are troublesome for curvature-based classification, and (b) it has a short execution time that is not dependent on the length of the stroke or the number of sample points acquired (assuming coordinates have been summed while drawing the stroke). The procedure can therefore be used to provide continuous feedback of the interpreted entity during drawing, in real time. However, in spite of this ability, it is evident that geometrical-based classification is inherently limited and a more general, context-sensitive approach must be pursued.

A new endpoint clustering scheme has also been presented based on adaptive tolerances at different parts of the sketch. The proposed formulation provides a framework for implementing various criteria for determining local thresholds, such as detail sensitive criteria, dynamic criteria, or other application specific criteria. Again, clustering can be improved using a context-sensitive approach.

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A New Criterion for Positive Mean Stress Fatigue Design

Shyh-Jen Wang[1] **and Marvin W. Dixon**[2]

The modified Findley line is proposed by using ultimate strength and endurance limit as parameters to obtain a good initial approximation of an acceptable design. Comparing with experimental data, the modified Findley line could be a strong candidate for fatigue design criterion for parts made of a nonferrous materials, and be conservative for ferrous material parts.

Introduction

Improperly designed engineering products may fail in fatigue causing losses in revenue and personal injury or death. Currently, these failures are avoided by either using expensive design techniques involving extensive modeling and testing or by over designing the part. The expense of testing and modification of the initial design is reduced if the design criterion gives a good initial approximation. Several design approaches have been developed to address the problem of fatigue damage of ductile metals loaded with positive mean and alternating stresses. The Bagci, Gerber, Nichihara, modified Goodman, Quadratic, and Soderberg lines are a few of the techniques that have been proposed to address the problem.

This paper presents the modified Findley line for designing parts which experience positive mean stress fatigue loading. The modified Findley line is based on the assumption that the critical shear decreases with an increase in the normal stress acting on the same plane, and is simple and less conservative than the modified Goodman line.

The Modified Findley Line

Flavenot and Skalli (1984) stated "the mechanism corresponding to the initiation of fatigue cracks is most often the shearing of crystallographic planes. It appears logical then to have a criterion relating the normal stress to alternating shear stress which might be local shear stress in most favorable oriented plane." This assumption was used before by Stanfield (1935), who suggested that both the shear and normal stresses on the fatigue plane should be considered in a fatigue failure criterion and proposed the relation

$$\tau_N = f - k\sigma_N, \tag{1}$$

where τ_N and σ_N are the shear and normal stresses components on the critical plane; f and k are materials constants. Stulen and Cummings (1954), and Findley et al. (1956) used similar forms as fatigue criteria to address the problem of absolutely reversing fatigue.

Findley (1959) used the linear relationship between shear stress and normal stress on a critical plane to include the effect of mean stress on the fatigue of metals under combined loading. Brown and Miller (1973) stated "Findley's work gives a first representation of the effects of mean stress given suitable values of the constants k. It also allows for the effects of hydrostatic pressure on torsion tests. . . . Little (1969) suggested that a reliable criterion of failure may be found from a similar approach to that of Findley." According to Findley, the fatigue criterion for uniaxial loading is

$$f = k\frac{\sigma_{\max}}{2} + \frac{1}{2}\sqrt{k^2\sigma_{\max}^2 + \sigma_a^2}, \tag{2}$$

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where $\sigma_{\max}$ and σ_a are the maximum and alternating fatigue stresses; f and k are experimental constants. Since these constants may vary with the design parameters, including materials, the actual design must be tested to determine the values of f and k. To experimentally find the values of these constants, the life of the part is determined, and at this point the values of f and k are of only academic interest. Thus, if the fatigue criterion presented in Eq. (2) is to be of any real value then a way, short of experimentation, must be found to determine the value of f and k. Using the procedure which follows, a good approximation of f and k can be found to match the limit points on the failure criterion.

Equation (2) can be rearranged as

$$-f^2 + kf\sigma_{\max} + \frac{\sigma_a^2}{4} = 0. \quad (3)$$

Since the maximum stress is equal to the sum of mean and alternating stresses ($\sigma_{\max} = \sigma_m + \sigma_a$), Eq. (3) also can to rewritten as

$$(kf\sigma_m - f^2) + kf\sigma_a + \frac{\sigma_a^2}{4} = 0. \quad (4)$$

From Eq. (4), the alternating stress can be obtained as

$$\sigma_a = -2fk + 2\sqrt{f^2k^2 + f^2 - fk\sigma_m}. \quad (5)$$

It should be noted that the negative root of Eq. (4) is rejected since the alternating stress should be positive, and $f^2k^2 + f^2 - fk\sigma_m$ must be greater than or equal to zero for a real number alternating stress.

