Definition of Pressure and Transmission Angles Applicable to Multi-Input Mechanisms

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Pressure angle is an important measure of the effectiveness with which a force is transmitted between a cam and follower. A pressure angle of zero degrees indicates that the force is transmitted with maximum effectiveness while a 90 deg pressure angle indicates that the force cannot make the desired contribution to the follower motion. There are a number of definitions available in the literature that can be used to determine pressure angle. These definitions are all consistent with the meaning of pressure angle described above when applied to followers driven by only a single cam. For followers driven by multiple inputs, however, we have found that none of these definitions provides a value of pressure angle that retains this same meaning. The purpose of this paper is to draw attention to this fact and to present a precise definition of pressure angle, as well as a discussion of its mathematical consequences, that properly characterizes the performance of either single-input or multi-input cam-follower mechanisms. For single-input systems, this definition is shown to be equivalent to the definitions for pressure angle found in the literature. The applicability of this definition to the determination of transmission angle for linkages with multiple inputs is also discussed.

1 Introduction

Standard texts on the kinematics of mechanisms such as Chen [1], Mabie and Reinholtz [2], Martin [3], Rothbart [4], and Shigley and Uicker [5] all provide definitions of pressure angle. Mabie and Reinholtz [2, p. 73], for example, state, "The pressure angle is the angle between the direction of motion of the trace point and the common normal (the line of action) to the contacting surfaces." They add, "The pressure angle is a measure of the instantaneous force transmission properties of the mechanism." Martin [3, p. 212], offers a similar definition stated in slightly different terms, "The angle which the common normal for the cam and follower makes with the path of the follower is called the pressure angle." The most succinctly stated definition is that of Shigley and Uicker [5, p. 111]. We shall use this definition as a point of departure for our subsequent developments and refer to it as "Definition (1)." Shigley and Uicker state

Definition (1): *The pressure angle is defined as the acute angle between the direction of the output force and the direction of the velocity of the point where the output force is applied.*

For cam-follower mechanisms, Shigley and Uicker point out that only the component of force along the "line of motion" of the follower is useful in overcoming the output load. Components of force that are not along the line of motion are resisted by the bearings supporting the follower. They state that these force components should be minimized to reduce system loads and friction forces between the follower and its supports. The pressure angle provides a measure of how much of the total force exerted by the cam on the follower contributes to the desired motion of the follower and how much of the force gives rise to extraneous support reactions. When the pressure angle is too large, undesirable effects can arise due to both the extraneous support reactions and the resulting friction forces, and the follower may chatter or jam.

Shigley and Uicker also mention that the pressure angle for a cam-follower mechanism is one of a class of "indices of merit" for mechanisms. Other such indices are the pressure angle for meshing gear teeth and the transmission angle for four-bar linkages. Another index of merit, proposed by Denavit et al. [6], makes use of the determinant of the coefficients of the simultaneous equations relating the dependent velocities of a mechanism. This index provides an overall measure of force transmission within a mechanism, but does not allow an individual evaluation of the effectiveness of any particular driving force.

When Definition (1) is applied to a system driven by only a single cam, the pressure angle is found to be directly related to the force that a cam exerts on a follower and the minimum force needed to achieve a given motion. When applied to multi-input mechanisms, however, this relationship is no longer valid. We propose an alternative definition that maintains the same intrinsic meaning as Definition (1) but can be applied to multiple cam systems. This definition is

Definition (2): *The pressure angle for each cam of a cam-follower mechanism is the acute angle whose cosine is the ratio*

