*Few-Body Problem: Theory and Computer Simulations*
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# Generation of chaos about a fast rotating and strongly elongated body

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**Abstract.** The results obtained in the current study are a supplement to those described by Chauvineau, Farinella and Mignard (1993), where the planar motion of a small satellite is considered close to a triaxially ellipsoidal asteroid (with semiaxis ratios $a : b : c = \sqrt{2} : 1 : 1/\sqrt{2}$), and where most computations are made for a long asteroid spin period ($T = 40$ h). To ease comparison, in the current study the same system of units and the same designations are adopted as used earlier. The planar dynamics of a small satellite is studied in the close vicinity of the asteroid 243 *Ida*, which is approximated by a triaxial ellipsoid of density $2.5\,\mathrm{g/cm^3}$ with semiaxes 28, 12, 10.5 km (Belton, 1995). The asteroid is fast rotating ($T = 4.63$ h) and strongly elongated (Belton, 1995). The triaxial problem (where *Ida* is approximated by a triaxial ellipsoid) is viewed as a perturbation of the Keplerian problem (where *Ida* is approximated by a sphere of the same mass and density). The results of numerical integrations are plotted using the method described in Chavineau et. al (1993). They point to the generation of a wide zone of chaotic motion in the triaxial case (Fig. 4). This zone is associated with orbits which are retrograde in both the rest and rotating frames, and it comes into contact with the zone of collisions.

## 1. General equations

Restricting ourselves to satellite orbits lying in the equatorial plane of the asteroid (which rotates uniformly at a rate $\omega$ around its shortest axis) we can write the equations of motion of a small satellite in a rotating reference frame as follows (Subbotin, 1937):

$$\ddot{x} = 2\omega\dot{y} + \omega^2 x + V_x, \qquad \ddot{y} = -2\omega\dot{x} + \omega^2 y + V_y. \tag{1}$$

In the chosen system of units, in which the mass of the asteroid, its mean radius and the Gauss constant are all equal to 1 (Chauvineau et. al, 1993), the gravitational potential of the asteroid with semiaxes $a > b > c$, approximated by a triaxial ellipsoid, and its first derivatives can be expressed in the following classical form (Subbotin, 1949):

$$V = \frac{3}{4}\int\limits_{\lambda}^{+\infty}\left(1 - \frac{x^2}{a^2+s} - \frac{y^2}{b^2+s}\right)\frac{ds}{R(s)}, \tag{2}$$

where $\lambda$ and $R(s)$ are determined from

$$\frac{x^2}{a^2+\lambda} + \frac{y^2}{b^2+\lambda} = 1\ , \qquad R(s) = \sqrt{(a^2+s)(b^2+s)(c^2+s)}\ . \tag{3}$$

The method of Chauvineau, Farinella, Mignard (1993) can be described as follows: for an orbit in four-dimensional phase space $(x,\ y,\ \dot{x},\ \dot{y})$ we consider only those of its points which satisfy the conditions $y = 0,\ \dot{y} > 0$. Associating with each of these points, a point $(x,\ s = sin\ \theta)$, where $\tan\ \theta = \dot{x}/\dot{y}$, we form the Poincare's section of the orbit. The initial point $(x, s = 0)$ in the section for a fixed value $C$ of the Jacobi constant corresponds to the point $(x,\ y = 0,\ \dot{x} = 0,\ \dot{y} = \sqrt{-C + \omega^2 x^2 + 2V})$ in four-dimensional space.

## 2. Description of results

In Figs. 1 and 2, the Poincare sections of orbits for a small satellite of the asteroid 243 *Ida* are represented for $C = 2.2$. The variables 10 arctan $(x/10),\ s)$ are used (Chauvineau et. al, 1993) instead of $(x,\ s)$ which allow us to consider the orbits passing at large distances from the asteroid. Because of symmetries of the problem the plots are limited to $s \geq 0$. The mapping is constructed by numerical integration of the orbits with initial conditions $(x \neq 0,\ y = 0,\ \dot{x} = 0,\ \dot{y} = \sqrt{-C + \omega^2 x^2 + 2V})$. The regions with $x > 0$ $(x < 0)$ correspond to orbits which are direct (retrograde) in the rotating frame fixed in the asteroid body. In the triaxial case (Fig. 2) the orbits with initial coordinates $(x,\ s = 0)$, where $x \gtrsim 6$ times the mean asteroid radii, are chaotic.

