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The Supersymmetric Standard Model

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ABSTRACT

This set of lectures introduces at an elementary level the supersymmetric Standard Model and discusses some of its phenomenological properties.*

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*We intend to regularly update this review. Therefore we would be very grateful for your corrections, comments and suggestions.

1 Introduction

The Standard Model of Particle Physics is an extremely successful theory which has been tested experimentally to a high level of accuracy [1, 2]. After the discovery of the top quark the Higgs boson which is predicted to exist by the Standard Model is the only 'missing' ingredient that has not been directly observed yet.

However, a number of theoretical prejudices suggest that the Standard Model is not the 'final answer' of nature but rather an effective description valid up to the weak scale of order $\mathcal{O}(100GeV)$. The arbitrariness of the spectrum and gauge group, the large number of free parameters, the smallness of the weak scale compared to the Planck scale and the inability to turn on gravity suggest that at higher energies (shorter distances) a more fundamental theory will be necessary to describe nature. Over the past 20 years various extensions of the Standard Model such as Technicolor [3, 4], Grand Unified Theories [5, 6], Supersymmetry [7, 8] or String Theory [9] have been proposed. In recent years supersymmetric extensions of the Standard Model became very popular also among experimentalists not necessarily because of their convincing solution of the above problems but rather because most other contenders have been (more or less) ruled out by now. Another reason for the popularity of supersymmetric theories among theorists is the fact that the low energy limit of superstring theory – a promising candidate for a unification of all interactions including gravity – is (by and large) supersymmetric.

This set of lectures give an elementary introduction to the supersymmetric Standard Model. Section 2 contains some of the necessary background on generic supersymmetric field theories while section 3 develops supersymmetric extensions of the Standard Model and discusses spontaneous breaking of supersymmetry. In section 4 extensions of the Standard Model with softly broken supersymmetry are presented and some of the phenomenological properties are discussed. Section 5 contains a summary and our conventions which follow rather closely ref. [8] are recorded in an appendix.

These lectures are not meant to review the latest developments of the supersymmetric Standard Model but rather attempts to give an elementary introduction from a "modern" point of view. Many excellent review articles on supersymmetry and the supersymmetric Standard Model do exist and have been heavily used in these lectures [10] – [20]. In addition, a collection of some of the classic papers concerning the subject can be found in ref. [21].

2 Introduction to Supersymmetry

Supersymmetry is a symmetry between bosons and fermions or more precisely it is a symmetry between states of different spin [7]. For example, a spin-0 particle is mapped to a spin-$\frac{1}{2}$ particle under a supersymmetry transformation. Thus, the generators $Q_\alpha, \overline{Q}_{\dot\alpha}$ of the supersymmetry transformation must transform in the spin-$\frac{1}{2}$ representations of the Lorentz group. These new fermionic generators form together with the four-momentum P_m and the generators of the Lorentz transformations M^{mn} a graded Lie algebra which features in addition to commutators also anticommutators in their defining relations. The simplest ($N = 1$) supersymmetry algebra reads:

$$\begin{aligned}
\{Q_\alpha, \overline{Q}_{\dot\beta}\} &= 2\sigma^m_{\alpha\dot\beta}\, P_m \\
\{Q_\alpha, Q_\beta\} &= \{\overline{Q}_{\dot\alpha}, \overline{Q}_{\dot\beta}\} = 0 \\
[\overline{Q}_{\dot\alpha}, P_m] &= [Q_\alpha, P_m] = 0 \\
[Q_\alpha, M^{mn}] &= \frac{1}{2}\,\sigma^{mn\beta}_{\ \ \alpha} Q_\beta \\
[\overline{Q}_{\dot\alpha}, M^{mn}] &= \frac{1}{2}\,\bar\sigma^{mn\dot\beta}_{\ \ \dot\alpha} Q_{\dot\beta}
\end{aligned} \tag{1}$$

where we used the notation and convention of ref. [8]. σ^m are the Pauli matrices and the σ^{mn} are defined in the appendix.†

The particle states in a supersymmetric field theory form representations (supermultiplets) of the supersymmetry algebra (1). We do not recall the entire representation theory here (see, for example, refs. [12, 8]) but only highlight a few generic features:

(a) There is an equal number of bosonic degrees of freedom n_B and fermionic degrees of freedom n_F in a supermultiplet

$$n_B = n_F\ . \tag{2}$$

† In general it is possible to have N sets of supersymmetry generators $Q^I_\alpha, \overline{Q}^I_{\dot\alpha}, I = 1, \ldots, N$, in which case one refers to N-extended supersymmetry. Such extended superalgebras have been classified by Haag, Lopuszanski and Sohnius [22] generalizing earlier work of Coleman and Mandula [23] who showed that the possible bosonic symmetries of the S-matrix of a four-dimensional, local, relativistic quantum field theory consist of the generators of the Poincaré group and a finite number of generators of a compact Lie group, which are Lorentz scalars. In ref. [22] this theorem was generalized to also include symmetry transformations generated by fermionic operators and all possible superalgebras were found. In extensions of the Standard Model N-extended supersymmetries have played no role so far since they cannot accommodate the chiral structure of the Standard Model.

(b) The masses of all states in a supermultiplet are degenerate. In particular the masses of bosons and fermions are equal[‡]

$$m_B = m_F \ . \tag{3}$$

(c) Q has mass dimension $\frac{1}{2}$ and thus the mass dimensions of the fields in a supermultiplet differ by $\frac{1}{2}$.

The two irreducible multiplets which are important for constructing the supersymmetric Standard Model are the chiral multiplet and the vector multiplet which we discuss in turn now.