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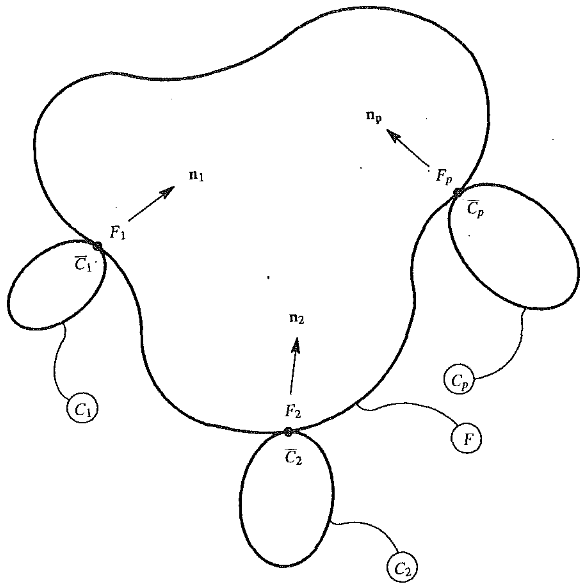

Fig. 1 Multi-input cam-follower mechanism

between the magnitude of the minimum force that can be applied by that cam to the follower to produce a given follower motion and the magnitude of the actual force applied by that cam to the follower.

We shall show that Definition (2) is equivalent to Definition (1) for single-input systems by first investigating its mathematical consequences and then applying it to the analysis of a single-input system. We will also demonstrate how it can be applied to a multi-input system in a straightforward and consistent manner. Finally, we will discuss several special concerns that arise in the application of Definition (2).

2 Mathematical Consequences

To investigate the mathematical consequences for the pressure angle defined in Definition (2), consider the case of a rigid follower F whose motion is completely determined by contact with a number of rigid bodies, as shown in Fig. 1. In general, the rigid bodies in contact with the follower can be any combination of cams and other rigid constraints. For the purposes of this derivation, however, we will refer to all the rigid bodies as "cams" and denote them by the symbols $C_1, \ldots, C_p$. For the motion of a follower in a three-dimensional space, the maximum value of p is six. The forces that the cams exert on the follower are $\mathbf{R}_i$ $(i=1, \ldots, p)$. The points of the cams that are in contact with the follower are $\overline{C}_i$ $(i=1, \ldots, p)$, and the points of the follower in contact with the cams are F_i $(i=1, \ldots, p)$. Unit vectors directed toward the surface of the follower along common normals to the surfaces of the cams and follower at their points of contact are $\mathbf{n}_i$ $(i=1, \ldots, p)$.

Without loss of generality, we focus on the determination of the pressure angle for cam C_p and assume that the motions of cams C_i $(i=1, \ldots, p-1)$ are prescribed. Note that this assumption allows the inclusion of cases in which one or more of the C_i $(i=1, \ldots, p-1)$ plays the role of a fixed rigid guide. We denote the pressure angle for C_p as ϕ_p. In accordance with Definition (2), the cosine of ϕ_p is the ratio between the minimum magnitude of $\mathbf{R}_p$ needed to produce a given motion of the follower and the magnitude of $\mathbf{R}_p$ itself. To determine the minimum magnitude of $\mathbf{R}_p$, we will develop expressions for the component of $\mathbf{R}_p$ that contributes to the motion of the follower using Kane's method [7].

The first step of Kane's method is to introduce generalized speeds. Generalized speeds are variables that are inherent to Kane's method and that describe the *motion* of a system in the same way that generalized coordinates describe *configuration*. Since the motions of all parts of the system under consideration here, except that of cam C_p, are regarded as prescribed, a single generalized speed u_p is sufficient to completely characterize the motion of the follower. For the purposes of this derivation, one can regard u_p as arbitrarily chosen so long as it represents an independent quantity and suffices to completely describe the motion of F. For any generalized speed that satisfies these conditions, one can express the velocities of points F_i $(i=1, \ldots, p)$ as

$$\mathbf{v}^{F_i}=\mathbf{v}_p^{F_i}u_p+\mathbf{v}_t^{F_i} \quad (i=1,\ldots,p), \tag{1}$$

where $\mathbf{v}_p^{F_i}$ is referred to in the terminology of Kane's method as the partial velocity of F_i with respect to u_p, and $\mathbf{v}_t^{F_i}$ is the component of the velocity of F_i due to the prescribed motion of cams C_i $(i=1,\ldots,p-1)$.

