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ACCURATE AND TOTAL ACCURATE DOMINATING SETS OF INTERVAL GRAPHS

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ABSTRACT

Interval graphs have drawn the attention of many researchers for over 30 years. They are extensively studied and revealed their practical relevance for modeling problems arising in the real world.

In this paper we discuss various cases in which the dominating set constructed by the algorithm becomes an accurate dominating set, total accurate dominating set and also the cases where it is not an accurate dominating set and a total accurate dominating set.

Keywords: Algorithm, Dominating Set, Accurate Dominating Set, Total Accurate Dominating Set, Clique, Interval Graph.

Subject Classification: 68R10

1. INTRODUCTION

The theory of domination in graphs introduced by Ore [1] and Berge[2] is an emerging area of research in graph theory today. The concept of accurate domination and total accurate domination was introduced by Kulli and Kattimani [3]. They have studied this concept for various standard graphs and obtained bounds.

A Dominating set D of a graph G(V, E) is called an accurate dominating set if $< V – D >$ has no dominating set of cardinality $|D|$. The accurate domination number γ_a is the number of vertices in a minimum accurate dominating set of G. A total dominating set T of G is called a total accurate dominating set if $< V - T >$ has no total dominating set of cardinality $|T|$. The total accurate domination number γ_{ta} is the number of vertices in a minimum total accurate dominating set of G, where G has no isolated vertices.

2. INTERVAL GRAPH

Let I = {1,2,……...,n} be an interval family where each i in I is an interval on the real line and i = [a_i, b_i] for i = 1, 2,…. n. Here a_i is called the left endpoint and b_i is called the right endpoint of i. Without loss of generality, we assume that all endpoints of the intervals in I are distinct numbers between 1 and 2n. The intervals are labeled with respect to their right end points. Two intervals i and j are said to **intersect** each other if they have non-empty intersection.

A graph G(V, E) is called an interval graph if there is a one-to-one correspondence between V and I such that two vertices of G are joined by an edge in E if and only if their corresponding intervals in I intersect. We construct a dominating set given by the following algorithm. The dominating set becomes minimum [4].

3. ALGORITHM : MDS - IG

Input	:	Interval family I = { 1,2,…………n}.
Output	:	Minimum dominating set of the interval graph G.
Step 1	:	Let S = {max (1)}.
Step 2	:	LI = The largest interval in S.
Step 3	:	Compute Next (LI).
Step 4	:	If Next (LI) = null then go to step 8.
Step 5	:	Find max(Next (LI)).
Step 6	:	If max(Next (LI)) does not exist then max(Next (LI)) = Next (LI).
Step 7	:	S = S ∪ max(Next (LI))) go to step 2.
Step 8	:	End.

4. ACCURATE DOMINATING SETS

We assume that G is always a connected graph. Also D is the minimum dominating set of G constructed by the above algorithm .

Theorem 1 : Let i and j be any two intervals in I such that $i \in D$, $j \subset i$ and there is no interval k in I such that k intersects j. Then D becomes an accurate dominating set of G.

Proof : Let D be the minimum dominating set of G constructed by the algorithm. Let $i \in D$ and $j \in I$ such that i, j satisfy the hypothesis of the theorem. Then clearly in < V - D >, j becomes an isolated vertex. Thus the dominating set of < V - D > must include the vertex j. In general $\gamma\ (V - D) \geq \gamma(G)$ for any subset D of V. Hence it follows in this case that $\gamma\ (V - D) \geq \gamma(G) + 1 > \gamma(G)$

Thus D becomes an accurate dominating set of G.

Theorem 2 : Let D = {$x_1, x_2 \ldots x_m$} be such that < N[x_1] –u_1 >, < N [x_2] –u_2 > ... < N[x_m] > are cliques, where u_1, u_2 … u_{m-1} are the last vertices dominated by x_1, x_2 … x_{m-1} respectively. If u_1, u_2 … u_{m-1} are also the first vertices in N[x_2], N[x_3], …N [x_m] respectively, then D is not an accurate dominating set of G.

Proof : Since N[x_1] is a clique, if we remove x_1 from N[x_1], then it is obvious that N (x_1) is also a clique. Similar is the case with the other N[x_i]'s.

