## Erik J. Salisbury*

K. Vinod Domala Kee S. Moon Michele H. Miller
John W. Sutherland
Department of Mechanical EngineeringEngineering Mechanics, Michigan Technological University, Houghton, MI 49931

# A Three-Dimensional Model for the Surface Texture in Surface Grinding, Part 1: Surface Generation Model 

A geometric-kinematic model for the generation of surface topography in a single-pass surface grinding process is presented. The model incorporates the effects of process parameters such as table speed and wheel speed and also takes into account the wheel topography and original workpiece surface texture. Comparisons between modelsimulated output and experimental results show a good match, supporting the validity of the model. [DOI: 10.1115/1.1391427]

## Introduction

The grinding process is one of the most common finishing operations. Although it has been a topic of extensive research, a complete understanding of the process has yet to be achieved. The inherent nature of the grinding wheel (i.e., the random distribution of the grits and the inability to completely characterize the grit cutting edge geometry) is regarded as the primary obstacle to describing the surface generation process. The complex geometry of the wheel complicates the kinematic analysis of the process.

Many variables contribute to ground surface texture. These factors include geometric or primary factors, noise or secondary factors and miscellaneous or tertiary factors as shown in Fig. 1.

The geometric factors include the cutting parameters such as wheel speed and table speed, workpiece geometry including initial surface texture and form errors, and grinding wheel topography characteristics such as grit size, wheel dressing and wear. The intrinsic nature of these factors in the surface generation mechanism distinguishes them from the secondary and tertiary factors which may or may not be present in the process. The noise factors are disturbances in the grinding environment and are not always significantly involved in the cutting process.

Surface finish is an important functional feature and quality characteristic of manufactured products. It has an important role in the wear, lubricating and optical properties of the manufactured products. Understanding how the grinding wheel topography is mapped onto the workpiece provides guidance for producing surfaces with desired characteristics. Two-dimensional analyses do not reveal the complete structure of the ground surface which is necessary in understanding the impact that surface finish has on a part's functionality. A three-dimensional study provides a more comprehensive understanding of the impact that surface finish plays in the functional properties of the product.

In this paper, a mechanistic model is developed for the single pass surface grinding process based on the structure of the grinding wheel, cutting conditions and workpiece geometry. The model yields a three-dimensional topographic map of the ground surface. The model can be easily extended to a multi-pass grinding process by feeding back the predicted surface of one pass to subsequent passes. The model also provides an understanding of how the grinding wheel texture is mapped onto the workpiece surface.

The remainder of Part 1 is organized into four sections. Following a summary of the relevant grinding research, a detailed de-

[^0]scription of the model development is presented. The next section details the experimental studies that were done to validate the model. The final section of Part 1 presents some conclusions. Part 2 of this paper will describe a modeling procedure for characterizing grinding wheel texture that can be used to supplement the surface texture model developed in Part 1.

## Literature Review

Research on surface roughness in grinding has yielded several models, each of these taking into account the structure of the wheel in a different manner. Modeling work by Chen et al. [1] considered the grain characteristics on the wheel in a onedimensional sense by taking into account the grain distance, width of the cutting edge and the grain diameter. Modeling efforts involving the prediction of parameters such as $R_{a}$ and $R_{q}$ associated with the ground surface, were done by Nakayama and Shaw [2]. However, a uniform spacing between the grains was assumed by these researchers. In reality, grain distribution is much more complicated. Reichenbach et al. [3] incorporated the two-dimensional nature of the grain distribution by considering grain count and width of cut vs. depth of cut. Many measurements were required to determine empirical parameters and coefficients in this model. Yoshikawa and Sata [4] developed a three-dimensional simulation procedure for the grinding process using Monte Carlo simulation techniques. They assumed a uniform distribution to describe the grain spacing in the axial and peripheral directions, an assumption which is not supported by experimental evidence. According to