Two failure conditions can be used to determine these two constants. Considering the end points of boundary between the infinite and finite life as defined by the modified Goodman diagram leads to: (a) the mean stress is zero ($\sigma_m = 0$), when the alternating stress is equal to the endurance limit ($\sigma_a = S_n$), and (b) when the alternating stress is zero ($\sigma_a = 0$), the mean stress can equal the ultimate strength of the material before failure occurs ($\sigma_m = S_{ut}$). Using condition (a), Eq. (5) becomes

$$S_n = -2fk + 2f\sqrt{k^2 + 1}. \quad (6)$$

Upon the application of condition (b), Eq. (4) becomes

$$f = kS_{ut}. \quad (7)$$

Equations (6) and (7) can be solved for constants, f and k, as

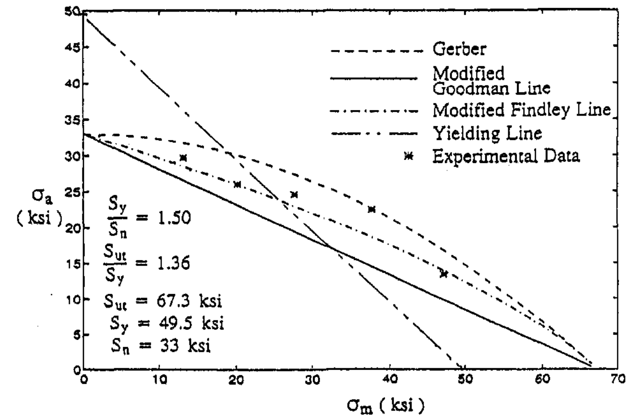

Fig. 2 Comparison of various fatigue criteria with actual failure data for 0.29 carbon steel, Forrest (1962)

$$f = \frac{S_{ut}}{2\sqrt{\frac{S_{ut}}{S_n}\left(\frac{S_{ut}}{S_n} - 1\right)}}; \quad (8)$$

$$k = \frac{1}{2\sqrt{\frac{S_{ut}}{S_n}\left(\frac{S_{ut}}{S_n} - 1\right)}}. \quad (9)$$

Substituting Eqs. (8) and (9) into Eq. (5), the fatigue criterion is

$$\sigma_a = \frac{S_n}{2(S_{ut} - S_n)}[-S_n + \sqrt{S_n^2 + 4(S_{ut} - S_n)(S_{ut} - \sigma_m)}], \quad (10)$$

which is called the modified Findley line.

Comparison With Actual Experimental Data

The modified Findley, Gerber, and modified Goodman lines were compared with the experimentally developed fatigue data found in the literature. Typical data showing the fatigue points of both ferrous and non-ferrous ductile materials are shown in Figs. 1–4. An extensive set of data used for additional comparisons was presented by Wang (1995). All of the experimental data is for uniaxial fatigue loading, and the yield strength was found by using 0.2 percent offset value. From these figures, it appears that the modified Findley line predicts the fatigue behavior for non-ferrous ductile materials very well, and is conser-

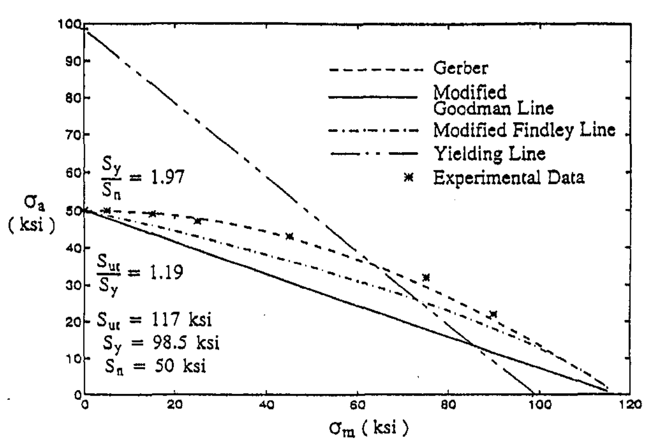

Fig. 1 Comparison of various fatigue criteria with actual failure data for 4130 steel, Grover et al. (1951)

