Linear Stability Analysis of a Tilting-Pad Journal Bearing System

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This paper describes a mathematical model to study the linear stability of a tilting-pad journal bearing system. By employing the Newton-Raphson method and the pad assembly technique, the full dynamic coefficients involving the shaft degrees of freedom as well as the pad degrees of freedom are determined. Based on these dynamic coefficients, the perturbation equations including self-excited motion of the rotor and rotational motion of the pads are derived. The complex eigenvalues of the equations are computed and the pad critical mass identified by eigenvalues can be used to determine the stability zone of the system. The results show that some factors, such as the preload coefficient, the pivot position, and the rotor speed, significantly affect the stability of tilting-pad journal bearing system. Correctly adjusting those parameter values can enhance the stability of the system. Furthermore, various stability charts for the system can be plotted.
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Introduction

Because tilting-pad journal bearings are more stable and efficient than conventional bearings, they have been commonly applied to many rotating machinery applications. The chief feature of tilting-pads is that they modify their configuration to adapt to every operating condition, creating several convergent-divergent gaps around the circumference and thus making the system highly stable [1]. Since Lund [2] developed a numerical method for calculating dynamic coefficients for tilting-pad journal bearings, extensive theoretical and experimental studies on dynamic and stability analysis have been conducted. In the course of the development of tilting-pad journal bearings, many effective methods have been applied, such as Newton-Raphson method, pad assembly technique, finite elements method, and Genetic Algorithm [3–6].

For tilting-pad bearings, when the deformation of the pads and the thermal effects are not taken into consideration, each pad pivoting around a point represents an additional degree of freedom. In past years, it was fairly complex to set up a model involving the degrees of pad freedom, and to carry out dynamic and stability analyses, because of the many variables and degrees of freedom. In most dynamic and stability analysis of rotor-bearing systems, reduced analytical methods are used. In general, in order to simplify the dynamic and stability analysis, both shaft and pads are restricted to synchronous motion with the same frequency of vibration around their static equilibrium positions. In this case, the number of degrees of pad freedom reduces to eight synchronous dynamic coefficients [7]. Usually, these coefficients are thought of as equivalent coefficients, functions of pad mass and excitation pulsation, not having actually a physical significance [8]. In fact, the frequency of free vibration of the system, which needs to be determined by the complex eigenvalues computed from the differential equations of the system, is not equal to the assumed frequency of vibration. Thus, employing synchronous reduced dynamic coefficients cannot effectively predict steady state and dynamic behavior of the tilting-pad bearing system.

It is well known that dynamic coefficients play an important role in linear dynamic and stability analysis of a rotor-bearing system. An accurate dynamic and stability analysis should be done utilizing the complete set of dynamic coefficients involving all of the shaft degrees of freedom as well as the pad degrees of freedom [7]. This paper presents a mathematical model to study the linear stability of a tilting-pad journal bearing system. In order to obtain the dynamic coefficients, the equilibrium position has to be found first [9]. The Newton-Raphson method, which has been proved to have a rapid rate of convergence, is used to find the equilibrium position of journal and pads. The pad assembly technique is employed to obtain the complete set of dynamic coefficients of the tilting-pad journal bearing system. Based on these dynamic coefficients, the perturbation equations, including self-excited motion of the rotor and rotational motion of the pads, are derived. By analyzing complex eigenvalues of the perturbation equations, the linear stability analysis of the system is carried out. The real parts of the eigenvalues ascertain the stability of the system and the imaginary parts represent the whirl frequency. The results show that some factors, such as the preload coefficient, the pivot position, and the rotor speed, significantly affect the stability threshold of the system.

Jacobian Matrix of the Subsystem

Figure 1 shows a schematic representation of a single pad. In the figure, there are three reference frames, the bearing reference frame (O,u,v), a local reference frame (O,x,y) with an angle γ measured from the u axis and a pad reference frame (A,ξ,η) fixed in the pad. O is the bearing center, C is the journal center and A is the pad center.

Using the pad reference frame, the oil-film forces acting on the journal generated by a single pad, can be expressed as

$$\mathbf{F}(\mathbf{a},\dot{\mathbf{a}})=\begin{Bmatrix} F_{\xi}(\xi,\eta,\dot{\xi},\dot{\eta}) \\ F_{\eta}(\xi,\eta,\dot{\xi},\dot{\eta}) \end{Bmatrix} \tag{1}$$

where $\mathbf{a}=(\xi,\eta)$ and $\mathbf{a}=(\dot{\xi},\dot{\eta})$ are, respectively, the displacement vector and velocity vector of the journal center in the pad reference frame, respectively.