As the choice of values for $C$ and $x$ determines the initial conditions completely, the global dynamics of the orbits can be analyzed in the $Cx$ plane. Figs. 3, 4 display such diagrams for the Keplerian and triaxial cases of motion close to the asteroid 243 *Ida*. The zones of initial conditions which lead to chaotic motion are diagonally dashed. The wider chaotic zone is associated with orbits which are retrograde in both the rotating (as $x < 0$) and rest frames. The initial conditions $(C,\ x)$ were assigned to the chaotic zone if the corresponding orbit was not regular and if it did not lead to collision with the asteroid during 10 years. The left part of the chaotic zone comes into contact with the zone of collisions (horizontally dashed) and its right part touches the zone of escaping orbits (vertically dashed). There is a wide zone of regular orbits (not dashed, with the curve marked "3" of almost circular orbits within it) inside the chaotic zone. It is seen (Fig. 4) that the chaotic zone does not allow the almost circular orbits corresponding to the curve "3" (retrograde in both frames) to pass closer than nearly 3 asteroid radii from the asteroid center, while in the Keplerian case the circular orbits corresponding to the curve "3" (Fig. 3) and regular orbits about them can exist at very short distances.

## References

Subbotin, M. F. 1937, Manual on Celestial Mechanics, v. II, Moscow

Subbotin, M. F. 1949, Manual on Celestial Mechanics, v. III, Moscow

Chauvineau, B., Farinella, P. and Mignard, F. 1993, Icarus, 105, 370

Belton, M., Chapman, C., Thomas, P., etc. 1995, Nature, 374, 785

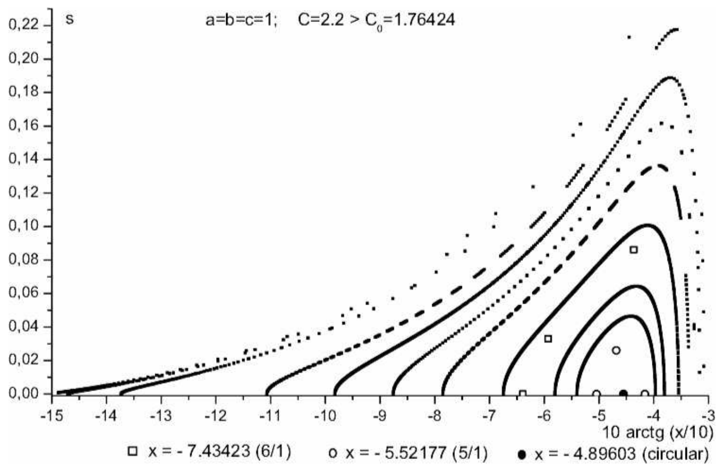


Figure 1. The Poincare's sections at $C = 2.2$ for the motion about the asteroid 243 *Ida* approximated by a sphere. The dark point is the section of circular orbit with initial conditions ($x = -4.89603,\ s = 0$); the sections of resonant orbits are marked with three open squares (resonance 6/1) and three open circles (resonance 5/1). The unit of length is mean radius of the asteroid ($\sqrt[3]{abc}$).