By the construction of D, it is clear that x_1, x_2……x_m are the last vertices in their respective cliques. Since $N(x_i)$s are cliques, any set of m vertices from these $N(x_i)$s respectively form a dominating set for < V - D >. Hence the dominating set of < V - D > contains exactly m vertices so that D cannot be an accurate dominating set.

Theorem 3 : Let $D = \{x_1, x_2 \ldots x_m\}$ where $u_1, u_2, \ldots u_m$ be the first vertices dominated by $x_1, x_2 \ldots x_m$ respectively. Suppose $< \{u_1 \ldots. x_1\} >$, $<\{ u_2 \ldots x_2\}>$, …. $<\{ u_m \ldots x_m\}>$ are cliques. If there are vertices between x_{i-1} and u_i where $i = 2,3, \ldots m$, then D is not an accurate dominating set.

Proof : Let $x_1, x_2 \ldots x_m$, $u_1, u_2 \ldots u_m$ satisfy the hypothesis of the theorem.
Then two cases arise.

Case 1 : Suppose there is a vertex between x_{i-1} and u_i say v_i, $i \neq 1$. Since G is connected we have x_{i-1} and v_i are adjacent and also u_i and v_i are adjacent. Then as in the proof of Theorem 2, in this case also < V - D > contains a dominating set of m vertices only. But not as in Theorem 2 that any set of m vertices from $N(x_i)$'s respectively forms a dominating set of < V - D >. We specify the dominating set of < V - D > as follows. Consider $D_1 = \{x_1\text{-}1, u_2, u_3, \ldots.u_m\}$ in < V - D >.

Since $x_1 \notin < V - D >$, the vertices in $N(x_1)$ are dominated by x_1-1 and v_2 is dominated by u_2. So x_1-1 and u_2 are included in a dominating set of < V - D >. Now $u_2 \in N(x_2)$ and so u_2 dominates the rest of vertices in $N(x_2)$. Since $x_2 \notin < V - D >$, the vertex v_3 is now dominated by u_3 and $u_3 \in N(x_3)$ and u_3 dominates the rest of the vertices in $N(x_3)$. Thus u_3 is included in a dominating set of < V - D >. The argument goes on like this and ultimately D_1 becomes a dominating set of < V - D > which contains exactly m vertices. So D cannot be an accurate dominating set of G.

Case 2 : Suppose there is one more vertex between u_i and v_i say w_i such that x_{i-1}, v_i, w_i, u_i, $i \neq 1$ are consecutive vertices which are adjacent consecutively. Then if we consider this interval family and construct the dominating set in accordance with the algorithm, then u_i enters into dominating set which is not the case. Hence this case does not exist, if D is given as per the hypothesis.

Remark : Suppose some vertices in $N(x_i)$ are adjacent with some vertices in $N(x_j)$, $i \neq j$ or no vertex in $N(x_i)$ is adjacent to a vertex in $N(x_j)$, $i \neq j$. In both the cases, we can show that the dominating set of < V - D > contains only m vertices, so that D cannot be an accurate dominating set of G.

Theorem 4 : Let $D = \{x_1, x_2 \ldots.. x_m\}$ be such that $<\{ u_1 \ldots\ldots x_1\}>$, $<\{ u_2 \ldots\ldots x_2\}>$,…… $<\{u_m \ldots\ldots x_m\}>$ are cliques, where $u_1, u_2, \ldots, u_m$ be the first vertices dominated by $x_1, x_2 \ldots.x_m$ respectively. Suppose there are vertices between x_{i-1} and u_i where $i \neq 1$. Then D is an accurate dominating set of G if all the intermediate vertices between x_{i-1} and u_i are dominated by x_{i-1}.

Proof : Let $x_1, x_2 \ldots\ldots x_m$, $u_1, u_2 \ldots\ldots u_m$ satisfy the hypothesis of the theorem.
Suppose there are intermediate vertices between x_{i-1} and u_i, the first vertex in the clique $N[x_i]$. By hypothesis, x_{i-1} dominates all these vertices. Let these vertices be $w_1, w_2 \ldots w_k$ say where $x_{i-1} < w_1 < w_2 < \ldots < w_k < u_i$. Since we have right endpoint labeling, x_{i-1} dominates w_k implies that w_k is adjacent to $w_1, w_2 \ldots w_{k-1}$. That is w_k dominates all these vertices.