Differentiating oil-film forces, Eq. (1) becomes

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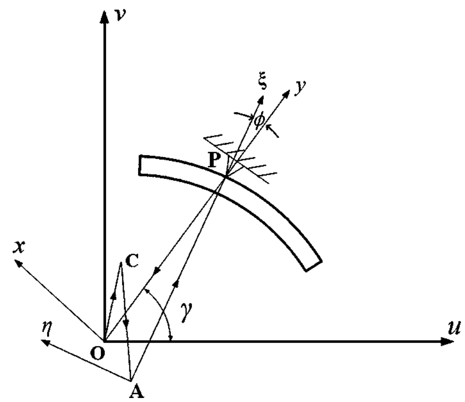

Fig. 1 Single-pad schematic and reference frames

$$d\mathbf{F}(\mathbf{a},\dot{\mathbf{a}}) = \mathbf{D}_a\mathbf{F}(\mathbf{a},\dot{\mathbf{a}})d\mathbf{a} + \mathbf{D}_{\dot{a}}\mathbf{F}(\mathbf{a},\dot{\mathbf{a}})d\dot{\mathbf{a}} \tag{2}$$

where $\mathbf{D}_a\mathbf{F}(\mathbf{a},\dot{\mathbf{a}})$ and $\mathbf{D}_{\dot{a}}\mathbf{F}(\mathbf{a},\dot{\mathbf{a}})$, represent the Jacobian matrix of the force vector $\mathbf{F}(\mathbf{a},\dot{\mathbf{a}})$, with respect to the coordinate vectors $\mathbf{a}$ and $\dot{\mathbf{a}}$, is as follows

$$\mathbf{J} = \mathbf{D}_a\mathbf{F}(\mathbf{a},\dot{\mathbf{a}}) = \begin{bmatrix} \dfrac{\partial F_\xi(\xi,\eta,\dot{\xi},\dot{\eta})}{\partial \xi} & \dfrac{\partial F_\xi(\xi,\eta,\dot{\xi},\dot{\eta})}{\partial \eta} \\ \dfrac{\partial F_\eta(\xi,\eta,\dot{\xi},\dot{\eta})}{\partial \xi} & \dfrac{\partial F_\eta(\xi,\eta,\dot{\xi},\dot{\eta})}{\partial \eta} \end{bmatrix} \tag{3}$$

$$\hat{\mathbf{J}} = \mathbf{D}_{\dot{a}}\mathbf{F}(\mathbf{a},\dot{\mathbf{a}}) = \begin{bmatrix} \dfrac{\partial F_\xi(\xi,\eta,\dot{\xi},\dot{\eta})}{\partial \dot{\xi}} & \dfrac{\partial F_\xi(\xi,\eta,\dot{\xi},\dot{\eta})}{\partial \dot{\eta}} \\ \dfrac{\partial F_\eta(\xi,\eta,\dot{\xi},\dot{\eta})}{\partial \dot{\xi}} & \dfrac{\partial F_\eta(\xi,\eta,\dot{\xi},\dot{\eta})}{\partial \dot{\eta}} \end{bmatrix} \tag{4}$$

Defining a subsystem composed by the journal and a single pad, the relationship of four coordinate vectors seen from the figure is

$$\mathbf{OC} + \mathbf{CA} + \mathbf{AP} + \mathbf{PO} = \mathbf{0} \tag{5}$$

where

$$\mathbf{OC} = x\vec{\mathbf{i}} + y\vec{\mathbf{j}}, \quad \mathbf{CA} = -\xi\vec{\boldsymbol{\xi}} - \eta\vec{\boldsymbol{\eta}}, \quad \mathbf{AP} = R_A\vec{\boldsymbol{\xi}}, \quad \mathbf{PO} = -R_p\vec{\mathbf{i}} \tag{6}$$

Substituting expressions (6) into (5) yields

$$(\mathbf{i},\mathbf{j})\begin{pmatrix} x - R_p \\ y \end{pmatrix} - (\boldsymbol{\xi},\boldsymbol{\eta})\begin{pmatrix} \xi - R_A \\ \eta \end{pmatrix} = \mathbf{0} \tag{7}$$

The transformation equation between the local reference frame and the pad reference frame is

$$(\mathbf{i},\mathbf{j}) = (\boldsymbol{\xi},\boldsymbol{\eta})\mathbf{P} \tag{8}$$

where

$$\mathbf{P} = \begin{bmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{bmatrix} \approx \begin{bmatrix} 1 & \phi \\ -\phi & 1 \end{bmatrix} \tag{9}$$

The quantity ϕ denotes the pad pitch angle measured by starting from the y axis. Substituting Eq. (8) into (7) and neglecting the terms of order of magnitude of c/R_B and the smaller, the following matrix equation is obtained

$$\mathbf{a} = \mathbf{A}\mathbf{b} \tag{10}$$

where

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix} \tag{11}$$

the vector $\mathbf{b}=\{x,y,\varepsilon\}^T$ is the displacement of the subsystem in the local reference frame. In addition, $\varepsilon = R_p\phi$ represents the displacement of the pad center along the direction of the η axis.

