) - x^*]^2 + \frac{1}{2}\int_0^T u^2(t)dt, \tag{12}$$

where $T = 0.25$, and $x^* = 10$ is a large value of $x(t)$ *a priori* unreachable for time T. In other words, the optimal control problem is to maximize the state $x(t)$ using the minimum energy of control u.

Let us first construct the regulator where the optimal control law and the matrix $Q(t)$ are calculated in the same manner as for the optimal linear regulator for a linear system without delays in control input, that is $u_{opt}(t) = (R(t))^{-1}B^T(t)Q(t)x(t)$ (see [1] for reference). Since $B(t) = 1$ in (11) and $R(t) = 1$ in (12), the optimal control is actually equal to

$$u(t) = Q(t)x(t), \tag{13}$$

where $Q(t)$ satisfies the Riccati equation

$$\dot{Q}(t) = -a^T(t)Q(t) - Q(t)a(t) + L(t) - Q(t)B(t)R^{-1}(t)B^T(t)Q(t),$$

with the terminal condition $Q(T) = \psi$. Since $a(t) = 1$, $B(t) = 1$ in (11), and $L = 0$ and $\psi = 1$ in (12), the last equation turns to

$$\dot{Q}(t) = -2Q(t) - (Q(t))^2, \quad Q(0.25) = 1. \tag{14}$$

Upon substituting the optimal control (13) into (11), the controlled system takes the form

$$\dot{x}(t) = x(t) + Q(t)x(t - 0.1). \tag{15}$$

The results of applying the regulator (13),(14) to the system (11) are shown in Fig. 1, which presents the graphs of the controlled state (15) $x(t)$ in the interval $[0, T]$, the shifted ahead by 0.1 criterion (12) $J(t-0.1)$ in the interval $[0.1, T+0.1]$, and the shifted ahead by 0.1 control (13) $u(t-0.1)$ in the interval $[0, T]$. The values of the state (15) and the criterion (12) at the final moment $T = 0.25$ are $x(0.25) = 1.5097$ and $J(0.25) = 36.2598$.

Let us now apply the optimal regulator (9)–(10) for linear systems with time delay in control input to the system (11). Since $a(t) = 1$ and $h = 0.1$ in (11) and, therefore, $\exp\left(\int_{t-h}^t a^T(s)ds\right) = \exp(0.1)$, the optimal control law takes the form

$$u_{opt}(t) = \exp(0.1)Q(t)x(t), \tag{16}$$

where $Q(t)$ satisfies the Riccati equation

$$\dot{Q}(t) = -2Q(t) - (\exp(0.1)Q(t))^2, \quad Q(0.25) = 1. \tag{17}$$

Upon substituting the optimal control (16) into (11), the optimally controlled system takes the form

$$\dot{x}(t) = x(t) + \exp(0.1)Q(t-0.1)x(t-0.1). \tag{18}$$

The results of applying the regulator (16),(17) to the system (11) are shown in Fig. 2, which presents the graphs of the optimally controlled state (18) $x(t)$ in the interval $[0, T]$, the shifted ahead by 0.1 criterion (12) $J(t-0.1)$ in the interval $[0.1, T+0.1]$, and the shifted ahead by 0.1 optimal control (16) $u_{opt}(t-0.1)$ in the interval $[0, T]$. The values of the state (18) and the criterion (12) at the final moment $T = 0.25$ are $x(0.25) = 1.668$ and $J(0.25) = 35.3248$. There is a definitive improvement in the values of the controlled state to be maximized and the criterion to be minimized, in comparison to the preceding case, due to the optimality of the regulator (16),(17) for the linear system (11) with time delay in control input.