Fig. 1 Classification of factors affecting ground surface texture

McCool [5], actual distributions of grain summits are not usually symmetrical, but exhibit negative skewness, wherein the valleys are deeper than the summits are high. König and Lortz [6] have illustrated the effect of increasing the number of active cutting edges, i.e., using a finer grit wheel, on the surface quality. The increased quality of the surface when using finer grits may be due to the smaller cutting edges in the axial direction of the wheel that produce thinner chips. König and Steffens [7] numerically simulated surface generation in surface grinding and analyzed the effect of node spacing in the table speed direction. Based on a series of axial wheel profiles, Bhateja [8] predicted 2-D workpiece surface profiles using an "enveloping profile approach." His analysis showed how a rough wheel can produce a smooth ground surface. Tönshoff et al. [9] reviewed the merits and deficiencies of a few of the surface roughness and topography models.

Work in grinding process modeling includes that of Law et al. [10], who suggested a modeling procedure where model parameters describing the distribution of the heights of grain summits and grain apex-angle were estimated. They proposed the estimation of model parameters through sample spectra of experimentally ground surfaces. They also classified grinding models as mechanistic, empirical-mechanistic and empirical. Using a semiempirical model for the grinding wheel, Cooper and Lavine [11] developed a numerical simulation that calculated grain depths of cut, grinding zone shape, percentage of active cutting grains, etc.

Research in the wheel and surface characterization area include the efforts of Peklenik [12] and Dong et al. [13]. Whitehouse [14] used beta functions to classify surface textures while Ramamoorthy and Radhakrishnan [15] characterized a few of the features of texture produced by various machining operations. Ito et al. [16] studied the depth distribution of cutting edges on a dressed grinding wheel, and considered the relation between the dressing feed and patterns produced by it. Chen and Rowe [17] simulated wheel surface generation by single point diamond dressing. Bhateja [18] proposed a parameter called successive cutting profile contribution to quantify the functional parameters of the grinding wheel. Saini [19] reviewed the efforts of several researchers to describe local elastic deflections of grinding abrasives. Such deflections result in an increase of the number of active cutting grains.

Kaliszer and Trmal [20] studied the surface topography generation for a conventional plunge grinding process, but only in twodimensions. Adequate work with regard to surface generation in grinding process has not been done in a three-dimensional sense. However, in other processes such as end-milling, the generation of three-dimensional surface texture has been modelled by Babin et al. [21]. The need for such a model with regard to grinding has been previously discussed.

## Model Development

The geometric surface generation model predicts the 3-D topography of a ground surface given the 3-D topography of the grinding wheel and the cutting conditions. The model maps the motion of the cutting tool (i.e., the grinding wheel surface) with respect to the workpiece and determines the resultant surface texture. The model input includes numerical values for the spindle RPM $N$, the table speed $f$, depth of cut $d$, and a 2-D array of grinding wheel surface heights $h_{i j}$, where $i$ and $j$ are indices for individual elements in the array. For an experimentally obtained grinding wheel surface map, the indices $i$ and $j$ can be made to coincide with the trace direction of the stylus instrument and the direction of the parallel traces respectively. The output of the model is another 2-D array of surface heights, this time representing the surface texture of the ground surface.