Differentiating matrix Eq. (10), the following relationships are obvious

$$d\mathbf{a} = \mathbf{A}d\mathbf{b}, \quad d\dot{\mathbf{a}} = \mathbf{A}d\dot{\mathbf{b}} \tag{12}$$

Furthermore

$$d\mathbf{a} = \mathbf{A}\mathbf{B}d\mathbf{u}, \quad d\dot{\mathbf{a}} = \mathbf{A}\mathbf{B}d\dot{\mathbf{u}} \tag{13}$$

where

$$\mathbf{B} = \begin{bmatrix} \cos\gamma & \sin\gamma & 0 \\ -\sin\gamma & \cos\gamma & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{14}$$

vectors $\mathbf{u}=\{u,v,\varepsilon\}^T$ and $\dot{\mathbf{u}}=\{\dot{u},\dot{v},\dot{\varepsilon}\}^T$ are, respectively, the displacement vector and velocity vector of the subsystem in the bearing reference frame.

The force F_ε that leads the rotation of the pad is

$$\mathbf{F}_\varepsilon = -\mathbf{F}_\eta \tag{15}$$

When F_x and F_y represent the components of the oil-film force vector $\mathbf{F}(\mathbf{a},\dot{\mathbf{a}})$ in the x-y frame, the following matrix equation can be written

$$\mathbf{F}(\mathbf{b},\dot{\mathbf{b}}) = \mathbf{C}^T\mathbf{F}(\mathbf{a},\dot{\mathbf{a}}) \tag{16}$$

where

$$\mathbf{F}(\mathbf{b},\dot{\mathbf{b}}) = \{F_x, F_y, F_\varepsilon\}^T \quad \mathbf{C} = \begin{bmatrix} 1 & \phi & 0 \\ -\phi & 1 & 1 \end{bmatrix} \tag{17}$$

If the components of the oil-film force vector $\mathbf{F}(\mathbf{a},\dot{\mathbf{a}})$ are denoted by F_u and F_v in the bearing reference frame, it is then necessary to express $\mathbf{F}(\mathbf{u},\dot{\mathbf{u}})$ with $\mathbf{F}(\mathbf{a},\dot{\mathbf{a}})$ by matrix $\mathbf{B}$ and Eq. (16). Thus, the matrix equation is written as

$$\mathbf{F}(\mathbf{u},\dot{\mathbf{u}}) = \mathbf{B}^T\mathbf{F}(\mathbf{b},\dot{\mathbf{b}}) = \mathbf{B}^T\mathbf{C}^T\mathbf{F}(\mathbf{a},\dot{\mathbf{a}}) \tag{18}$$

where force vector $\mathbf{F}(\mathbf{u},\dot{\mathbf{u}})$ is stated in the following form

$$\mathbf{F}(\mathbf{u},\dot{\mathbf{u}}) = \begin{Bmatrix} F_u(u,v,\varepsilon,\dot{u},\dot{v},\dot{\varepsilon}) \\ F_v(u,v,\varepsilon,\dot{u},\dot{v},\dot{\varepsilon}) \\ F_\varepsilon(u,v,\varepsilon,\dot{u},\dot{v},\dot{\varepsilon}) \end{Bmatrix} \tag{19}$$

Differentiating matrix Eq. (18) and integrating leads to

$$d\mathbf{F}(\mathbf{u},\dot{\mathbf{u}}) = \mathbf{B}^T\mathbf{A}^T d\mathbf{F}(\mathbf{a},\dot{\mathbf{a}}) + \mathbf{B}^T\mathbf{Z}d\mathbf{q} \tag{20}$$

where

$$\mathbf{Z} = \begin{bmatrix} 0 & -\phi & -F_\eta/R_p \\ \phi & 0 & F_\xi/R_p \\ 0 & 0 & 0 \end{bmatrix} \quad d\mathbf{q} = \{dF_\xi, dF_\eta, d\varepsilon\}^T \tag{21}$$

It should be noted that the second term could be neglected in comparison with the first term on the right side of Eq. (20), so the matrix equation further simplifies to

$$d\mathbf{F}(\mathbf{u},\dot{\mathbf{u}}) = (\mathbf{A}\mathbf{B})^T d\mathbf{F}(\mathbf{a},\dot{\mathbf{a}}) \tag{22}$$