6 Appendix

Proof of the optimal control problem solution. Define the Hamiltonian function [19, 8] for the optimal control problem (1),(2) as

$$H(x, u, q, t) = u^T R(t)u + x^T L(t)x + q^T[a_0(t) + a_1(t)x + B(t)u_1(u)], \tag{19}$$

where $u_1(u) = u(t-h)$. Applying the maximum principle condition $\partial H/\partial u = 0$ to this specific Hamiltonian function (19) yields

$$\partial H/\partial u = 0 \Rightarrow R(t)u(t) + (\partial u_1(t)/\partial u)^T B^T(t)q(t) = 0.$$

Upon denoting $(\partial u_1(t)/\partial u) = M(t)$, the optimal control law is obtained as

$$u^*(t) = -R^{-1}(t)M^T(t)B^T(t)q(t).$$

Taking linearity and causality of the problem into account, let us seek $q(t)$ as a linear function in $x(t)$

$$q(t) = -Q(t)x(t), \tag{20}$$

where $Q(t)$ is a square symmetric matrix of dimension n. This yields the complete form of the optimal control

$$u^*(t) = R^{-1}(t)M^T(t)B^T(t)Q(t)x(t). \tag{21}$$

Note that the transversality condition [19, 8] for $q(T)$ implies that $q(T) = -\partial J/\partial x(T) = -\psi x(T)$ and, therefore, $Q(T) = \psi$.

Using the co-state equation $dq(t)/dt = -\partial H/\partial x$, which gives

$$-dq(t)/dt = L(t)x(t) + a_1^T(t)q(t), \tag{22}$$

and substituting (20) into (22), we obtain

$$\dot{Q}(t)x(t) + Q(t)d(x(t))/dt = L(t)x(t) - a_1^T(t)Q(t)x(t). \tag{23}$$

Substituting the expression for $\dot{x}(t)$ from the state equation (1) into (23) yields

$$\dot{Q}(t)x(t) + Q(t)a_1(t)x(t) + Q(t)B(t)u(t-h) = L(t)x(t) - a_1^T(t)Q(t)x(t). \tag{24}$$

In view of linearity of the problem, differentiating the last expression in x does not imply loss of generality. Upon taking into account that $(\partial u(t-h)/\partial x(t)) = (\partial u(t-h)/\partial u(t))(\partial u(t)/\partial x(t)) = M(t)R^{-1}(t)M^T(t)B^T(t)Q(t)$ and differentiating the equation (24) in x, it is transformed into the Riccati equation

$$\dot{Q}(t) = L(t) - Q(t)a_1(t) - a_1^T(t)Q(t) - Q(t)B(t)M(t)R^{-1}(t)M^T(t)B^T(t)Q(t). \tag{25}$$

Let us find the value of matrix $M(t)$ for this problem. First of all, let us note [1] that the Hamiltonian function $H(x^*, u^*, q^*, t)$ is constant in t for the optimal control (21) $u^*(t)$, the corresponding optimal state (1) $x^*(t)$ and co-state $q^*(t)$ satisfying (20), and $Q(t)$ satisfying the equation (25), and equal to

$$H(x^*, u^*, q^*, t) = u^{*T}R(t)u^* + x^{*T}L(t)x^* + d(x^{*T}Q(t)x^*)/dt = C = const. \tag{26}$$

Integrating the last equality from $t-h$ to t yields

$$\int_{t-h}^{t}[u^{*T}(s)R(s)u^*(s) + x^{*T}(s)L(s)x^*(s)]ds + x^{*T}(t)Q(t)x^*(t) - x^{*T}(t-h)Q(t-h)x^*(t-h) = Ch.$$

Differentiating the obtained formula respect to $x^*(t)$ and $u^*(t)$ and taking into account the optimal control expressions for $u^*(t)$ and $u^*(t-h)$ given by (21), we obtain

$$(R^{-1}(t)M^T(t)B^T(t)) = (M(t))^{-1}(R^{-1}(t-h)M^T(t-h)B^T(t-h))\exp\left(\int_{t-h}^{t} a^T(s)ds\right), \tag{27}$$

also using that

$$\partial x(t)/\partial x(t-h) = \exp\left(\int_{t-h}^{t} a(s)ds\right).$$

The last formula follows from the Cauchy formula for the solution of the linear state equation (1)

$$x(t) = \Phi(t, t-h)x(t-h) + \int_{t-h}^{t} \Phi(t,\tau)a_0(\tau)d\tau + \int_{t-h}^{t} \Phi(t,\tau)B(\tau)u(\tau-h)d\tau,$$

where $\Phi(t,\tau)$ is the matrix of fundamental solutions of the homogeneous equation (1), that is solution of the matrix equation

$$\frac{d\Phi(t,\tau)}{dt} = a(t)\Phi(t,\tau), \quad \Phi(t,t) = I,$$

where I is the identity matrix. In other words, $\Phi(t, t-h) = \exp \int_{t-h}^{t} a(s)ds$.