2.1 The chiral supermultiplet

The chiral supermultiplet Φ [7] contains a complex scalar field $A(x)$ of spin 0 and mass dimension 1, a Weyl fermion $\psi_\alpha(x)$ of spin $\frac{1}{2}$ and mass dimension $\frac{3}{2}$ and an auxiliary complex scalar field $F(x)$ of spin 0 and mass dimension 2

$$\Phi = (A(x), \psi_\alpha(x), F(x)) \ . \tag{4}$$

Φ has off-shell four real bosonic degrees of freedom ($n_B = 4$) and four real fermionic degrees of freedom ($n_F = 4$) in accord with (2). The supersymmetry transformations act on the fields in the multiplet as follows:

$$\begin{aligned} \delta_\xi A &= \sqrt{2}\xi\psi \\ \delta_\xi \psi &= \sqrt{2}\xi F + i\sqrt{2}\sigma^m \bar{\xi}\partial_m A \\ \delta_\xi F &= i\sqrt{2}\bar{\xi}\bar{\sigma}^m \partial_m \psi \end{aligned} \tag{5}$$

where we used the conventions of ref. [8] and the appendix. The parameters of the transformation ξ^α are constant, complex anticommuting Grassmann parameters obeying

$$\xi_\alpha \xi_\beta = -\xi_\beta \xi_\alpha . \tag{6}$$

The transformations (5) can be thought of as generated by the operator

$$\delta_\xi = \xi Q + \bar{\xi}\overline{Q} \tag{7}$$

with Q and $\overline{Q}$ obeying (1). This can be explicitly checked by evaluating the commutators $[\delta_\xi,\, \delta_\eta]$ on the fields A, ψ and F.

[‡]This follows immediately from the fact that P^2 is a Casimir operator of the supersymmetry algebra (1) $[P^2, Q] = [P^2, M^{mn}] = 0$.

Exercise: *Show* $[\delta_\xi, \delta_\eta] = 2i(\eta\sigma^m\bar{\xi} - \xi\sigma^m\bar{\eta})\partial_m$ *by using (7) and (1).*

Exercise: *Evaluate the commutator* $[\delta_\xi, \delta_\eta]$ *using (5) for all three fields* A, ψ *and* F *and show that this is consistent with the results of the previous exercise.*

The field F has the highest mass dimension of the members of the chiral multiplet and therefore is called the highest component. As a consequence it cannot transform into any other field of the multiplet but only into their derivatives. This is not only true for the chiral multiplet (as can be seen explicitly in (5)) but holds for any supermultiplet. This fact can be used to construct Lagrangian densities which transform into a total derivative under supersymmetry transformations leaving the corresponding actions invariant. We do not review here the method for systematically constructing supersymmetric actions which is done most efficiently using a superspace formalism. Since these lectures focus on the phenomenological properties of supersymmetry we refer the reader to the literature [8] for further details and only quote the results.

For the chiral multiplet a supersymmetric and renormalizable Lagrangian is given by [8]

$$\begin{aligned}
\mathcal{L}(A, \psi, F) &= -i\bar{\psi}\bar{\sigma}^m\partial_m\psi - \partial_m\bar{A}\partial^m A + F\bar{F} \\
&\quad + m(AF + \bar{A}\bar{F} - \frac{1}{2}(\psi\psi + \bar{\psi}\bar{\psi})) \\
&\quad + Y(A^2F + \bar{A}^2\bar{F} - A\psi\psi - \bar{A}\bar{\psi}\bar{\psi}) ,
\end{aligned} \tag{8}$$

where m and Y are real parameters. This action has the peculiar property that no kinetic term for F appears. As a consequence the equations of motion for F are purely algebraic

$$\frac{\delta\mathcal{L}}{\delta\bar{F}} = F + m\bar{A} + Y\bar{A}^2 = 0, \qquad \frac{\delta\mathcal{L}}{\delta F} = \bar{F} + mA + YA^2 = 0.$$

Thus F is a non-dynamical, 'auxiliary' field which can be eliminated from the action algebraically by using its equation of motion. This yields

$$\begin{aligned}
\mathcal{L}(A, \psi, F = -m\bar{A} - Y\bar{A}^2) &= -i\bar{\psi}\bar{\sigma}^m\partial_m\psi - \partial_m\bar{A}\partial^m A \\
&- \frac{m}{2}(\psi\psi + \bar{\psi}\bar{\psi}) - Y(A\psi\psi + \bar{A}\bar{\psi}\bar{\psi}) - V(A, \bar{A})
\end{aligned} \tag{9}$$

where $V(A, \bar{A})$ is the scalar potential given by

$$\begin{aligned}
V(A, \bar{A}) &= |\, mA + YA^2 \,|^2 \\
&= m^2A\bar{A} + mY(A\bar{A}^2 + \bar{A}A^2) + Y^2A^2\bar{A}^2 \\
&= F\bar{F}\,|_{\frac{\delta\mathcal{L}}{\delta F} = \frac{\delta\mathcal{L}}{\delta \bar{F}} = 0} \ .
\end{aligned} \tag{10}$$

As can be seen from (9) and (10) after elimination of F a standard renormalizable Lagrangian for a complex scalar A and a Weyl fermion ψ emerges. However (9) is not the most general renormalizable Lagrangian for such fields. Instead it satisfies the following properties:

- $\mathcal{L}$ only depends on two independent parameters, the mass parameter m and the dimensionless Yukawa coupling Y. In particular, the $(A\bar{A})^2$ coupling is not controlled by an independent parameter (as it would be in non-supersymmetric theories) but determined by the Yukawa coupling Y.
- The masses for A and ψ coincide in accord with (3).[§]
- V is positive semi-definite, $V \geq 0$.

2.2 The vector supermultiplet

The vector supermultiplet V contains a gauge boson v_m of spin 1 and mass dimension 1, a Weyl fermion (called the gaugino) λ of spin $\frac{1}{2}$ and mass dimension $\frac{3}{2}$, and a real scalar field D of spin 0 and mass dimension 2

$$V = (v_m(x), \lambda_\alpha(x), D(x)) \ . \tag{11}$$

Similar to the chiral multiplet also the vector multiplet has $n_B = n_F = 4$.