Combining Eqs. (2), (13), and (22), the differential form of the oil-film forces in the bearing reference frame is obtained

$$d\mathbf{F}(\mathbf{u},\dot{\mathbf{u}}) = (\mathbf{A}\mathbf{B})^T\mathbf{D}_a\mathbf{F}(\mathbf{a},\dot{\mathbf{a}})\mathbf{A}\mathbf{B}d\mathbf{u} + (\mathbf{A}\mathbf{B})^T\mathbf{D}_{\dot{a}}\mathbf{F}(\mathbf{a},\dot{\mathbf{a}})\mathbf{A}\mathbf{B}d\dot{\mathbf{u}} \tag{23}$$

Thus, according to Eqs. (3) and (4), the Jacobian matrix of the force vector $\mathbf{F}(\mathbf{u},\dot{\mathbf{u}})$ with respect to vectors $\mathbf{u}$ and $\dot{\mathbf{u}}$ can be obtained

$$\mathbf{D}_{\hat{\mathbf{u}}}\mathbf{F}(\mathbf{u},\dot{\mathbf{u}}) = (\mathbf{AB})^T\mathbf{JAB} = \begin{bmatrix} \mathbf{P}^T\mathbf{JP} & \mathbf{P}^T\mathbf{JQ} \\ \mathbf{Q}^T\mathbf{JP} & \mathbf{Q}^T\mathbf{JQ} \end{bmatrix}$$

$$\mathbf{D}_{\dot{\hat{u}}}\mathbf{F}(\mathbf{u},\dot{\mathbf{u}}) = (\mathbf{AB})^T\hat{\mathbf{J}}\mathbf{AB} = \begin{bmatrix} \mathbf{P}^T\hat{\mathbf{J}}\mathbf{P} & \mathbf{P}^T\hat{\mathbf{J}}\mathbf{Q} \\ \mathbf{Q}^T\hat{\mathbf{J}}\mathbf{P} & \mathbf{Q}^T\hat{\mathbf{J}}\mathbf{Q} \end{bmatrix} \quad (24)$$

where

$$\mathbf{P} = \begin{bmatrix} \cos\gamma & \sin\gamma \\ -\sin\gamma & \cos\gamma \end{bmatrix} \quad \mathbf{Q} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad (25)$$

and $\mathbf{u}=\{u,v,\varepsilon\}^T$ and $\dot{\mathbf{u}}=\{\dot{u},\dot{v},\dot{\varepsilon}\}^T$ represent the subsystem's motion state.

Dynamic Coefficients of the Rotor-Bearing System

A rotor-bearing system with n pads consists of n subsystems. The displacement and velocity vectors of each subsystem can be written as $\mathbf{u}_i=\{u,v,\varepsilon_i\}^T$ and $\dot{\mathbf{u}}_i=\{\dot{u},\dot{v},\dot{\varepsilon}_i\}^T$ $(i=1,\ldots,n)$, respectively, where i indicates the sequential number of pad. Thus, $\hat{\mathbf{u}}=\{u,v,\varepsilon_1,\ldots,\varepsilon_n\}^T$ and $\dot{\hat{\mathbf{u}}}=\{u,v,\dot{\varepsilon}_1,\ldots,\dot{\varepsilon}_n\}^T$ can be used to define the system's motion state.

Augmenting and adding up the Jacobian matrix of each subsystem, one obtains the Jacobian matrix of the rotor-bearing system as follows

$$\mathbf{D}_{\hat{\mathbf{u}}}\hat{\mathbf{F}}(\hat{\mathbf{u}}^*,\dot{\hat{\mathbf{u}}}) = \begin{bmatrix} \sum_{i=1}^{n}\mathbf{P}_i^T\mathbf{J}_i\mathbf{P}_i & \mathbf{P}_1^T\mathbf{J}_1\mathbf{Q}_1 & \cdots & \mathbf{P}_n^T\mathbf{J}_n\mathbf{Q}_n \\ \mathbf{Q}_1^T\mathbf{J}_1\mathbf{P}_1 & \mathbf{Q}_1^T\mathbf{J}_1\mathbf{Q}_1 & & \\ \vdots & & \ddots & \\ \mathbf{Q}_n^T\mathbf{J}_n\mathbf{P}_n & & & \mathbf{Q}_n^T\mathbf{J}_n\mathbf{Q}_n \end{bmatrix} \quad (26)$$