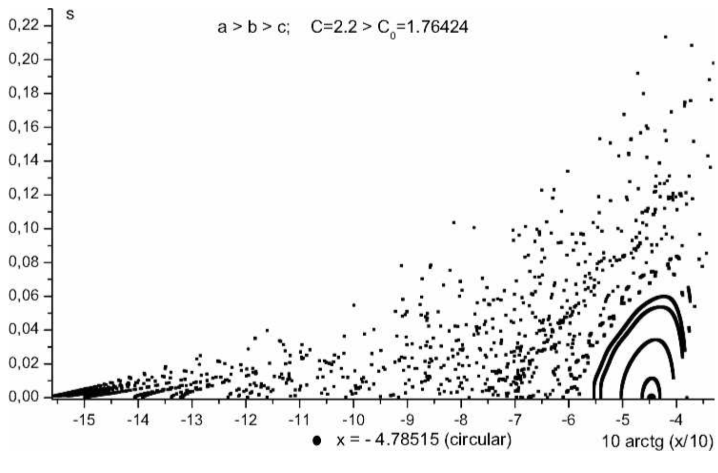


Figure 2. The Poincare's sections at $C = 2.2$ for the motion about the asteroid 243 *Ida* approximated by a triaxial ellipsoid. The initial conditions for the Keplerian resonant orbits (Fig. 1) generate chaotic motion. The dark point is the section of almost circular orbit with initial conditions ($x = -4.78515,\ s = 0$) obtained by numerical continuation of the Keplerian circular orbit (Fig. 1) with initial conditions ($x = -4.89603,\ s = 0$).

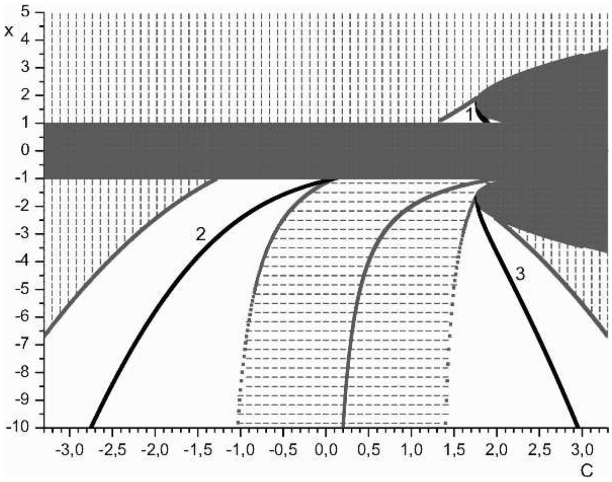


Figure 3. Dynamical zones about the asteroid 243 *Ida* approximated by a sphere. The *white area* is associated with regular orbits. The *vertically dashed* regions correspond to escape orbits. The *dark bar* corresponds to initial conditions lying inside the asteroid: $x < a$ for the triaxial problem (Fig. 4) and $x < \sqrt[3]{abc}$ for the Keplerian problem (Fig. 3). The *dark regions* lying above and below the dark bar are the zones of imaginary velocity. The *horizontally dashed* area corresponds to orbits which lead to collision with the surface of the asteroid (spherical or ellipsoidal). The *black curves* marked **1, 2, 3** correspond to circular orbits in the Keplerian problem (Fig. 3) or almost circular orbits in the triaxial problem (Fig. 4). The *dark curve lying inside the collisional area* corresponds to the boundary between the orbits (retrograde in the rotating frame) that are either direct or retrograde in the fixed frame. *Diagonally dashed* regions correspond to chaotic orbits.

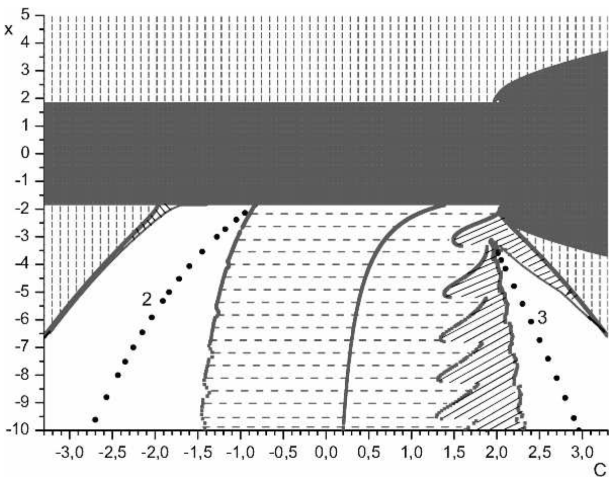


Figure 4. Dynamical zones about the asteroid 243 *Ida* approximated by a triaxial ellipsoid. Regions are marked as in figure 3.