The vector multiplet can be used to gauge the action of the previous section. An important consequence of the theorems of refs. [23, 22] is the fact that the generators T^a of a compact gauge group G have to commute with the supersymmetry generators

$$[T^a, Q_\alpha] = [T^a, \overline{Q}_{\dot{\alpha}}] = 0 \ . \tag{12}$$

Therefore all members of a chiral multiplet (A, ψ, F) have to reside in the same representation of the gauge group. Similarly, the members of the vector multiplet have to transform in the adjoint representation of G and thus they all are Lie-algebra valued fields

$$v_m = v_m^a T^a \ , \qquad \lambda_\alpha = \lambda_\alpha^a T^a \ , \qquad D = D^a T^a \ . \tag{13}$$

The supersymmetry transformations of the components of the vector multiplet are [8]:

$$\begin{aligned} \delta_\xi v_m^a &= -i\bar{\lambda}^a \bar{\sigma}^m \xi + i\bar{\xi}\bar{\sigma}^m \lambda^a \ , \\ \delta_\xi \lambda^a &= i\xi D^a + \sigma^{mn}\xi F_{mn}^a \ , \\ \delta_\xi D^a &= -\xi\sigma^m D_m \bar{\lambda}^a - D_m \lambda^a \sigma^m \bar{\xi} \ . \end{aligned} \tag{14}$$

[§]As immediate consequence of this feature one notes that supersymmetry must be explicitly or spontaneously broken in nature.

The field strength of the vector bosons F^a_{mn} and the covariant derivative $D_m\lambda^a$ are defined according to

$$\begin{aligned} F^a_{mn} &:= \partial_m v^a_n - \partial_n v^a_m - g f^{abc} v^b_m v^c_n \ , \\ D_m\lambda^a &:= \partial_m \lambda^a - g f^{abc} v^b_m \lambda^c \ , \end{aligned} \tag{15}$$

where f^{abc} are the structure constants of the Lie algebra and g is the gauge coupling. A gauge invariant, renormalizable and supersymmetric Lagrangian for the vector multiplet is given by

$$\mathcal{L} = -\frac{1}{4} F^a_{mn} F^{mn\,a} - i\bar{\lambda}^a \bar{\sigma}^m D_m \lambda^a + \frac{1}{2} D^a D^a \ . \tag{16}$$

As before the equation of motion for the auxiliary D-field is purely algebraic $D^a = 0$.

A gauge invariant, renormalizable Lagrangian containing a set of chiral multiplets (A^i, ψ^i, F^i) coupled to vector multiplets is found to be [8]

$$\begin{aligned} \mathcal{L}(A^i, \psi^i, F^i, v^a_m, \lambda^a, D^a) \;=\; & -\frac{1}{4} F^a_{mn} F^{mn\ a} - i\bar{\lambda}^a \bar{\sigma}^m D_m \lambda^a + \frac{1}{2} D^a D^a \\ & - D_m A^i D^m \bar{A}^i - i\bar{\psi}^i \bar{\sigma}^m D_m \psi^i + \bar{F}^i F^i \\ & + i\sqrt{2} g (\bar{A}^i T^a_{ij} \psi^j \lambda^a - \bar{\lambda}^a T^a_{ij} A^i \bar{\psi}^j) \\ & + g D^a \bar{A}^i T^a_{ij} A^j - \frac{1}{2} W_{ij} \psi^i \psi^j - \frac{1}{2} \bar{W}_{ij} \bar{\psi}^i \bar{\psi}^j \\ & + F^i W_i + \bar{F}^i \bar{W}_i \ , \end{aligned} \tag{17}$$

where the covariant derivatives are defined by

$$\begin{aligned} D_m A^i &:= \partial_m A^i + i g v^a_m T^a_{ij} A^i \ , \\ D_m \psi^i &:= \partial_m \psi^i + i g v^a_m T^a_{ij} \psi^j \ . \end{aligned} \tag{18}$$

W_i and W_{ij} in (17) are the derivatives of a holomorphic function $W(A)$ called the superpotential

$$\begin{aligned} W(A) &= \frac{1}{2} m_{ij} A^i A^j + \frac{1}{3} Y_{ijk} A^i A^j A^k \ , \\ W_i &\equiv \frac{\partial W}{\partial A^i} = m_{ij} A^j + Y_{ijk} A^j A^k \ , \\ W_{ij} &\equiv \frac{\partial^2 W}{\partial A^i \partial A^j} = m_{ij} + 2 Y_{ijk} A^k \ . \end{aligned} \tag{19}$$

By explicitly inserting (19) into (17) one observes that the m_{ij} are mass parameters while the Y_{ijk} are Yukawa couplings. Supersymmetry forces W to be a holomorphic function of the scalar fields A while renormalizability restricts W to be at most a cubic polynomial of A. Finally, the parameters m_{ij} and Y_{ijk} are further constrained by gauge invariance.

As before, F^i and D^a obey algebraic equations of motion which read

$$\begin{aligned}
\frac{\delta\mathcal{L}}{\delta F} = 0 \quad &\Rightarrow \quad \bar{F}_i + W_i = 0 \ , \\
\frac{\delta\mathcal{L}}{\delta \bar{F}} = 0 \quad &\Rightarrow \quad F_i + \bar{W}_i = 0 \ , \\
\frac{\delta\mathcal{L}}{\delta D^a} = 0 \quad &\Rightarrow \quad D^a + g\bar{A}^i T^a_{ij} A^j = 0 \ .
\end{aligned} \tag{20}$$