$$\mathbf{D}_{\dot{\hat{\mathbf{u}}}}\hat{\mathbf{F}}(\hat{\mathbf{u}}^*,\dot{\hat{\mathbf{u}}}) = \begin{bmatrix} \sum_{i=1}^{n}\mathbf{P}_i^T\hat{\mathbf{J}}_i\mathbf{P}_i & \mathbf{P}_1^T\hat{\mathbf{J}}_1\mathbf{Q}_1 & \cdots & \mathbf{P}_n^T\hat{\mathbf{J}}_n\mathbf{Q}_n \\ \mathbf{Q}_1^T\hat{\mathbf{J}}_1\mathbf{P}_1 & \mathbf{Q}_1^T\hat{\mathbf{J}}_1\mathbf{Q}_1 & & \\ \vdots & & \ddots & \\ \mathbf{Q}_n^T\hat{\mathbf{J}}_n\mathbf{P}_n & & & \mathbf{Q}_n^T\hat{\mathbf{J}}_n\mathbf{Q}_n \end{bmatrix} \quad (27)$$

where $\hat{\mathbf{F}}(\hat{\mathbf{u}},\dot{\hat{\mathbf{u}}})=\{\Sigma_{i=1}^{n}F_u,\Sigma_{i=1}^{n}F_v,F_{\varepsilon_1},\cdots,F_{\varepsilon_n}\}^T$ represents the oil-film forces acting on the shaft and leading to the rotation of the pads. Once the static equilibrium position of the system is determined by Newton-Raphson method, the dynamic coefficients of the system can be obtained as follows

$$\mathbf{K} = -\mathbf{D}_{\hat{\mathbf{u}}}\hat{\mathbf{F}}(\hat{\mathbf{u}}^*,\mathbf{0}) \quad (28)$$

$$\mathbf{C} = -\mathbf{D}_{\dot{\hat{\mathbf{u}}}}\hat{\mathbf{F}}(\hat{\mathbf{u}}^*,\mathbf{0}) \quad (29)$$

where $\hat{\mathbf{u}}^*=\{u^*,v^*,\varepsilon_1^*,\ldots,\varepsilon_n^*\}^T$ is the static equilibrium position of the system including the journal and the pads.

Linear Stability Analysis

For a symmetric rotor-bearing system, the perturbation equations of the system with a rigid rotor, after obtaining dynamic coefficients, can be expressed in the following form

$$\mathbf{M}\ddot{\tilde{\mathbf{u}}} + \mathbf{C}\dot{\tilde{\mathbf{u}}} + \mathbf{K}\tilde{\mathbf{u}} = \mathbf{0} \quad (30)$$

The mass matrix $\mathbf{M}$ is

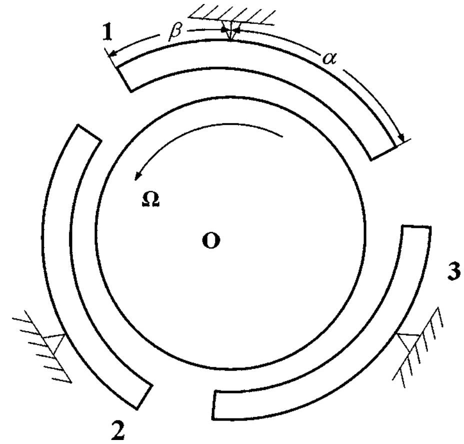

Fig. 2 Schematic of a three-pad journal bearing system

$$\mathbf{M} = \begin{bmatrix} m_b & & & & \\ & m_b & & & \\ & & m_p & & \\ & & & \ddots & \\ & & & & m_p \end{bmatrix} \quad (31)$$

where m_b is the half-mass of the rotor, m_p is the pad equivalent mass that is expressed in terms of the pad moment of inertia J_p as

$$m_p = J_p/R_A^2 \quad (32)$$

The linear equations consist of a set of total $n+2$ equations, the first two equations describe motion of the journal and the others describe the tilting-pads. In the matrix equation, the unknown $\tilde{\mathbf{u}}$ represents small-perturbed displacements with respect to the static equilibrium position $\hat{\mathbf{u}}^*$.

Equation (30), a second-order linear differential equation, has the following general solution

$$\tilde{\mathbf{u}} = \boldsymbol{\varphi}e^{\lambda t} \quad (33)$$

where eigenvalues λ and the corresponding normal mode φ generally come in complex conjugate pairs.