Figure 2 shows a schematic of the surface grinding process on which the model is based. Figure 2(a) shows the position of the wheel, the workpiece and the model parameters associated with them at an initial time, $t_{0}$. Figure 2(b) shows a snapshot of the same process at $t_{1}$, where $t_{1}=t_{0}+\Delta t$. The model uses two frames of reference $\{\mathbf{O}\}$ and $\{\mathbf{W}\}$ that can be seen in Fig. 2. The
origin of frame $\{\mathbf{O}\}$ is fixed in space (i.e., no machine-tool vibration) at the center and front of the grinding wheel. The origin of frame $\{\mathbf{W}\}$ is located at the front left end of the workpiece and is being translated with respect to frame $\{\mathbf{O}\}$ by the motion of the work table. These frames are initially separated by distances $y_{0}$ and $(R-d)$ along the $\mathbf{Y}$ and $\mathbf{Z}$ axes respectively, where $y_{0}$ is an arbitrary starting distance, $R$ is the nominal radius of the grinding wheel and $d$ is the depth of cut. The workpiece surface texture is being created on the workpiece in frame $\{\mathbf{W}\}$, by the grinding wheel texture $h_{i j}$ that is rotating about $\{\mathbf{O}\}$ while $\{\mathbf{O}\}$ is translating with respect to $\{\mathbf{W}\}$. The grinding wheel array, $h_{i j}$, has indices $i$ and $j$, where $i$ is in the axial direction of the wheel or $X$ direction and $j$ corresponds to a position on the grinding wheel's circumference. The angle $\theta_{j}$ describes the angular position of the $h_{i j}$ points in the grinding wheel array. The distances in the $X, Y$ and $Z$ directions from the workpiece frame $\{\mathbf{W}\}$ to any point $h_{i j}$ on the grinding wheel surface are $x, y$ and $z$, respectively.
Consider a point on the wheel $h_{i j}$ at an initial angle $\theta_{0 j}$ before the wheel starts rotating, i.e. at time instant $t_{0}$. At time $t_{1}$, after some $\Delta t$, the same point makes an angle $\theta_{1 j}=\theta_{0 j}-\omega \Delta t$, where $\omega$, the angular speed of the grinding wheel, is given by $2 \pi N$. The position of a point on the surface of the grinding wheel in the $X_{\{\mathbf{W}\}}$ direction is simply equal to the $i$ index while the positions in the $Y_{\{\mathbf{W}\}}$ and $Z_{\{\mathbf{W}\}}$ directions depend on $\theta_{1 j}$ and the relative position of the frames of reference at $t_{1}$.
At any instant of time, the position of any point on the wheel surface with respect to the frame $\{\mathbf{W}\}$ origin is described by the following set of relations:

$$
\begin{gather*}
x=i  \tag{1}\\
y=y_{0}-f \cdot \Delta t+\left(R+h_{i j}\right) \sin \left(\theta_{0 j}-\omega \Delta t\right)  \tag{2}\\
z=R-d-\left(R+h_{i j}\right) \cos \left(\theta_{0 j}-\omega \Delta t\right) \tag{3}
\end{gather*}
$$

where,
$i=$ wheel height array index in the radial direction,
$j=$ wheel height array index in the circumferential direction,
$h_{i j}=$ height of the grinding wheel surface array at element (i,j),
$\theta_{0 j}=$ initial angle of the $j$ th row of the grinding wheel array,
$=\pi / 2$ radians for $i=1 \& j=1$,
$y_{0}=$ initial distance between the $\{\mathbf{O}\}$ and $\{\mathbf{W}\}$ origins in the $Y$ direction,
$R=$ nominal radius of the wheel.
Having formulated the relations as a function of time, it would appear on a cursory glance that 3-D topography of the ground surface can be determined by incrementing time, and generating the resultant surface texture using Eqs. (1-3) in a time basis. However, due to the geometry of the process and the desired output surface represented as a 2-D array, this is not the case. The following discussion explains the problems involved in determining the time evolution of the surface and the solutions.

The surface topography of the workpiece surface consists of an array of points with varying surface heights (similar to how the grinding wheel topography is represented). Equations (1-3) define the position of a point on the grinding wheel after time $\Delta t$, with respect to the original position. A point on the surface of the workpiece is altered by a grinding wheel array element $h_{i j}$, only if the wheel array element has a $z$ value (from Eq. (3)) less than the $Z_{\{W\}}$ value of the workpiece at that position. Therefore, as $\Delta t$ increases and the wheel array passes over/through the workpiece, the $Z_{\{W\}}$ coordinates of the workpiece due to some wheel array points $h_{i j}$ must be compared to the $Z_{\{\mathbf{W}\}}$ coordinates due to previous wheel array points. Since the desired output of the model is a 2-D workpiece array, the $Z_{\{\mathbf{W}\}}$ heights for specific and predetermined values of $x$ and $y$ must be calculated. Therefore, for each $h_{i j}$ that will impact some $(x, y)$ workpiece location, a calculation is done to determine $\Delta t$. The time $t_{0}+\Delta t$ is the instant at which a grinding wheel data array element $h_{i j}$ has the same $y$ location as a point


Fig. 2 A Schematic of the surface grinding process
in the workpiece array. The calculated $\Delta t$ is then used to update the workpiece surface array using Eq. (3). The final ground surface, therefore, is the envelope of all the updated $Z_{\{\mathbf{W}\}}$ coordinates at all workpiece array elements $(x, y)$. An updating technique of a similar nature was used for the creation of end-milled surfaces by Babin et al. [21].