They can be used to eliminate the auxiliary fields F^i and D^a from the Lagrangian (17) and one obtains

$$\begin{aligned}
&\mathcal{L}(A^i, \psi^i, v^a_m, \lambda^a, F_i = -\bar{W}_i, D^a = -g\bar{A}^i T^a_{ij} A^j) = \\
&\quad -\frac{1}{4} F^a_{mn} F^{mn\ a} - i\bar{\lambda}^a \bar{\sigma}^m D_m \lambda^a - D_m A^i D^m \bar{A}^i - i\bar{\psi}^i \bar{\sigma}^m D_m \psi^i \\
&\quad + i\sqrt{2} g (\bar{A}^i T^a_{ij} \psi^j \lambda^a - \bar{\lambda}^a T^a_{ij} A^i \bar{\psi}^j) - \frac{1}{2} W_{ij} \psi^i \psi^j - \frac{1}{2} \bar{W}_{ij} \bar{\psi}^i \bar{\psi}^j - V(A, \bar{A})
\end{aligned} \tag{21}$$

where

$$\begin{aligned}
V(A, \bar{A}) &= W_i \bar{W}_i + \frac{1}{2} g^2 (\bar{A}^i T^a_{ij} A^j)(\bar{A}^i T^a_{ij} A^j) \\
&= (F^i \bar{F}^i + \frac{1}{2} D^a D^a) \mid_{\frac{\delta\mathcal{L}}{\delta F} = 0, \frac{\delta\mathcal{L}}{\delta D^a} = 0} \\
&\geq 0 \ .
\end{aligned} \tag{22}$$

As before the scalar potential $V(A, \bar{A})$ is positive semi-definite.

Exercise: *Insert (20) into (17) and derive (21).*

3 A Supersymmetric Extension of the Standard Model

3.1 The Standard Model

In this section we briefly review some basic features of the Standard Model. The Standard Model is a quantum gauge field theory with a chiral gauge group $G_{\rm SM} = SU(3) \times SU(2) \times U(1)_Y$. The spectrum of particles includes three families of quarks and leptons, the gauge bosons (gluons, $W^{\pm}, Z^0$, photon) of $G_{\rm SM}$ and one spin-0 Higgs doublet. In table 1 the particle content and the corresponding

		SU(3)	SU(2)	$U(1)_Y$	$U(1)_{\text{em}}$
quarks	$q_L^I = \begin{pmatrix} u_L^I \\ d_L^I \end{pmatrix}$	3	2	$\frac{1}{6}$	$\begin{pmatrix} \frac{2}{3} \\ -\frac{1}{3} \end{pmatrix}$
	u_R^I	$\bar{3}$	1	$-\frac{2}{3}$	$-\frac{2}{3}$
	d_R^I	$\bar{3}$	1	$\frac{1}{3}$	$\frac{1}{3}$
leptons	$l_L^I = \begin{pmatrix} \nu_L^I \\ e_L^I \end{pmatrix}$	1	2	$-\frac{1}{2}$	$\begin{pmatrix} 0 \\ -1 \end{pmatrix}$
	e_R^I	1	1	1	1
Higgs	$h = \begin{pmatrix} h^0 \\ h^- \end{pmatrix}$	1	2	$-\frac{1}{2}$	$\begin{pmatrix} 0 \\ -1 \end{pmatrix}$
gauge bosons	G	8	1	0	0
	W	1	3	0	$(0, \pm 1)$
	B	1	1	0	0

Table 1: The particle content of the Standard Model. The index $I = 1, 2, 3$ labels the three families of chiral quarks q_L^I, u_R^I, d_R^I and chiral leptons l_L^I, e_R^I. All of them are Weyl fermions and transform in the $(\frac{1}{2}, 0)$ representation of the Lorentz group (they have an undotted spinor index α). The subscripts R, L do not specify the representation of the Lorentz group but instead are used to indicate the different transformation properties under the chiral gauge group $SU(2) \times U(1)$. This somewhat unconventional notation is used to make a smooth transition to the supersymmetric Standard Model later on. The electromagnetic charge listed in the last column is defined by $Q_{em} = T^3_{SU(2)} + Q_Y$.

gauge quantum numbers are displayed. The Lagrangian of the Standard Model reads

$$
\begin{aligned}
\mathcal{L} \;=\; & -\frac{1}{4}\sum_{(a)=1}^{3}\Big((F^{b}_{mn}F^{mn\ b})_{(a)}\Big) - D_m h D^m \bar{h} \\
& + \sum_{I=1}^{3}\Big(-i\bar{q}^I_L \not{D}\, q^I_L - i\bar{u}^I_R \not{D}\, u^I_R - i\bar{d}^I_R \not{D}\, d^I_R - i\bar{l}^I_L \not{D}\, l^I_L - i\bar{e}^I_R \not{D}\, e^I_R\Big) \qquad (23) \\
& - \sum_{IJ=1}^{3}\Big((Y_u)_{IJ}\bar{h} q^I_L u^J_R + (Y_d)_{IJ} h q^I_L d^J_R + (Y_l)_{IJ} h l^I_L e^J_R + h.c.\Big) - V(h,\bar{h}),
\end{aligned}
$$

where $\not{D} = \sigma^m D_m$ and the index (a) labels the 3 different factors in the gauge group. $V(h,\bar{h})$ is the scalar potential for the Higgs doublet which is chosen to be

$$V(h,\bar{h}) = \mu^2 h\bar{h} + \lambda(h\bar{h})^2 \ . \qquad (24)$$

In order to have a bounded potential $\lambda > 0$ has to hold. For $\mu^2 < 0$ the electroweak gauge group $SU(2)\times U(1)_Y$ is spontaneously broken down to $U(1)_{\rm em}$. In this case the minimum of the potential is not at $\langle h\rangle = 0$, but at $\langle h\bar{h}\rangle = -\frac{\mu^2}{2\lambda}$.