Substituting Eq. (33) into the differential Eqs. (30), the corresponding second-order eigenvalue problem is

$$(\lambda^2\mathbf{M} + \lambda\mathbf{C} + \mathbf{K})\boldsymbol{\varphi} = \mathbf{0} \quad (34)$$

The stability of the system depends on the eigenvalues. The real parts of the system eigenvalues can ascertain whether the system is stable. The $n+2$ modal damping ratios $\bar{\varsigma}_k$ are related to the complex eigenvalues $\lambda_k=\varsigma_k\pm i\omega_k$ $(k=1,2,\ldots,n+2)$ by

$$\bar{\varsigma}_k = \frac{-\varsigma_k}{\sqrt{\varsigma_k^2+\omega_k^2}} \quad (35)$$

The modal damping ratios ς_k can be used to show a general tendency of stability of the system. The system will become unstable if one of the modal damping ratios tends to be negative. The complex mode shapes $\boldsymbol{\varphi}_k=\boldsymbol{\alpha}_k\pm i\boldsymbol{\beta}_k$ reflect the amplitude ratio and the phase relationship of the system.

Results

An ideal model generated from a small bearing used in the light-duty air compressor is chosen for the study. The symmetrical bearing system has three tilting pads and a rigid rotor, as illustrated in Fig. 2. The values of some parameters are listed as follows: $\bar{\lambda}=1.0$, $\hat{\beta}=100$ deg, $\mu=0.012$ Pa s, $R_p=5$ cm, $m_b=50$ kg, and $c/R_B=0.0055$. A computer program was developed to compute the static equilibrium position, the dynamic coefficients and the complex eigenvalues of the system. The dimensionless pad

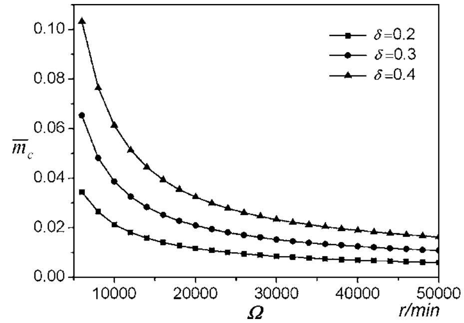

Fig. 3 Nondimensional pad critical mass $\bar{m}_c$ versus Ω (Ψ =0.5)

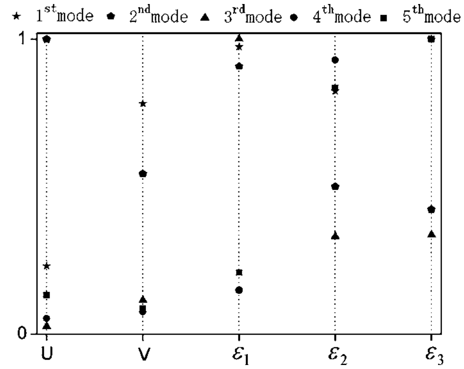

Fig. 6 The associated amplitude ratio of complex modes (δ =0.2, Ω=10,000 rpm)

critical mass $\bar{m}_c = m_c/2m_b$ is defined. Here, m_c represents the mass of the pad when one of the complex eigenvalues has zero real part and others have negative real parts.

For different values of preload, the correlation between rotational speed and the pad critical mass is illustrated in Fig. 3. From the figure it is found that the pad critical mass gradually converges to a certain value with the increase of the rotational speed. The curves also indicate that the pad critical mass will be insensitive to the increase of rotational speed in the higher speed section. Conversely, the critical mass of the pad is strongly influenced by the rotational speed as it is in lower speed section. As we know, preload is significant in controlling system performance. Seeing the figure, with increasing δ, $\bar{m}_c$ increases. The curves show that the preload coefficient greatly affects the stability of the system.

For a tilting-pad journal bearing, the pivot positon of the pad can have a large influence on its dynamic properties. Figures 4 and 5 present the nondimensional pad critical mass $\bar{m}_c$ versus the pivot position $\Psi = \alpha/\beta$. In these figures, it can be seen that the pad critical mass increases in a continuous manner when the pivot is displaced toward the outlet.

Figures 6–8 display the amplitude ratio and the real and imaginary portions of complex mode shapes corresponding to each eigenvalue for $m_p = 1$ kg, $c/R_B = 0.0058$, and $\Psi = 0.5$. These plots show the coupling relationship between the vibration of rotor and pads. The vibrations of the pads have high coupling with the principal vibration of the journal. However, the principal vibration of a pad is coupled loosely with other vibrations.

For $m_p = 1.0$ kg, $\Psi = 0.5$, and $\Omega = 1000$ rpm, Figs. 9 and 10 show that the variations of damping ratios and frequencies for different preload coefficients. These pictures show that the system will easily lose its stability when it is in lower preload range and that the preload has a great influence on the higher modal frequencies.