Kane's method stipulates that the only components of force that contribute to the motion of a body are those that make nonzero contributions to the generalized active forces. For the forces exerted by the cams on the follower, the single generalized active force K_p is given by

$$K_p=\sum_{i=1}^{p}\mathbf{R}_i\cdot\mathbf{v}_p^{F_i}. \tag{2}$$

Equation (2) can be simplified by making use of the fact that points F_i and $\overline{C}_i$ $(i=1,\ldots,p)$ must remain in contact throughout all motions of the mechanism. Mathematically, this is expressed as

$$\mathbf{v}^{F_i}\cdot\mathbf{n}_i=\mathbf{v}^{\overline{C}_i}\cdot\mathbf{n}_i \quad (i=1,\ldots,p). \tag{3}$$

Equation (3) can be expressed in terms of the partial velocities of F_i and $\overline{C}_i$ after first noting that the velocities of $\mathbf{v}^{\overline{C}_i}$ $(i=1,\ldots,p)$ can be written in a form similar to that used for $\mathbf{v}^{F_i}$ in equation (1); that is,

$$\mathbf{v}^{\overline{C}_i}=\mathbf{v}_p^{\overline{C}_i}u_p+\mathbf{v}_t^{\overline{C}_i} \quad (i=1,\ldots,p). \tag{4}$$

Since u_p is an independent quantity, substitution of equations (1) and (4) into equation (3) yields

$$\mathbf{v}_p^{F_i}\cdot\mathbf{n}_i=\mathbf{v}_p^{\overline{C}_i}\cdot\mathbf{n}_i \quad (i=1,\ldots,p). \tag{5}$$

If one neglects friction at the point of contact between F and the cams C_i $(i=1,\ldots,p-1)$, the forces $\mathbf{R}_i$ $(i=1,\ldots,p-1)$ are parallel to $\mathbf{n}_i$ and one can make use of equation (5) to write

$$\mathbf{v}_p^{F_i}\cdot\mathbf{R}_i=\mathbf{v}_p^{\overline{C}_i}\cdot\mathbf{R}_i \quad (i=1,\ldots,p-1). \tag{6}$$

Substitution of equation (6) into equation (2) produces

$$K_p=\sum_{i=1}^{p-1}\mathbf{R}_i\cdot\mathbf{v}_p^{\overline{C}_i}+\mathbf{R}_p\cdot\mathbf{v}_p^{F_p}. \tag{7}$$

After again noting the motions of cams C_i $(i=1,\ldots,p-1)$ are prescribed, one realizes that all of the $\mathbf{v}^{\overline{C}_i}$ $(i=1,\ldots,p-1)$ are independent of u_p and thus all the $\mathbf{v}_p^{\overline{C}_i}$ $(i=1,\ldots,p-1)$ of equation (4) are equal to zero. As a result, equation (7) reduces to

$$K_p=\mathbf{R}_p\cdot\mathbf{v}_p^{F_p}. \tag{8}$$

The dot product in equation (8) shows that the only component of the cam force $\mathbf{R}_p$ that contributes to the generalized active force, and hence to the motion of the follower, is the component that is parallel to $\mathbf{v}_p^{F_p}$. Thus, the minimum force $\mathbf{R}_p$ that produces a given follower motion is a force solely in the direction of $\mathbf{v}_p^{F_p}$. In accordance with Definition (2), the cosine of the pressure angel ϕ_p is then the ratio between the magnitude of the component of $\mathbf{R}_p$ in the direction of $\mathbf{v}_p^{F_p}$ and the magnitude of $\mathbf{R}_p$. Based on the results presented in equation (8),

$$\phi_p=\cos^{-1}\left\{\frac{|\mathbf{R}_p\cdot\mathbf{v}_p^{F_p}|}{|\mathbf{R}_p||\mathbf{v}_p^{F_p}|}\right\} \quad (0\leq\phi_p\leq 90°). \tag{9}$$