Consider < V - D >. Since $N(x_i)$s are cliques any set of m vertices from these $N(x_i)$s dominate the vertices in these $N(x_i)$'s respectively. Hence if we consider the set $D_1 = \{ x_1\text{-}1, x_2\text{-}1, \ldots x_{i-1}\text{-}1, w_k, x_i\text{-}1, \ldots x_m\text{-}1\}$ then clearly D_1 is a dominating set of < V - D > . Further < V - D > has no dominating set of cardinality lower than m+1 , because, from the m cliques, a set of m vertices (which is minimum) are chosen and a single vertex w_k, which dominates all intermediate vertices between x_{i-1} and u_i which were not dominated by any of these m vertices is included. So $|D_1| = m+1$ whereas $|D| = m$.

Therefore D becomes an accurate dominating set of G.

Remark : For the interval family, considered in the above theorem, there may be a possibility that $< V - D >$ has isolated vertices. Also the hypotheses of Theorems 2 and 3 is satisfied between x_{i-1} and u_i for any i but not for all i. But for at least one i, the hypothesis of Theorem 4 must be satisfied. For all such possibilities we get an accurate dominating set.

Theorem 5 : Let $D = \{x_1, x_2 \ldots\ldots x_m\}$ and $< V - D >$ has no isolated vertices. Then D is not an accurate dominating set of G if and only if $\gamma_a = \gamma + 1$.

Proof : Let $D = \{x_1, x_2 \ldots\ldots\ldots\ldots x_m\}$ be the minimum dominating set of G constructed by the algorithm. Suppose D is not an accurate dominating set of G. Let $D_1 = \{x_1, x_2\text{-}1, x_2, x_3, \ldots\ldots\ldots\ldots x_m\}$. Clearly D_1 is a dominating set of G. In $< V - D_1 >$, consider the set $D_2 = \{x_1\text{-}1, x_2\text{-}2, x_3\text{-}1, \ldots\ldots\ldots x_m\text{-}1\}$. By the construction of dominating set D in G it is clear that the vertices in D_2 dominate the vertices in $< V - D >$. Since $< V - D >$ has no isolated vertices and $|D_1| = m+1$ and $|D_2| = m$, it follows that D_1 is an accurate dominating set of G. Hence $\gamma_a = m+1 = \gamma + 1$.

Conversely, let $\gamma_a = \gamma + 1$. Then it is obvious that D is not an accurate dominating set of G. Otherwise $|D| \geq \gamma_a = \gamma + 1$ which is not possible, since $|D| = \gamma$.

We can easily prove the following theorem.

Theorem 6 : Let G be an interval graph with n vertices such that G is a path from vertex 1 to vertex n. If n is a multiple of 3, then the minimum dominating set constructed by the algorithm becomes an accurate dominating set. That is $\gamma_a = \gamma$. Otherwise $\gamma_a = \gamma + 1$.

5. TOTAL ACCURATE DOMINATING SET

Theorem 7 : We assume that G is always a connected graph and G has no isolated vertices. Let i and j be any two intervals in G such that $j \subset i$ and there is no interval k in I such that k intersects j. Then G has no total accurate dominating set.

Proof : Let T be a total dominating set of G and i, j satisfy the hypothesis of the theorem. Then clearly $i \in T$ and in $< V - T >$, j becomes an isolated vertex. Therefore we cannot construct a total dominating set in $< V - T >$. Hence G has no total accurate dominating set.

Theorem 8 : Let $D = \{x_1, x_2 \ldots x_m\}$ be such that $< N[x_1]\text{-}u_2 >$, $< N[x_2]\text{-}u_3 > \ldots\ldots < N[x_m] >$ are cliques, where u_i is the first vertex dominated by x_i. Then G posseses a total accurate dominating set if there are no vertices between x_i and u_{i+1} for $i = 1,2,\ldots.m\text{-}1$.

Proof : Let $x_1, x_2 \ldots x_m$, $u_1, u_2 \ldots u_m$ satisfy the hypothesis of the theorem. Since G is a connected graph, x_i and u_{i+1} are adjacent. Since $< N[x_i] - u_{i+1} >$ is a clique, x_i dominates all vertices in this clique. Further, since u_m is the first vertex dominated by x_m, u_m dominates all vertices in $N[x_m]$. Therefore if we consider $T = \{x_1, u_2, x_2, u_3, \ldots.. x_{m-1}, u_m\}$, then clearly T is a total dominating set of G. Further

$|T| = 2m - 2$, since we have not included the vertices u_1 and x_m into T.