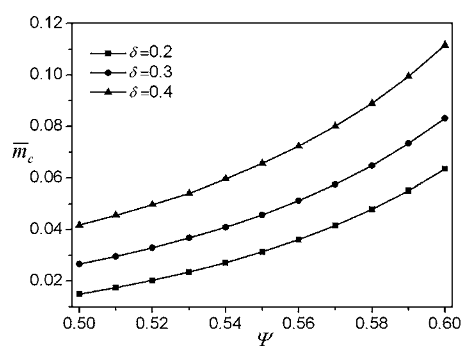

Fig. 4 Nondimensional pad critical mass $\bar{m}_c$ versus Ψ (a) (Ω =15,000 rpm)

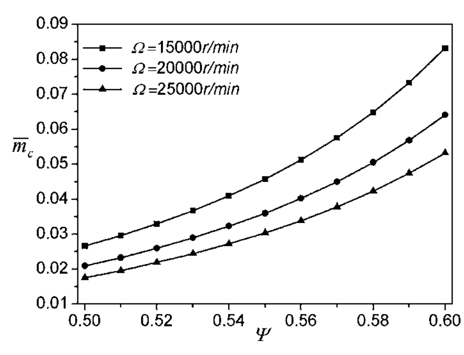

Fig. 5 Nondimensional pad critical mass $\bar{m}_c$ versus Ψ (b) (δ =0.3)

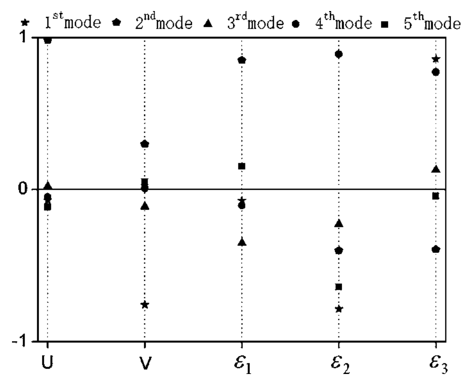

Fig. 7 The real portion of complex mode shapes (δ=0.2, Ω =10,000 rpm)

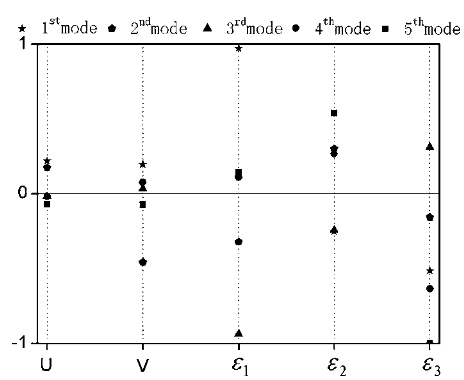

Fig. 8 The imaginary portion of complex mode shapes (δ =0.2, Ω=10,000 rpm)

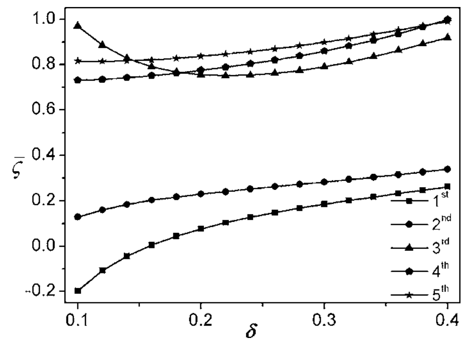

Fig. 9 Modal damping ratios versus δ (Ω=10,000 rpm)

Figures 10–12 illustrate the variations of the damping ratios and frequencies versus rotational speed in cases of δ=0.2. Notice that the stability of the system will be threatened as the rotational speed increases. The results presented in these figures support the conclusions shown by previous figures.

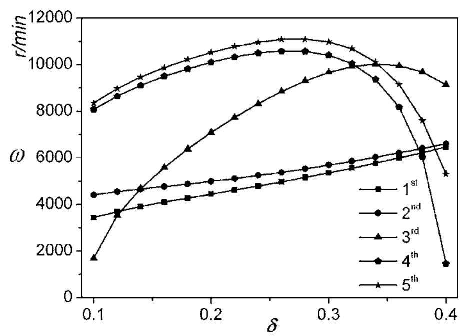

Fig. 10 Natural frequencies versus δ (Ω=10,000 rpm)

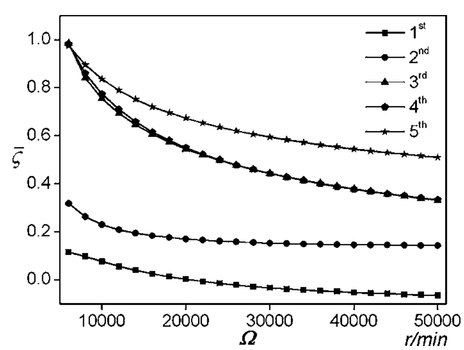

Fig. 11 Damping ratios versus Ω (δ=0.2)