Equation (9) can be simplified further by defining a unit vector $\mathbf{f}_p$ in the direction of $\mathbf{v}_p^{F_p}$ and a unit vector $\mathbf{n}_{R_p}$ in the direction of $\mathbf{R}_p$. This allows equation (9) to be rewritten as

$$\phi_p = \cos^{-1} |\mathbf{n}_{R_p} \cdot \mathbf{f}_p| \quad (0 \le \phi_p \le 90^\circ). \tag{10}$$

One should note that $\mathbf{f}_p$ indicates the direction of motion of point F_p due to the motion of *only* cam C_p. This observation is based on the fact that the partial velocity of any point of a system associated with a particular generalized speed of the system is in the direction of the total velocity of the point when all other generalized speeds are set equal to zero. Or in terms more specific to the analysis of multi-input cam-follower mechanisms, the direction of the partial velocity of any point of the follower associated with the contribution of a particular cam to the motion of the follower is in the direction of the total velocity of the point when all other cams are held fixed. Thus, $\mathbf{f}_p$ is independent of a specific choice for the generalized speed u_p and equation (10) enables pressure angle to be determined through purely kinematical considerations and does not require explicit use of the partial velocities of a system.

In light of equation (9), it is interesting to consider the results that would be obtained with a direct application of Definition (1). Definition (1) states that the pressure angle is the acute angle between the direction of the "output force," here $\mathbf{R}_p$, and the velocity of the point where the output force is applied, here $\mathbf{v}^{F_p}$. Mathematically, this can be stated as

$$\phi_p = \cos^{-1} \left\{ \frac{|\mathbf{R}_p \cdot \mathbf{v}^{F_p}|}{|\mathbf{R}_p| |\mathbf{v}^{F_p}|} \right\} \quad (0 \le \phi_p \le 90^\circ). \tag{11}$$

Based on equations (1), (9), and (11), Definitions (1) and (2) only produce the same value for ϕ_p when the component of the velocity of F_p due to the prescribed motions of cams C_i $(i = 1,\ldots,p-1)$ [the term $\mathbf{v}_t^{F_i}$ in equation (1)] is zero. In general, this only occurs when cams C_i $(i = 1,\ldots,p-1)$ are stationary, and it thus becomes apparent that Definition (1) only provides a physically meaningful definition of pressure angle for systems driven by a single input and thus does not properly accommodate the interactions between the inputs for systems driven by multiple inputs.

3 Single-input Example

In this section, a simple single-input example is presented both to illustrate the use of equation (10) and to demonstrate the compatibility of Definition (2) with other definitions of pressure angle found in the literature. The notation used in this example follows that presented in Sec. 2 when the number of cams p is set equal to one.

The system under consideration is shown in Fig. 2 and consists of a rigid oscillating follower F driven by a single rigid cam C_1. The follower and the cam rotate about parallel axes passing through points O and A, respectively. The point of F in contact with C_1 is denoted F_1, and the distance from O to F_1 is L. A unit vector in the direction of motion of F_1 is $\mathbf{f}_1$, as shown in Fig. 2. A unit vector directed toward the surface of the follower along a common normal to the surfaces of the cam and follower at their point of contact is $\mathbf{n}_1$. The contact force between the cam and the follower is $\mathbf{R}_1$.

In accordance with equation (10), the determination of pressure angle first requires the determination of the direction of motion of point F_1 due to the motion of cam C_1. Since C_1 is the only cam acting on the system, the direction of motion of F_1 is along $\mathbf{f}_1$ for all motions of C_1. Next, the contact force $\mathbf{R}_1$ can be expressed as

$$\mathbf{R}_1 = R_1 \mathbf{n}_{R_1}, \tag{12}$$

where R_1 is the magnitude of $\mathbf{R}_1$ and $\mathbf{n}_{R_1}$ is a unit vector in the direction of $\mathbf{R}_1$. Now making use of equation (10), one obtains

$$\phi_1 = \cos^{-1} |\mathbf{n}_{R_1} \cdot \mathbf{f}_1|. \tag{13}$$

Once details of the cam and follower geometries, as well as the direction of $\mathbf{R}_1$, have been established, equation (13) can be used to determine a numerical value for ϕ_1. One should