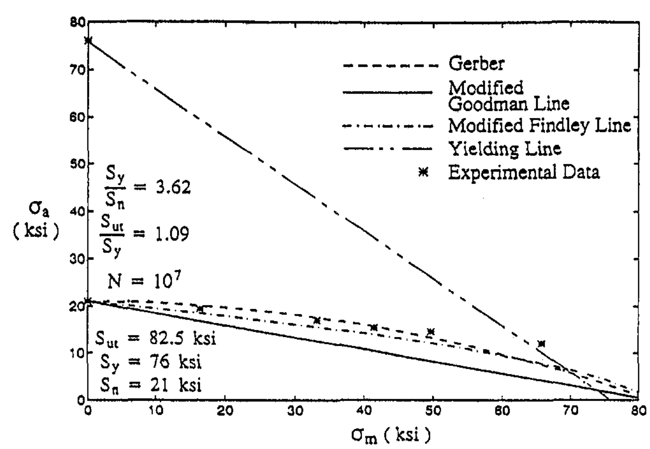

Fig. 3 Comparison of various fatigue criteria with actual failure data for 75S-T6 aluminum alloy, Schwartz (1948)

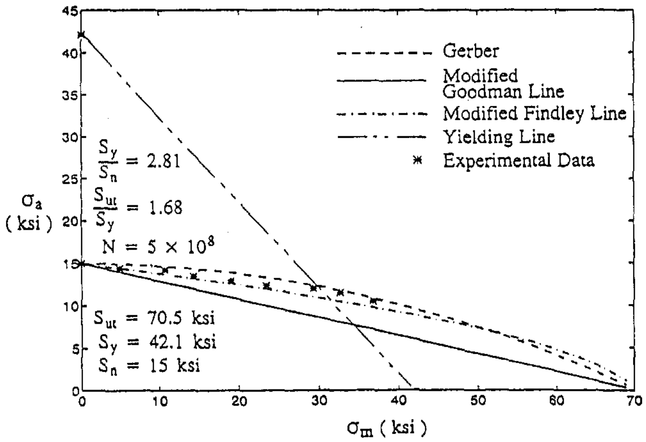

Fig. 4 Comparison of various fatigue criteria with actual failure data for 24S-T4 aluminum alloy, Schwartz (1948)

vative for ferrous materials. Note that in all of the cases presented, the modified Goodman and modified Findley lines are conservative, while in some cases the Gerber criterion parabola is not a good model for non-ferrous materials. Hence, without experimental data, one can not rely on the Gerber criterion to predict fatigue failure.

As shown in these figures and comparison done by Wang (1995), the modified Findley line falls between the modified Goodman line and Gerber parabola, which is supported by Forrest (1962), "Summarizing all the results for ductile metals, . . . two-third between the modified Goodman line and Gerber parabola." The modified Findley is less conservative than the modified Goodman line, and is a good model to predict fatigue of parts loaded with a positive mean stress.

Conclusion

The modified Findley line is based on the assumption that the critical shear decreases with an increase in the normal stress acting on the same plane, then by using ultimate strength and endurance limit as parameters to obtain a good initial approximation. Limited fatigue data is available in the open literature, and more comparison should be made before the modified Findley line is universally adopted. However, form the references found, it appears that the modified Findley line is a strong candidate for fatigue criterion for parts made of non-ferrous ductile materials, and is conservative for ferrous parts. For a design engineer, the modified Findley line is simple and easy to use, and represents a very promising approach for leading to reasonable starting designs involving positive mean stress fatigue.

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Improved Calculation of Eigenvector Sensitivities Using Matrix Perturbation Analysis

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Derivatives of eigenvalues and eigenvectors have become increasingly important in the development of modern numerical methods for areas such as structural design optimization, dynamic system identification and dynamic control, and the development of effective and efficient methods for the calculation of such derivatives has remained to be an active research area for several decades. Based on the concept of matrix perturbation, this paper presents a new method for the improved calculation of eigenvector derivatives in the case where only few of the lower modes of a system under study have been computed. By using this new proposed method, considerable improvement on the accuracy of the estimation of eigenvector derivatives can be achieved at the expense of very tiny extra computational effort since only few matrix vector operations are required. Convergency criterion of the method has been established and the required accuracy can be controlled by including more higher order terms. Numerical results from practical finite element model have demonstrated the practicality of the proposed method. Further, the proposed method can be easily incorporated into commercial finite element packages to improve the accuracy of eigenderivatives needed for practical applications.

1 Introduction

Derivatives of eigenvalues and eigenvectors have become increasingly important in the development of modern numerical methods for areas such as structural design optimization, dynamic system identification and dynamic control and methods for computing such derivatives have been investigated by many researchers from different scientific disciplines for several decades. Since the late 70's, research effort was much focused on the applications of sensitivity analysis to systematic structural design optimization as a practical tool for design engineers when large and complicated engineering structures are considered (Arora and Haug, 1979). Because such practice as design optimizations of large structural systems involves enormous computational cost due to the necessary calculation of structural sensitivities for a large number of eigenvectors, much interest has recently been directed towards the development of efficient computational procedures.

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