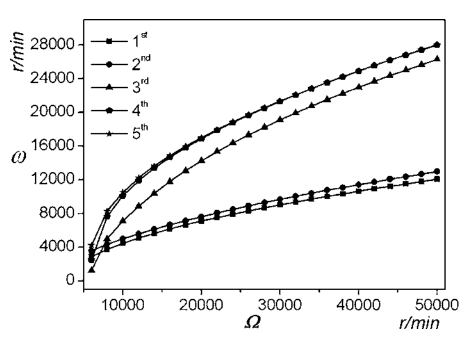

Fig. 12 Natural frequencies versus Ω (δ=0.2)

Conclusion

In this paper a model has been presented to study the linear stability of a tilting-pad journal bearing. By making a series of coordinate transformations, the stiffness and damping matrices of the system were obtained from the dynamic coefficients of each subsystem consisting of the rotor and a single pad. After attaining the stiffness and damping matrices, the perturbation equations of the system were derived by linearizing the oil-film forces. In the numerical analysis, the Newton-Raphson method was used to find the static equilibrium position of each pad and the dynamic coefficients of each subsystem were obtained simultaneously. Compared to models developed in the past years, in this model the mass of the pad as well as the pad degrees of freedom are taken into account. As an example, a three-tilting-pad journal bearing system with a rigid rotor was investigated. The following conclusions were obtained from the reults.

The stability of the system as well as the pad critical mass is greatly influenced by the preload. As the value of the preload increases, both improve. Therefore, correctly adjusting the pivot position is beneficial to enhance the stability of the system. If the principal vibration is the rotor's, there is a strong coupling relationship between the rotor and the pads. The system more easily loses its stability when the preload is lower or when the rotational speed is larger.

Nomenclature

A	=	pad center
C	=	journal center
R_J	=	shaft radius
R_A	=	outside radius of pad
R_B	=	inside radius of pad
R_p	=	radius of pivot's circle
O	=	bearing center
$c=R_B-R_J$	=	machined pad clearance
c/R_B	=	clearance ratio
m_b	=	half-mass of the rotor
m_i	=	mass of the pad
m_c	=	the critical mass of the pad
Ψ	=	pivot position
Ω	=	rotational speed
$\hat{\beta}$	=	pad arc angle
δ	=	preload coefficient
ε	=	displacement of the pad center
$\hat{\lambda}$	=	length to diameter ratio
μ	=	oil viscosity of lubricant
ϕ	=	pad pitch angle

Appendix

Oil-Film Forces. The calculation of oil-film pressure distribution p is based on the following Reynolds equation

$$\frac{1}{R_J}\frac{\partial}{\partial\psi}\left(\frac{h^3}{12\mu}\frac{\partial p}{\partial\psi}\right)+\frac{\partial}{\partial z}\left(\frac{h^3}{12\mu}\frac{\partial p}{\partial z}\right)=-\frac{\Omega}{2}\frac{\partial h}{\partial\psi}-\frac{\partial h}{\partial t} \quad \text{(A1)}$$

where

$$h=c-\xi\cos\psi-\eta\sin\psi \quad \text{(A2)}$$

the Reynolds boundary condition is

$$p=\frac{\partial p}{\partial n}=0 \text{ on } \Sigma^{+} \quad \text{(A3)}$$

where ψ is circumferential coordinate, z the axial coordinate, h is the oil-film thickness, t is the time, Σ^{+} is the boundary where cavitation takes place, and n is the outward normal vector to the boundary Σ^{+} (Fig. 13).

Arising from the pressure, the oil-film forces generated by a single pad can be obtained by

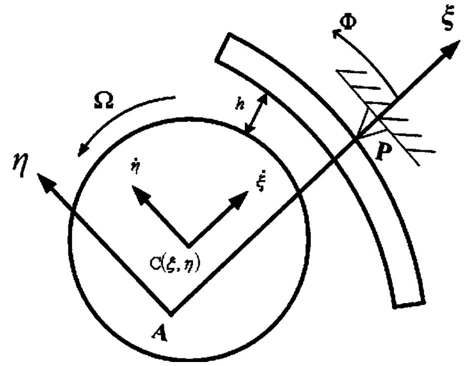

Fig. 13 Coordinates of a pad and journal in a tilting-pad journal bearing system

$$F_{\xi} = -\int\int_{\sigma} P(\psi,z)\cos\psi\, d\psi\, dz,$$

(A4)$F_{\eta}=-\int\int_{\sigma}P(\psi,z)\cos\psi\, d\psi\, dz$ where σ is the oil-film region. According to the Reynolds equation and its boundary condition, by using a variational method, the oil-film forces and their Jacobian matrices can be solved simultaneously. The detailed calculating process of the oil-film forces has been given by Klit and Lund [10].

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