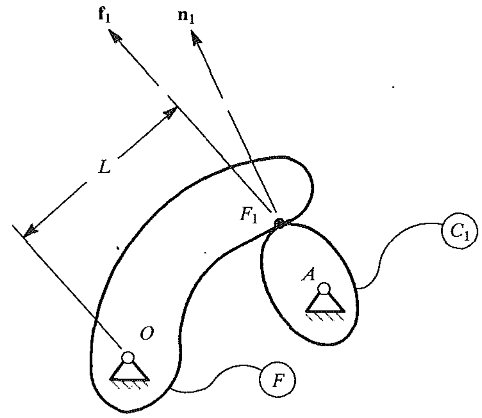

Fig. 2 Single-input cam-follower mechanism

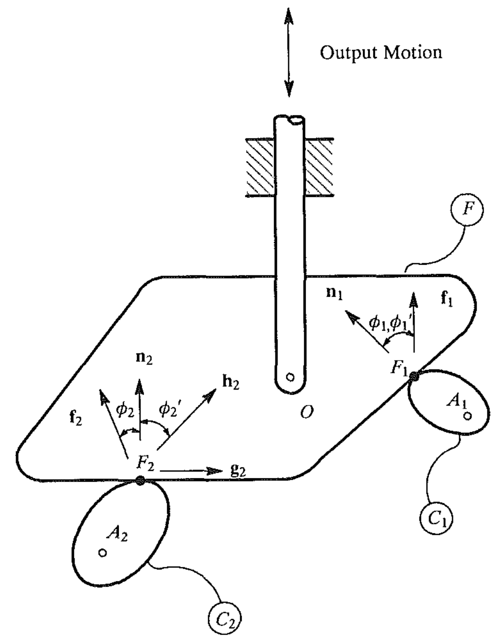

Fig. 3 Planar, two-input, cam-follower mechanism

note that if friction is neglected at the point of contact between F and C_1, $\mathbf{n}_{R_1}$ is parallel to $\mathbf{n}_1$, and ϕ_1 is simply the angle between $\mathbf{n}_1$ and $\mathbf{f}_1$. The salient feature of equation (13) is that it gives an expression for the pressure angle of the single-input mechanism of Fig. 2 that is identical to that which would be obtained when using any one of the other definitions of pressure angle in the current literature.

4 Multi-Input Example

This section presents a simple multi-input example to illustrate the use of equation (10) for a multi-input system. As in the preceding section, the notation again follows that presented in Sec. 2, in this case with p equal to two.

Figure 3 depicts a planar mechanism in which a rigid follower F is driven by two separate rigid cams C_1 and C_2. The follower and cams rotate about parallel axes passing through points O, A_1, and A_2, respectively. The point of F in contact with C_1 is F_1, and the point of F in contact with C_2 is F_2. To describe the motion of F_1, it is convenient to introduce a unit vector $\mathbf{f}_1$ in the direction of the velocity of F_1 when C_2 is regarded as stationary. Note that since the system of Fig. 3 reduces to a single input system when C_2 is stationary, the direction of $\mathbf{f}_1$ can be determined by standard kinematical methods, such as the method of instant centers. A unit vector $\mathbf{f}_2$ can be similarly introduced in the direction of the velocity of F_2 when C_1 is regarded as stationary. Unit vectors $\mathbf{n}_i$ $(i=1,2)$ are in directions normal to the surfaces of cams C_1 and C_2, respectively, at their points of contact with the follower. The contact forces exerted between the follower and the two cams are $\mathbf{R}_1$ and $\mathbf{R}_2$, respectively.