Exercise: *Give explicitly all covariant derivatives in (23).*
Exercise: *Check that the Lagrangian (23) is gauge and Lorentz invariant.*

3.2 Supersymmetric Extensions

Let us now turn to the supersymmetric generalization of the Standard Model.¶ The idea is to promote the Lagrangian (23) to a supersymmetric Lagrangian. As we learned in the previous section supersymmetry requires the presence of additional states which form supermultiplets with the known particles. Since all states of a supermultiplet carry the same gauge quantum numbers we need at least a doubling of states: For every field of the SM one has to postulate a superpartner with the exact same gauge quantum numbers and a spin such that it can form an appropriate supermultiplet. More specifically, the quarks and leptons are promoted to chiral multiplets by adding scalar (spin-0) squarks $(\tilde{q}^I_L, \tilde{u}^I_R, \tilde{d}^I_R)$ and sleptons $(\tilde{l}^I_L, \tilde{e}^I_R)$ to the spectrum. The gauge bosons are promoted to vector multiplets by adding the corresponding spin-$\frac{1}{2}$ gauginos $(\tilde{G}, \tilde{W}, \tilde{B})$ to the spectrum. Finally, the Higgs boson is also promoted to a chiral multiplet with a spin-$\frac{1}{2}$ Higgsino superpartner. However, the supersymmetric version of the Standard Model cannot 'live' with only one Higgs doublet and at least a

¶See also [10] - [21].

second Higgs doublet has to be added. This can be seen from the fact that one cannot write down a supersymmetric version of the Yukawa interactions of the Standard Model without introducing a second Higgs doublet. The reason is the definite chirality of the Higgsino. Another way to see the necessity of a second Higgs doublet is the fact that the Higgsino is a chiral fermion which carries $U(1)$ hypercharge and hence it upsets the anomaly cancellation condition. Thus a second Higgsino with opposite $U(1)$ charge is necessary and supersymmetry then also requires a second spin-0 Higgs doublet.[‖] The precise spectrum of the supersymmetric Standard Model is summarized in table 2.

The Lagrangian for the supersymmetric Standard Model has to be of the form (17) with an appropriate superpotential W. It has to be chosen such that the Lagrangian of the non-supersymmetric Standard Model (23) is contained. This is achieved by

$$W = \sum_{IJ} \Big((Y_u)_{IJ} h_u \tilde{q}_L^I \tilde{u}_R^J + (Y_d)_{IJ} h_d \tilde{q}_L^I \tilde{d}_R^J + (Y_l)_{IJ} h_d \tilde{l}_L^I \tilde{l}_R^I \Big) + \mu\, h_u h_d \ . \qquad (25)$$

Once W it specified also the scalar potential is fixed. Of particular importance is the scalar potential for the Higgs fields since it controls the electroweak symmetry breaking. Using (22) and (25) one derives the Higgs potential for the two neutral Higgs fields h_d^0, h_u^0 by setting all other scalars to zero[**]

$$V(h_d^0, h_u^0) = |\mu|^2 \left(|h_d^0|^2 + |h_u^0|^2\right) + \frac{1}{8}\left(g_1^2 + g_2^2\right)\left(|h_u^0|^2 - |h_d^0|^2\right)^2 \ . \qquad (26)$$

The coupling of the terms quartic in the Higgs fields is not an independent parameter but instead determined by the gauge couplings g_1 of $U(1)_Y$ and g_2 of $SU(2)$. Thus it seems that the number of parameters is reduced. However, now there are two possible vacuum expectation values $\langle h_u^0 \rangle, \langle h_d^0 \rangle$ – one more than in the Standard Model.

Exercise: *Derive (26) from (22) and (25).*

In the last section we learned that the potential of any supersymmetric theory is positive semi-definite and the Higgs potential of eq. (26) is no exception as can be seen explicitly: $|\mu|^2$ cannot be chosen negative. Thus the minimum of V necessarily sits at $\langle h_u^0 \rangle = \langle h_d^0 \rangle = 0$ which corresponds to a vacuum with unbroken $SU(2) \times U(1)$. Therefore, the supersymmetric version of the Standard

[‖] Of course, extensions with more Higgs doublets are also possible, but two is the minimal number.

[**] Note that the scalars can only be set to zero in the potential V but not in the superpotential W since the computation of the potential requires taking appropriate derivatives of W.

	supermultiplet	F	B	SU(3)	SU(2)	$U(1)_Y$	$U(1)_{\rm em}$
quarks	$Q_L^I = \begin{pmatrix} U_L^I \\ D_L^I \end{pmatrix}$	q_L^I	$\tilde{q}_L^I$	3	2	$\frac{1}{6}$	$\begin{pmatrix} \frac{2}{3} \\ -\frac{1}{3} \end{pmatrix}$
	U_R^I	u_R^I	$\tilde{u}_R^I$	$\bar{3}$	1	$-\frac{2}{3}$	$-\frac{2}{3}$
	D_R^I	d_R^I	$\tilde{d}_R^I$	$\bar{3}$	1	$\frac{1}{3}$	$\frac{1}{3}$
leptons	$L_L^I = \begin{pmatrix} \mathcal{N}_L^I \\ E_L^I \end{pmatrix}$	l_L^I	$\tilde{l}_L^I$	1	2	$-\frac{1}{2}$	$\begin{pmatrix} 0 \\ -1 \end{pmatrix}$
	E_R^I	e_R^I	$\tilde{e}_R^I$	1	1	1	1
Higgs	$H_d = \begin{pmatrix} H_d^0 \\ H_d^- \end{pmatrix}$	$\begin{pmatrix} \tilde{h}^0 \\ \tilde{h}^- \end{pmatrix}$	$\begin{pmatrix} h_d^0 \\ h_d^- \end{pmatrix}$	1	2	$-\frac{1}{2}$	$\begin{pmatrix} 0 \\ -1 \end{pmatrix}$
	$H_u = \begin{pmatrix} H_u^+ \\ H_u^0 \end{pmatrix}$	$\begin{pmatrix} \tilde{h}^+ \\ \tilde{h}^0 \end{pmatrix}$	$\begin{pmatrix} h_u^+ \\ h_u^0 \end{pmatrix}$	1	2	$\frac{1}{2}$	$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$
gauge bosons	G	$\tilde{G}$	G	8	1	0	0
	W	$\tilde{W}$	W	1	3	0	$(0, \pm 1)$
	B	$\tilde{B}$	B	1	1	0	0

Table 2: Particle content of the supersymmetric Standard Model. The column below 'F' ('B') denotes the fermionic (bosonic) content of the model.