Consider < V-T >. Since $N[x_i]$s are cliques, $N(x_i)$ is also a clique. So < V - T > contains m components, each of which is a clique. In order to get a total dominating set in < V- T >, we have to choose two vertices from each such component which is also a clique, so that a total dominating set in < V - T > contains at least 2m vertices as there are m such components.

Therefore a total dominating set T_1 in < V - T > contains at least 2m vertices. That is $|T| \neq |T_1|$.

Hence T becomes a total accurate dominating set in G.

Theorem 9 : We assume that G is always a connected graph and G has no isolated vertices. Let $D = \{x_1, x_2 \ldots x_m\}$ be such that $< \{u_1, \ldots x_1\} >$, $< \{u_2 \ldots x_2\} >$ ……..$< \{u_m \ldots x_m\} >$ are cliques, where u_1, u_2 ……….u_m are the first vertices dominated by x_1, x_2 ……….x_m respectively. If there is one vertex between x_i and u_{i+1}, then G possesses a total accurate dominating set.

Proof : Let x_1, x_2 ……….. x_m, u_1, u_2 ……….. u_m satisfy the hypothesis of the theorem.

Case 1 : Suppose there is one vertex between x_i and u_{i+1} say v_i. Since G is a connected graph x_i, v_i are adjacent and v_i, u_{i+1} are adjacent. Consider $T = \{x_1, u_2, x_2, u_3 \ldots\ldots x_i, v_i, u_{i+1}, x_{i+1}, u_{i+2} \ldots\ldots x_{m-1}, u_m\}$. Since $< \{u_i \ldots\ldots x_i\} >$ is a clique, x_i dominates all vertices between u_i and x_i, including u_i. Further x_i and v_i are adjacent. Therefore T becomes total dominating set of G. Further $|T| = 2m - 1$, since we have excluded the vertices u_1, x_m and included the vertex v_i.

Consider < V - T >. Since $N[x_i]$s are cliques, $N(x_i)$ is also a clique. Then < V - T > contains m components, each of which is a clique. In order to get a total dominating set, we have to choose two vertices from each such component which is also a clique, so that a total dominating set in < V - T > contains at least 2m vertices as there are m such components. Therefore a total dominating set T_1 in < V - T > contains at least 2m vertices. That is $|T| \neq |T_1|$. Hence T becomes a total accurate dominating set in G.

Remark : Suppose there is one more vertex between x_i and u_{i+1} say w_i such that x_i, v_i, w_i, u_{i+1} are consecutive vertices which are adjacent consecutively. Then if we consider this interval family and construct the dominating set in accordance with the algorithm, then u_{i+1} enters into dominating set instead of x_{i+1}. Hence this possibility does not arise, if D is given as per the hypothesis.

ILLUSTRATIONS

Theorem 5

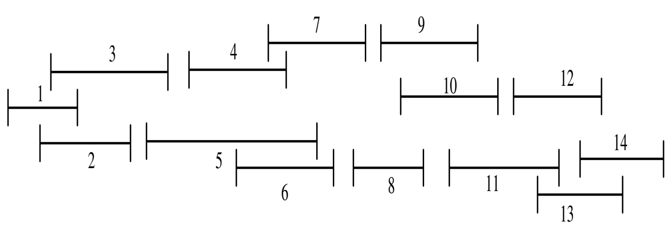

Fig. 1 : Interval Family I

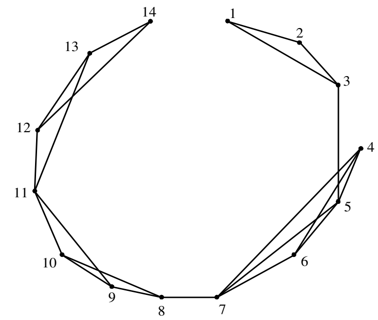

Fig. 2 : Interval Graph G

The dominating set according to the algorithm is D = {3, 7, 11, 14}
Here $x_1 = 3$, $x_2 = 7$, $x_3 = 11$, $x_4 = 12$ and $\gamma = 4$
Consider < V - D >. It is given by