Determining first the pressure angle for cam C_1, we note that unit vector $\mathbf{f}_1$ is in the direction of the component of the velocity of point F_1 due to the motion of cam C_1. As in the single-input example, the contact force $\mathbf{R}_1$ can be expressed as

$$\mathbf{R}_1 = R_1 \mathbf{n}_{R_1}, \tag{14}$$

where R_1 is the magnitude of $\mathbf{R}_1$ and $\mathbf{n}_{R_1}$ is a unit vector in the direction of $\mathbf{R}_1$. With the aid of equation (10), one thus obtains

$$\phi_1 = \cos^{-1} | \mathbf{n}_{R_1} \cdot \mathbf{f}_1 |. \tag{15}$$

One can follow a similar procedure for the determination of ϕ_2 in terms of $\mathbf{n}_{R_2}$ and $\mathbf{f}_2$. Note that as in the single-input example, when friction is neglected at points F_1 and F_2, the pressure angles ϕ_1 and ϕ_2 are simply equal to the angles between $\mathbf{n}_1$ and $\mathbf{f}_1$ and $\mathbf{n}_2$ and $\mathbf{f}_2$, respectively, as shown in Fig. 3.

It is interesting to consider the results that one would obtain for this example if one were to make use of the traditional definition for pressure angle given in Definition (1). Definition (1) defines pressure angle as the angle between the force applied by a cam to the follower and the *total* velocity of that point of the follower where the force is applied. For the problem at hand, therefore, each pressure angle ϕ'_i $(i=1,2)$, corresponding to Definition (1), would be the angle between the contact force $\mathbf{R}_i$ and total velocity $\mathbf{v}^{F_i}$ $(i=1, 2)$, where each of these total velocities depends in general on the motion of both cams C_1 and C_2.

For the instant in which the system is in the configuration depicted in Fig. 3, the motion of cam C_2 contributes nothing to the total velocity of F_1. Consequently, the total velocity of F_1 is along $\mathbf{f}_1$ and ϕ'_1 is equal to ϕ_1, and the two definitions of pressure angles give the same result. (The fact that the total velocity of F_1 must be in the direction of $\mathbf{f}_1$ can be seen by noting that the velocity of point O has no component along the line connecting O and F_1, and thus the velocity of F_1 can have no component along the line connecting O and F_1.)

The total velocity of F_2, however, depends on the motion of both cams C_1 and C_2. The part of the total velocity of F_2 due to the motion of cam C_2 is in the direction $\mathbf{f}_2$. The part of the total velocity of F_2 due to the motion of C_1 must be in a direction tangent to the surfaces of both C_2 and F at F_2. (Unit vector $\mathbf{g}_2$ is a vector tangent to these two surfaces.) The total velocity of F_2 is thus the vector sum of components along $\mathbf{f}_2$ and $\mathbf{g}_2$. Denoting the direction of the total velocity by $\mathbf{h}_2$, one obtains for the pressure angle, as defined by Definition (1),

$$\phi'_2 = \cos^{-1} | \mathbf{n}_{R_2} \cdot \mathbf{h}_2 |. \tag{16}$$

When friction is neglected at the point of contact between F and C_2, ϕ'_2 is as given in Fig. 3.

Note that the direction of $\mathbf{h}_2$ depends on the relative magnitudes of the components of the total velocity of F_2 in the directions $\mathbf{f}_2$ and $\mathbf{g}_2$. These components of velocities depend in turn on the design of the cams and their relative angular velocities. Thus, the pressure angle determined with Definition (1) could be much different (either lesser or greater) from the value determined with Definition (2). In the extreme case in which cam C_1 is lifting and cam C_2 is just beginning the transition from a dwell to a lifting phase, the pressure angle ϕ'_2, calculated with Definition (1), would approach 90 deg, since the component of the total velocity of F_2 contributed by cam C_2 would be small and the angle between the unit vectors $\mathbf{n}_2$ and $\mathbf{h}_2$ would thus be close to 90 deg.