Furthermore, it can be noted, differentiating twice the formula (26) with respect to $x^*(t)$, that the expression $R^{-1}(t)M^T(t)B^T(t))$ does actually not depend on $B(t)$ or $R^{-1}(t)$ as functions of time t. Thus, the value of the matrix $M(t)$ for this problem can be determined from (27) assuming that time $t-h$ is equal to t in the matrix function $R^{-1}(t-h)M^T(t-h)B^T(t-h))$. Finally, the formula (27) gives the following equality for calculating $M(t)$

$$M^T(t)B^T(t) = B^T(t)\exp\big(\int_{t-h}^{t} a^T(s)ds\big). \quad (28)$$

Substituting the formula (28) into (21) and (25) yields the desired formulas (9) and (10) for the optimal control law $u^*(t)$ and the matrix function $Q(t)$. The optimal control problem solution is proved.

References

[1] H. Kwakernaak, R. Sivan, Linear Optimal Control Systems, Wiley-Interscience, New York, 1972.

[2] W. H. Fleming, R.W. Rishel, Deterministic and Stochastic Optimal Control, Springer-Verlag, New York, 1975.

[3] V. B. Kolmanovskii, L. E. Shaikhet, Control of Systems with Aftereffect. American Mathematical Society, Providence, 1996.

[4] V. B. Kolmanovskii, A. D. Myshkis, Introduction to the Theory and Applications of Functional Differential Equations, Kluwer, New York, 1999.

[5] M. Malek-Zavarei, M. Jamshidi, Time-Delay Systems: Analysis, Optimization and Applications, North-Holland, Amsterdam, 1987.

[6] J. M. Dion, Linear Time Delay Systems, Pergamon, London, 2001.

[7] E.-K. Boukas, Z.K. Liu, Deterministic and Stochastic Time-Delay Systems, Birkhauser, 2002.

[8] G. L. Kharatashvili, A maximum principle in external problems with delays, in Mathematical Theory of Control, A. V. Balakrishnan and L. W. Neustadt (Eds.), Academic Press, New York, 1967.

[9] M. N. Oguztoreli, Time-Lag Control Systems, Academic Press, New York, 1966.

[10] M. N. Oguztoreli, A time optimal control problem for systems described by differential difference equations, SIAM J. Control, 1 (1963) 290–310.

[11] D. H. Eller, J. K. Aggarwal, H. T. Banks, Optimal control of linear time-delay systems, IEEE Trans. Aut. Contr. AC-14 (1969) 678–687.

[12] M. C. Delfour, The linear quadratic control problem with delays in space and control variables: a state space approach, SIAM. J. Contr. Optim. 24 (1986) 835–883.

[13] K. Uchida, E. Shimemura, T. Kubo, N. Abe, The linear-quadratic optimal control approach to feedback control design for systems with delay, Automatica, 24 (1988) 773-780.

[14] C. T. Abdallah, K. Gu (Eds.). Proc. 3rd IFAC Workshop on Time Delay Systems. OMNIPRESS, Madison, 2001.

[15] J. L. Dugard, Stability and Control of Time-Delay Systems, Springer, 1998.

[16] M. S. Mahmoud, Robust Control and Filtering for Time-Delay Systems, Marcel Dekker, 2000.

[17] M. V. Basin, M. A. Alcorta-Garcia, Optimal control for third degree polynomial systems and its automotive application, in Proc. 41st Conf. on Decision and Control (Las Vegas, NV, December 10–13, 2002), 2002, Ref. no. WeM05-6.

[18] M. V. Basin, L. M. Fridman, P. Acosta, J. Rogriduez-Gonzalez, Optimal and robust integral sliding mode filters over observations with delay, in Proc. 7th International Workshop on Variable Structure Systems (Sarajevo, July 17–19, 2002), 2002, pp. 163–174.