A simulation program was coded in FORTRAN that uses the model Eqs. (1)-(3) according to the flowchart shown in Fig. 3, to calculate the final surface topography of the workpiece. Equation (2) is used to determine the $\Delta t$ value for a given $h_{i j}$ and ( $x, y$ ) combination. Since Eq. (2) is nonlinear, a Newton-Raphson routine is used to estimate $\Delta t$. In this routine, an initial guess value for time, $\Delta t$, is specified, which is then used to iteratively estimate $\Delta t$ until the error in estimation approaches a predetermined limit. The error convergence criterion is set at, $\varepsilon \leqslant 1 \times 10^{-10}$ for the estimation of $\Delta t$ in the program. The value of estimated $\Delta t$, tnew, is then used to find znew (the wheel-point position along $Z_{\{\mathbf{W}\}}$. according to Eq. (3)). The height of the workpiece-point along $Z_{\{\mathbf{W}\}}, z \operatorname{surf}$, at the same $(x, y)$ position, is then compared to znew. The updating is done based on the logic discussed previously.

The model inputs are the wheel data, cutting conditions and the desired $x$ and $y$ spatial resolution for the simulated ground surface. Wheel data that is input to the program may be obtained either by measurement or by simulating data from a wheel topography char-
acterization model (to be described in Part 2 of this paper). Threedimensional surface data measured from a $7 \times 1 / 2 \times 1-1 / 4 \mathrm{in}$. wheel (type: 32A120-I6VBE) was used for model simulation and verification in this paper. Data was sampled using a Mahr-Perthen profilometer interfaced with an IBM-PC. The diamond probe tip has a $3 \mu \mathrm{~m}$ radius and 82.5 degree included angle. A plot of the grinding wheel surface is shown in Fig. 4. The $R_{a}$ and $R_{\max }$ for the wheel surface (for the whole area shown in Fig. 4) are approximately 28 microns and 180 microns, respectively.
The program begins by calculating the initial angle $\theta_{0 j}$, that each point on the wheel makes with respect to the origin of $\{\mathbf{O}\}$ and the distance between the data points and the origin of $\{\mathbf{O}\}$ i.e., $R+h_{i j}$. Relative motion is then introduced between the wheel and the workpiece, and the program proceeds through the following two routines:
(1) Newton-Raphson routine, and
(2) the surface-updating routine.

The output of the program is a matrix of updated $Z_{\{\mathbf{W}\}}$ coordinates (znew), which may then be plotted or used to calculate the various roughness parameters associated with the surface. For simulating a multi-pass grinding operation, the output of the program, i.e., the workpiece surface generated by the program can be fed back into the simulation software as the initial workpiece surface texture.


Fig. 3 Flowchart of the simulation model

Fig. 4 Measured topographic map of grinding wheel surface

## Model Verification

For the purpose of model verification, a grinding experiment was conducted with a wheel speed of 2900 rpm and a table speed of $0.373 \mathrm{~m} / \mathrm{sec}$ on a Brown \& Sharpe-Micromaster surface grinding machine. The choice of experimental conditions for verification of this model is crucial - they must emulate ideal conditions (i.e., conditions without secondary and tertiary factors influencing the process), since the model provides the output of such a process. Aluminum was chosen as the workpiece material so that forces would be low thus minimizing vibrations and wheel grit deflection. The sample was polished and the depth of cut was set at 200 microns, large enough to remove any previous surface texture. A semi-synthetic water soluble cutting fluid was used during grinding. The wheel was not trued. The ensuing runout limited the active cutting region of the wheel to a size comparable to that used in the simulation. The surface resulting from one upgrinding pass of the grinding wheel was measured with a profilometer and is shown in Fig. 5. The $R_{a}$ and $R_{\max }$ for the experimentally ground surface are 15 microns and 67 microns, respectively.