5 Discussion

Frictional Effects. There are several issues mentioned briefly in the derivation presented in Sec. 2 that deserve additional comment. First, one should note that the derivation neglects the effects of friction at the points of contact between the follower F and cams C_i $(i=1,\ldots,p-1)$. The fact that friction is not neglected at the point of contact between F and C_p may at first seem contradictory, particularly if one were to use equation (10) to successively calculate the pressure angles for each cam of a multi-cam system. Frictional effects are divided in this way to express equation (10) in the most general form possible and to allow a meaningful determination of pressure angle for systems in which friction plays an important role for only a single cam.

Although equation (10) can no longer be applied when one considers the effects of friction at *all* points of contact between the follower and the cams, Definition (2) is still valid, both for single-input and multi-input mechanisms, since it is written in terms of the total forces acting on the follower. Fortunately, most mechanisms are designed so that friction is minimized at the points of contact between the follower and the bodies that guide its motion, and thus, in most cases, the effects of friction on pressure angle should be small and equation (10) should generally produce sufficiently accurate results.

Transmission Angle. Pressure angle as it applies to cam-follower mechanisms has been emphasized in this paper since the presented results are derived from research on a multi-input cam-follower system [8]. One should note, however, that the derivation of Sec. 2 is also directly applicable to the determination of the transmission angles at all joints of planar or spatial linkages with single or multiple inputs, since the interaction of the links is kinematically equivalent to the interaction of a cam and follower. For a pin joint, for example, point F_p is defined as being at the center of the pin at the joint for which the transmission angle is desired. The transmission angle is then the complement of ϕ_p, as given in equation (10).

Rolling Contact. Although it has not been proven here, a derivation similar to that given in Sec. 2 would arrive at the result given by equation (10) when the motion of a follower is produced by rolling contact with one or more rigid bodies.

Design Considerations. When Definition (2), and thus equation (10), is used for design purposes, one should be aware that it applies to each cam of a mechanism based on a given geometrical layout of all the cams acting on the follower. When any one cam is redesigned so as to reduce its pressure angle, the overall geometry of the mechanism changes. This affects the reaction forces at the other cam-follower interfaces and may lead to increases (or decreases) in the values of the other pressure angles. A procedure for optimizing the cam and follower geometries to minimize the pressure angles, or the cam-follower contact forces, at *all* the cam-follower interfaces, either for any given configuration of a mechanism or throughout its complete range of motion, is beyond the scope of this paper. Such an optimization procedure requires the development of an optimization criterion (minimal maximum force magnitude at any cam, minimal maximum pressure angle at any cam, etc.), the determination and quantization of all the constraints on the problem, and the specification of an optimization algorithm.

Finally, as regards the use of Definition (2) in design, it should be emphasized that Definition (2) provides a measure of the instantaneous force transmission properties of *each* cam of a multi-input cam-follower mechanism, in contrast to other indices of merit for multi-input systems that consider the mechanism as a whole. The use of Definition (2) thus allows each cam to be evaluated individually and avoids situations in which an overall index of merit may indicate acceptable force transmission properties for a mechanism as a whole while failing to indicate poor force transmission at any one particular cam.

Conclusions. We have shown that all the definitions of pressure angle in common use are inadequate for use in the analysis of systems with multiple inputs. An alternative definition has been presented that can be applied to both single-input and multi-input systems. The mathematical consequences of this new definition have been investigated, and a simple formula has been derived that enables pressure angle to be easily determined. The use of this formula has been illustrated with both single-input and multi-input examples. These examples show that the determination of pressure angle can be reduced to simply the determination of the direction $\mathbf{n}_{R_i}$, corresponding to direction of the force applied by a cam to the follower, and the direction $\mathbf{f}_i$, corresponding to the direction of the component of the velocity of the point of the follower in contact with the cam due to the motion of that cam, and that thus the pressure angle ϕ_i for multi-input mechanisms is easily calculated from the relation

$$\phi_i = \cos^{-1} |\mathbf{n}_{R_i} \cdot \mathbf{f}_i| \quad (0 < \phi_p \leq 90°). \tag{17}$$

We have also discussed how the definition and formula can be applied to the determination of transmission angle for linkages driven by single or multiple inputs.

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