Model as it is defined so far – the spectrum of table 2 with interactions specified by the Lagrangian (17) with the W of (25) – cannot accommodate a vacuum with spontaneously broken electroweak symmetry. A second phenomenological problem is the presence of all the new supersymmetric states which have the same mass as their superpartners but are not observed in nature. As we said before, supersymmetry itself necessarily has to appear in its broken phase and as we will see electroweak symmetry breaking is closely tied to the breakdown of supersymmetry.

Before we close this section let us note that in addition to the couplings of (25) gauge and Lorentz invariance also allows terms in W which are of the form

$$h_u \tilde{l}_L, \quad \tilde{l}_L \tilde{q}_L \tilde{d}_R, \quad \tilde{d}_R \tilde{d}_R \tilde{u}_R, \quad \tilde{l}_L \tilde{l}_L \tilde{e}_R \ . \tag{27}$$

These terms violate baryon or lepton number conservation and thus easily lead to unacceptable physical consequences (for example the proton could become unstable [24]). Such couplings can be excluded by imposing a discrete R-parity [25]. Particles of the Standard Model (includuing both Higgs doublets) are assigned R-charge 1 while all new supersymmetric particles are assigned R-charge -1. This eliminates all terms of (27) while the superpotential of (25) is left invariant. An immediate consequence of this additional symmetry is the fact that the lightest supersymmetric particle (often denoted by the 'LSP') is necessarily stable. However, one should stress that R-parity is not a phenomenological necessity. Viable models with broken R-parity can be constructed and they also can have some phenomenological appeal [26].

Exercise: *Check the gauge and Lorentz invariance for each term in (27) and compute their R-charge.*

3.3 Spontaneous breaking of supersymmetry

In the previous section we learned that in the simplest supersymmetric extension of the Standard Model the electroweak symmetry is unbroken. However, so far we constructed a manifestly supersymmetric extension but from the mass degeneracy of each multiplet (3) it is already clear that supersymmetry cannot be an exact symmetry in nature but has to be either spontaneously or explicitly broken. Therefore we now turn to the question of spontaneous supersymmetry breaking and return to the electroweak symmetry breaking afterwards.

Let us first recall the order parameter for supersymmetry breaking. Multiplying the anticommutator $\{Q_\alpha, \overline{Q}_{\dot{\alpha}}\} = 2\sigma^m_{\alpha\dot{\alpha}} P_m$ of the supersymmetry-algebra (1) with $\bar{\sigma}^n$ and using $Tr(\sigma^m\bar{\sigma}^n) = -2\eta^{mn}$ results in

$$\bar{\sigma}^{n\alpha\dot{\alpha}}\{Q_\alpha, \overline{Q}_{\dot{\alpha}}\} = -4P^n \ .$$

Thus the Hamiltonian H of a supersymmetric theory is expressed as the 'square' of the supercharges

$$H = P_0 = \frac{1}{4}\left(Q_1\overline{Q}_1 + \overline{Q}_1 Q_1 + Q_2\overline{Q}_2 + \overline{Q}_2 Q_2\right) \ . \tag{28}$$

This implies that H is a positive semi-definite operator on the Hilbert space

$$\langle\psi|H|\psi\rangle \ \geq 0 \quad \forall\psi \ . \tag{29}$$

Supersymmetry is unbroken if the supercharges annihilate the vacuum $Q_\alpha|0\rangle = \overline{Q}_{\dot{\alpha}}|0\rangle = 0$. From (28) we learn that also H annihilates a supersymmetric vacuum $H|0\rangle = 0$. This in turn implies that the scalar potential V of a supersymmetric field theory which has a supersymmetric ground state has to vanish at its minimum

$$\langle H\rangle = 0 \quad \Rightarrow \quad \langle V\rangle \equiv V(A,\bar{A})|_{\rm min} = 0 \ . \tag{30}$$

The general form of the scalar potential $V = F^i\bar{F}^i + \frac{1}{2}D^aD^a$ was given in (22). Since V is positive semi-definite one immediately concludes from (30) that in a supersymmetric ground state

$$\langle F^i\rangle \equiv F^i|_{\rm min} = 0 \quad \text{and} \quad \langle D^a\rangle \equiv D^a|_{\rm min} = 0 \tag{31}$$

has to hold. The converse is also true

$$\langle F^i\rangle \neq 0 \quad \text{or} \quad \langle D^a\rangle \neq 0 \quad \Rightarrow \quad V|_{min} > 0 \quad \Rightarrow \quad Q_\alpha|0> \neq 0 \tag{32}$$

and supersymmetry is spontaneously broken. Thus $\langle F^i\rangle$ and $\langle D^a\rangle$ are the order parameters of supersymmetry breaking in that non-vanishing F- or D-terms signal spontaneous supersymmetry breaking.

Specific potentials which do lead to non-vanishing D- or F-terms have been constructed [27, 28]. For example, the O'Raifeartaigh model [28] has three chiral superfields A_0, A_1, A_2 and the following superpotential:

$$W = \lambda A_0 + mA_1A_2 + gA_0A_1^2 \ , \qquad m^2 > 2\lambda g \ . \tag{33}$$

By minimizing V it can be shown that $F_0|_{min} \neq 0$ and therefore supersymmetry is broken. Furthermore the mass spectrum of the 6 real bosons and the 3 Weyl fermions is found to be

$$\begin{aligned} \text{Bosons}: &\qquad (0,0,m^2,m^2,m^2 \pm 2\lambda g) \\ \text{Fermions}: &\qquad (0,m^2,m^2) \ . \end{aligned} \tag{34}$$

Thus the mass degeneracy between bosons and fermions is lifted but nevertheless a 'mass sum rule' still holds

$$\sum_{\text{bosons}} M_b^2 = 2 \sum_{\text{fermions}} M_f^2 \ . \tag{35}$$

Exercise: *Minimize V using (22), (33) and compute $F_i|_{\min}$. Verify the mass spectrum (34) and the sum rule (35).*