[19] L. S. Pontryagin, V. G. Boltyanskii, R. V. Gamkrelidze, E. F. Mishchenko, The Mathematical Theory of Optimal Processes, Interscience, New York, 1962.

[20] R. Bellman, Dynamic Programming, Princeton University Press, Princeton, 1957.

[21] A. F. Filippov, Differential Equations with Discontinuous Right-Hand Sides, Kluwer, 1989.

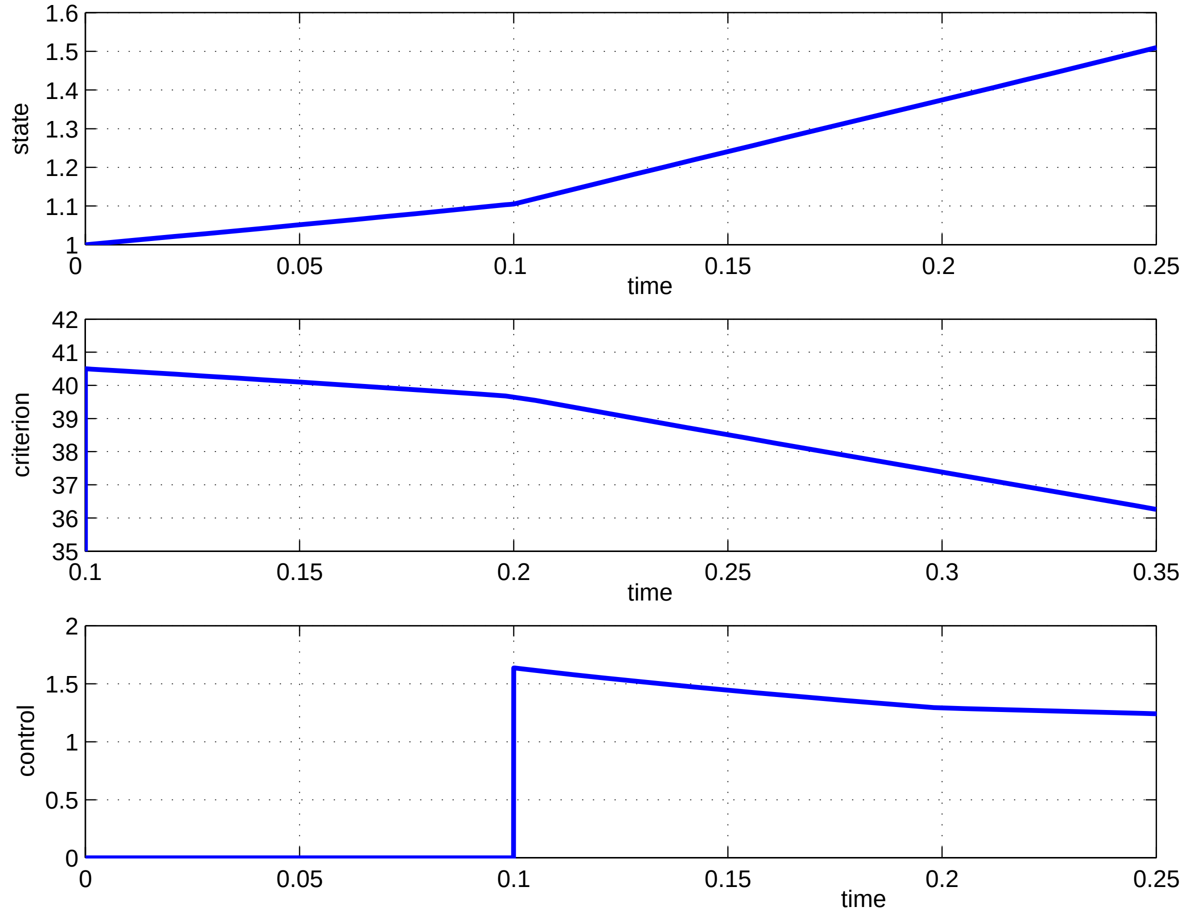

Figure 1: Best linear regulator available for linear systems without delays. Graphs of the controlled state (15) $x(t)$ in the interval $[0, 0.25]$, the shifted ahead by 0.1 criterion (12) $J(t-0.1)$ in the interval $[0.1, 0.35]$, and the shifted ahead by 0.1 control (13) $u(t-0.1)$ in the interval $[0, 0.25]$.