A simulation was performed with the same set of cutting conditions and using data from the same wheel. The initial geometry of the workpiece was specified as an array matrix with all its elements as zeros. Updating is done by comparison between the coordinates on the workpiece and the coordinates of the wheel points. The plot of the simulated surface is shown in Fig. 6. The direction of lay is evident in both the simulated surface and the


Fig. 5 Measured topographic map of workpiece surface


Fig. 6 Topographic map of simulated workpiece surface


Fig. 7 Power spectrum of simulated and machined surface profile


Fig. 8 Wheel profile \& simulated workpiece profile
experimentally generated surface. The $R_{a}$ and $R_{\max }$ for the simulated surface are 13 microns and 59 microns, which are comparable to the experimentally ground surface. It may be noted that the part of the wheel involved in the actual cutting is not necessarily the measured portion of the wheel. It was assumed that the measured portion of the wheel is representative of the general characteristics of the wheel.

To further verify the model, the frequency components of the simulated surface and the experimentally ground surface were compared. A plot of the power-spectrum of the profile in the axial direction of both the surfaces is shown in Fig. 7. As can be seen from the figure, these surfaces exhibit a nearly identical frequency content; most of the dominant frequencies are aligned for both the surfaces, indicating that the simulated surface is a good representation of the experimentally ground surface.

It is to be noted that the surface finish is the result of many more phenomena than the model accounts for, viz., the effects of wheel wear, thermal damage, the presence of coolant, the plowing action of the grits and the dynamics of the machine tool. The effects of these secondary and tertiary factors is evident in the experimentally ground specimen. However, the primary factors seem to have a dominant role in the surface creation process and each of the above secondary and tertiary phenomena has only a role in altering the geometry created by the said primary inputs. Moreover, once each of the secondary and tertiary phenomena has been characterized, it may be incorporated into the model.

In the example above, the surface is created by the highest points (points with the maximum $Z$ coordinates) of the wheel at each axial position. A close look at the simulation results reveals that the number of active cutting points is very small. Figure 8 shows a single simulated surface profile (in the table speed direction) along with the grinding grits that caused the final surface texture. It can be seen that only the material removal action of five grits results in the final surface texture.

In this experiment and simulation the active cutting region was small. Truing the grinding wheel to create a larger active cutting region would reduce the surface roughness. Simulating a larger cutting region is difficult due to the large amount of wheel topographic data that would have to be collected. This problem can be resolved by utilizing a model based grinding wheel approach, such as the one described in Part 2, as opposed to the measured data approach.

## Summary and Conclusions

A model for generating the surface topography based on the cutting conditions, wheel topography and initial workpiece geometry has been developed. The output of the model shows a good match with the experimentally ground surface. It is possible to calculate and predict roughness parameters such as $R_{a}, R_{q}$ and $R_{\text {max }}$ associated with the generated surface. Further, information acquired from the simulated surface may be used to obtain various
other statistical parameters such as the profile height distribution, spectral plots and autocorrelation functions. This 3-D model can also be used for friction, lubrication and wear studies.

The model is capable of handling a wide combination of work and wheel speeds, and is flexible enough to incorporate the effects of secondary and tertiary variables once they have been characterized. Simulations may be run to study the effects of various levels of table speed and wheel speed and wheel types on the surface finish without doing actual grinding.

Further research needs to be done on the characterization of the wheel topography. The role of vibration, cutting fluid, workpiece material, wheel wear, and grit deflection in altering the surface finish of the ground surface need to be studied not only by considering the variables independently, but also by taking into account their interactive effect. Part 2 of this paper will describe a modeling procedure for efficiently characterizing grinding wheel texture that can be used in conjunction with the surface texture model developed in this part of the paper.

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[^0]:    *Currently at Varatech Engineering Consultants in Holland, MI.
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