Unfortunately, the sum rule (35) is not a coincidence but a special case of a general sum rule which holds in any theory with spontaneously broken supersymmetry. Let us therefore proceed and derive this sum rule. In general the mass matrix of the bosons has the following form

$$V(A,\bar{A})|_{\text{mass terms}} \ = \ \mu^2_{i\bar{j}} A^i \bar{A}^j + \mu^2_{ij} A^i A^j + \mu^2_{\bar{i}\bar{j}} \bar{A}^i \bar{A}^j \ = \ (\bar{A} \quad A) M_0^2 \begin{pmatrix} A \\ \bar{A} \end{pmatrix} \tag{36}$$

where

$$M_0^2 = \begin{pmatrix} \frac{1}{2}\mu^2_{i\bar{j}} & \mu^2_{ij} \\ \mu^2_{\bar{i}\bar{j}} & \frac{1}{2}\mu^2_{i\bar{j}} \end{pmatrix} \ . \tag{37}$$

The entries in the mass matrix are determined by the appropriate derivatives of the potential evaluated at its minimum

$$\mu^2_{i\bar{j}} = V_{i\bar{j}}|_{min} \ , \qquad \mu^2_{ij} = V_{ij}|_{min} \ , \qquad \mu^2_{\bar{i}\bar{j}} = V_{\bar{i}\bar{j}}|_{min} \ , \tag{38}$$

where $V_{i\bar{j}} \equiv \frac{\partial^2 V}{\partial A^i \partial \bar{A}^j}$ etc. Using (22) one derives

$$\begin{aligned} V &= W_i \bar{W}_i + \frac{1}{2} D^a D^a \ , \\ V_j &= W_{ij} \bar{W}_i + D^a_j D^a \ , \\ V_{j\bar{k}} &= W_{ij} \bar{W}_{ik} + D^a_j D^a_{\bar{k}} + D^a D^a_{j\bar{k}} \ , \end{aligned} \tag{39}$$

where again the indices $i, j, \ldots$ denote derivatives with respect to $A^i, A^j, \ldots$. Inserted into (37) one obtains for the trace of the mass matrix:

$$\text{Tr} M_0^2 = \text{Tr}\mu^2_{i\bar{j}} = \text{Tr} V_{i\bar{j}}|_{min} = \text{Tr}(W_{ij}\bar{W}_{ik} + D^a_j D^a_{\bar{k}} + D^a D^a_{j\bar{k}})|_{min} \ . \tag{40}$$

For the fermion masses the relevant pieces of the Lagrangian (17) are

$$\mathcal{L} = i\sqrt{2} g \ (\ \bar{A}^i T^a_{ij} \psi^j \lambda^a - \bar{\lambda}^a T^a_{ij} A^i \bar{\psi}^j \ ) - \frac{1}{2} W_{ij} \psi^i \psi^j - \frac{1}{2} \bar{W}_{ij} \bar{\psi}^i \bar{\psi}^j + \ldots \ . \tag{41}$$

This can be rewritten as

$$\mathcal{L} = -\frac{1}{2} (\psi^i \ \lambda^a) M_{1/2} \begin{pmatrix} \psi^j \\ \lambda^b \end{pmatrix} \ + \text{h.c.} \ + \ldots \tag{42}$$

where

$$M_{1/2} = \begin{pmatrix} W_{ij} & -i\sqrt{2}g\bar{A}^i T^a_{ij} \\ -i\sqrt{2}g\bar{A}^i T^b_{ij} & 0 \end{pmatrix} = \begin{pmatrix} W_{ij} & i\sqrt{2}D^a_j \\ i\sqrt{2}D^b_i & 0 \end{pmatrix} . \qquad (43)$$

Thus, we obtain

$$\mathrm{Tr} M_{1/2}\bar{M}_{1/2} = \mathrm{Tr}(W_{ik}\bar{W}_{kj} + 4D^a_i D^a_j)|_{min} . \qquad (44)$$

Already at this point we learn from (40) and (44) that for $D^a|_{min} = 0$ we have a sum rule

$$\sum_{bosons} M_b^2 \ = \ 2 \sum_{fermions} M_f^2 \qquad (45)$$

where in the sum real bosons are counted.

For $D^a|_{min} \neq 0$ also the gauge symmetry is necessarily broken and some of the gauge bosons become massive. From (18)-(21) one obtains the mass matrix of the gauge bosons

$$M_1^2 = 2g^2 \bar{A}^j T^a_{jl} T^b_{lk} A^k = 2D^a_l D^b_l \qquad (46)$$

Combining (40), (44) and (46) one arrives at the mass sum rule [29]:

$$\mathrm{Str} M^2 \equiv \sum_{J=0}^{1} (-)^{2J}(2J+1)\mathrm{Tr} M_J^2 = -2g(\mathrm{Tr} T^a) D^a , \qquad (47)$$

where J is the spin of the particles. The right hand side of (47) vanishes for any non-Abelian factor in the gauge group while for $U(1)$ factors it is proportional to the sum of the $U(1)$ charges $\sum Q_{U(1)}$. Whenever this sum is non-vanishing the theory has a $U(1)$ trace-anomaly. (In the supersymmetric Standard Model this trace-anomaly vanishes.) Finally, by repeating the steps of this section one can show that (47) holds over all field space and not only at the minimum of V. This will play a rôle in deriving the soft supersymmetry breaking terms.

Exercise: *Verify (46) and (47).*
Exercise: *Compute* $\sum Q_{U(1)_Y}$ *in the supersymmetric Standard Model.*
Exercise: *Show that (47) holds over all field space and not only at the minimum of* V.

The sum rule (47) is problematic for the supersymmetric Standard Model. Since non of the supersymmetric partners has been observed yet they must be heavier than the particles of the Standard Model. Close inspection of (47) shows that this cannot be arranged within a spontaneously broken supersymmetric Standard Model.