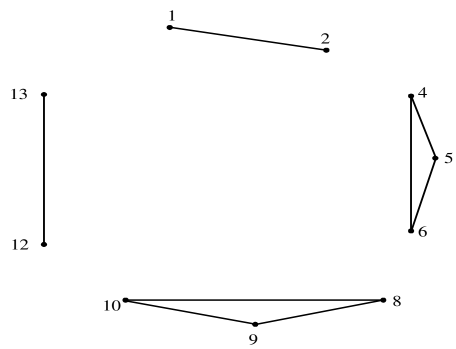

Fig. 3: Vertex induced subgraph < V - D >

The dominating set in < V - D > is D′ = {2, 5, 9, 12}
That is $|D| = |D'|$
Therefore D is not an accurate dominating set in G.
Let us take a dominating set with cardinality $\gamma + 1$
Consider $D_1 = \{3, 6, 7, 11, 14\}$
Here $x_1 = 3$, $x_2 - 1 = 6$, $x_2 = 7$, $x_3 = 11$, $x_4 = 14$
Clearly D_1 is a dominating set of G
Consider $< V - D_1 >$. It is given by

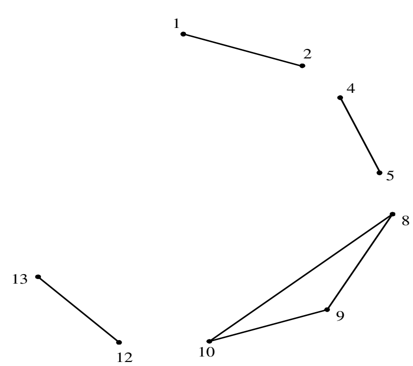

Fig. 4 : Vertex induced subgraph $< V - D_1 >$

The dominating set in $< V - D_1 >$ is $D_2 = \{2, 5, 10, 13\}$
Here $x_1 - 1 = 2$, $x_2 - 2 = 5$, $x_3 - 1 = 10$, $x_4 - 1 = 13$
That is $|D_1| \neq |D_2|$
Therefore D_1 becomes an accurate dominating set in G.
Thus $\gamma_a = |D_1| = 5 = 4 + 1 = \gamma + 1$.

Theorem 8

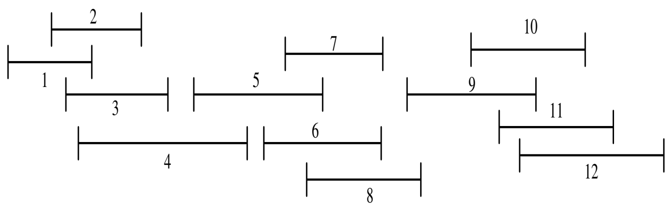

Fig. 1 : Interval Family I

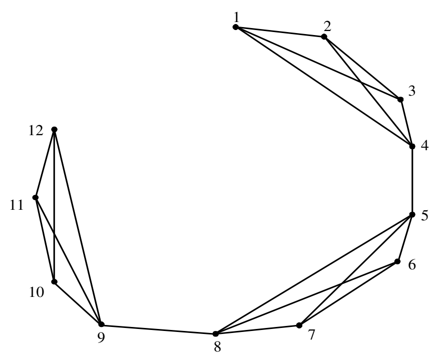

Fig. 2 : Interval Graph G

The dominating set according to the algorithm is $D = \{4, 8, 12\}$
Here $x_1 = 4$, $x_2 = 8$, $x_3 = 12$
$u_1 = 1$, $u_2 = 5$, $u_3 = 9$
The total dominating set is $T = \{4,5,8,9\}$
Consider $< V - T >$. It is given by

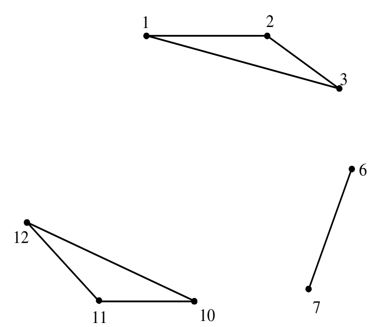

Fig. 3 : Vertex induced subgraph $< V - T >$

The total dominating set in $< V - T >$ is $T_1 = \{2, 3, 6, 7, 11, 12\}$
We get that $|T| \neq |T_1|$
Thus T becomes a total accurate dominating set in G.