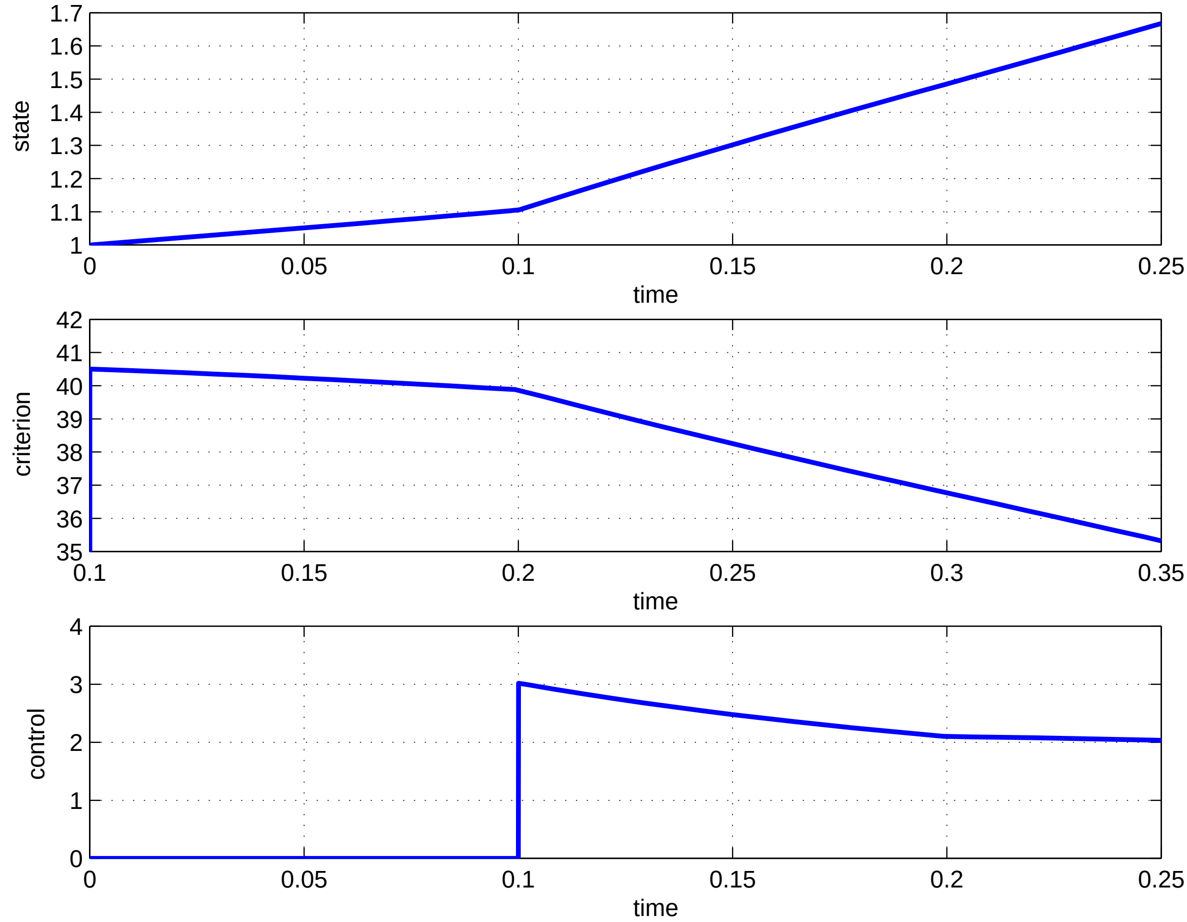

Figure 2: Optimal regulator obtained for linear systems with time delay in control input. Graphs of the optimally controlled state (18) $x(t)$ in the interval $[0, 0.25]$, the shifted ahead by 0.1 criterion (12) $J(t-0.1)$ in the interval $[0.1, 0.35]$, and the shifted ahead by 0.1 optimal control (16) $u_{opt}(t-0.1)$ in the interval $[0, 0.25]$.