Theorem 9

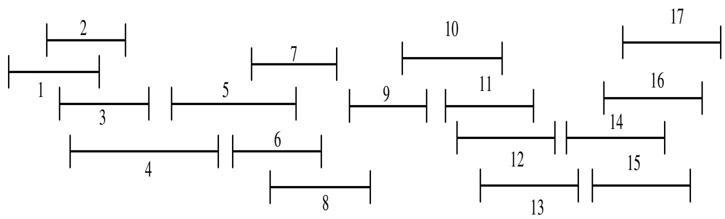

Fig . 1 : Interval Family I

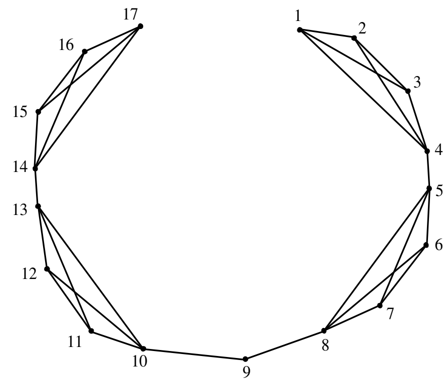

Fig. 2: Interval Graph G

The dominating set according to the algorithm is D = {4, 8, 13, 17}
Here $x_1 = 4, x_2 = 8, x_3 = 13, x_4 = 17$
$u_1 = 1, u_2 = 5, u_3 = 10, u_4 = 14, v_2 = 9$
The total dominating set is T = {4, 5, 8, 9, 10, 13, 14}

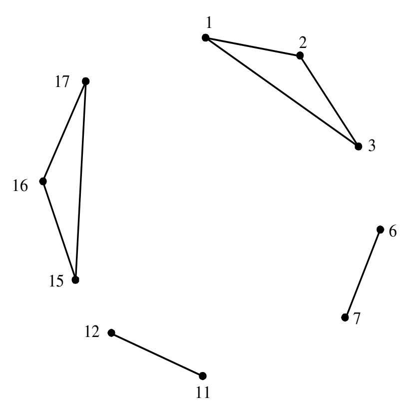

Fig. 3: Vertex induced subgraph < V - T >

Consider < V- T >. It is given by
The total dominating set in < V - T > is T_1 = {2, 3, 6, 7, 11, 12, 16, 17}
We get that $|T| \neq |T_1|$

Thus T becomes a total accurate dominating set in G.

REFERENCES

[1] Ore. O "Theory of Graphs", Amer. Math. Soc. Colloq. Publ. 38, Providence, (1962).

[2] Berge. C "Theory of Graphs and its applications", Methuen, London (1962).

[3] Kattimani. M.B. "Some New Contributions to Graph Theory", Ph.D. thesis, Gulbarga University, India (2000).

[4] Laksmi Naidu Y. "Some studies of Algorithms on Interval graphs and Circular –arc-graphs", Ph.D.thesis, S.P.Mahila Visvavidyalayam, India,(1999).

[5] M.Siva Parvathi and B.Maheswari, "Minimal Dominating Functions of Corona Product Graph of a Cycle with a Complete Graph", International Journal of Computer Engineering & Technology (IJCET), Volume 4, Issue 4, 2013, pp. 248 - 256, ISSN Print: 0976 – 6367, ISSN Online: 0976 – 6375.

[6] Makke Siva Parvathi and Bommireddy Maheswari, "Minimal Dominating Functions of Corona Product Graph of a Path with a Star", International Journal of Information Technology and Management Information Systems (IJITMIS), Volume 5, Issue 1, 2014, pp. 1 - 11, ISSN Print: 0976 – 6405, ISSN Online: 0976 – 6413.

[7] M.Manjuri and B.Maheswari, "Strong Dominating Sets of Strong Product Graph of Cayley Graphs with Arithmetic Graphs", International Journal of Computer Engineering & Technology (IJCET), Volume 4, Issue 6, 2013, pp. 136 - 144, ISSN Print: 0976 – 6367, ISSN Online: 0976 – 6375.