An additional problem is the presence of a massless Goldstone fermion. Goldstone's theorem implies that any spontaneously broken global symmetry leads to a massless state in the spectrum. This also holds for supersymmetry where the broken generator is a Weyl spinor and thus there is an additional massless Goldstone fermion. The presence of this state can be seen explicitly from the condition that at the minimum of the potential one has

$$V_j = W_{ij}\bar{W}_i + D^a_j D^a = 0 \ . \tag{48}$$

Let us consider for simplicity the case that supersymmetry is broken by a non-vanishing F-term $\langle F_i \rangle = -\bar{W}_i|_{\rm min} \neq 0$ while $\langle D^a \rangle = 0$.[††] From (48) one learns immediately that now $W_{ij}|_{\rm min}$ has to have a zero eigenvalue. Using (44) this implies that also the mass matrix of the fermions has to have a zero eigenvalue which is the Goldstone fermion.

To summarize, the lesson of this section is that also spontaneously broken supersymmetry runs into phenomenological difficulties. The only way out is an explicit breaking of (global) supersymmetry.

4 Extensions of the Standard Model with Softly Broken Supersymmetry

4.1 The Hierarchy and Naturalness Problem

Before we continue in our endeavor to construct a phenomenologically viable extension of the Standard Model let us briefly review what is called the hierarchy and naturalness problem in the Standard Model.[‡‡]

Consider the following (non-supersymmetric) Lagrangian of a complex scalar A and a Weyl fermion χ

$$\begin{aligned}\mathcal{L} = \ & - \ \partial_m \bar{A} \partial^m A - i\bar{\chi}\bar{\sigma}^m \partial_m \chi - \frac{1}{2}\, m_f\, (\chi\chi + \bar{\chi}\bar{\chi}) - m_b^2\, \bar{A}A \\ & - \ Y\, (A\chi\chi + \bar{A}\bar{\chi}\bar{\chi}) \ - \ \lambda\, (\bar{A}A)^2 \ .\end{aligned} \tag{49}$$

From (9) we learn that this Lagrangian is supersymmetric if $m_f = m_b$ and $Y^2 = \lambda$ but let us not consider this choice of parameters at first. $\mathcal{L}$ has a chiral symmetry for $m_f = 0$ given by

$$A \rightarrow e^{-2i\alpha}\, A \ , \qquad \chi \rightarrow e^{i\alpha}\, \chi \ . \tag{50}$$

[††]The general case is discussed in ref. [29].

[‡‡]The discussion of this section follows ref. [16].

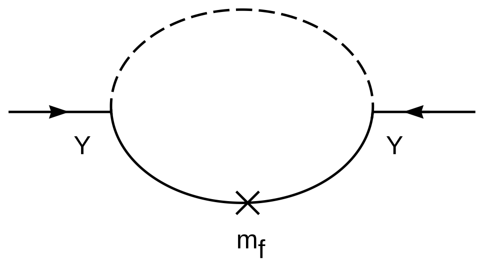

Figure 1: The one-loop correction to the fermion mass.

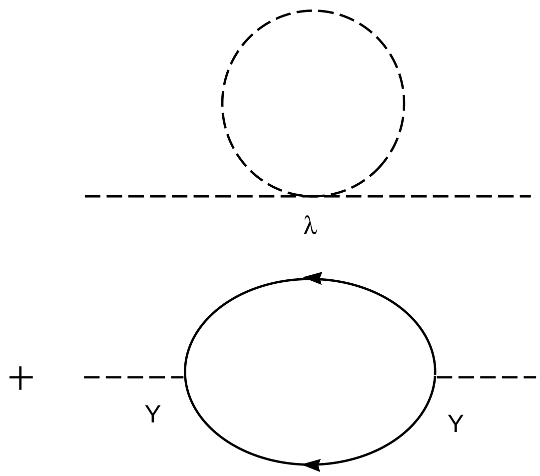

Figure 2: The one-loop corrections to the boson mass.

This symmetry prohibits the generation of a fermion mass by quantum corrections. For $m_f \neq 0$ the fermion mass does receive radiative corrections, but all possible diagrams have to contain a mass insertion as can be seen from the one-loop diagram shown in Fig. 1. Since the propagator of the boson (upper dashed line in the diagram) is $\sim \frac{1}{k^2}$ while the propagator of the fermion (lower solid line) is $\sim \frac{1}{k}$ one obtains a mass correction which is proportional to m_f

$$\delta m_f \sim Y^2 m_f \ln \frac{m_f^2}{\Lambda^2} \ , \tag{51}$$

where Λ is the ultraviolet cutoff. Hence the mass of a chiral fermion does not receive large radiative corrections if the bare mass is small. For that reason 't Hooft calls fermion masses "natural" – an extra symmetry appears when the mass is set to zero which in turn leads to a protection of the fermion mass by an approximate chiral symmetry [30].

This state of affairs is different for scalar fields. The diagrams giving the one-loop corrections to m_b are shown in Fig. 2. Both diagrams are quadratically divergent but they have an opposite sign because in the second diagram fermions are running in the loop. One finds

$$\delta m_b^2 \sim (\lambda - \ Y^2)\,\Lambda^2 \ . \tag{52}$$

Thus, in non-supersymmetric theories scalar fields receive large mass corrections (even if the bare mass is set to zero) and small scalar masses are "unnatural" [30, 31, 3]. They can only be arranged by delicately fine-tuning the bare mass and the couplings λ, Y. This problem becomes apparent in extensions of the Standard Model which apart from the weak scale M_Z do have a second larger scale, say $M_{\rm GUT}$ with $M_{\rm GUT} \gg M_Z$ [31, 3]. In such theories the mass of the scalar boson is naturally of the order of the largest mass parameter in the theory. This discussion applies to the Higgs boson of the Standard Model and it is difficult to understand the smallness of M_Z and how it can be kept stable against quantum corrections whenever the Standard Model is the low energy limit of a theory with a large mass scale.

A concrete example of this problem occurs in Grand Unified Theories (GUTs) [6] where the Standard Model is embedded into a single simple gauge group $G_{\rm GUT}$ (eg. $G_{\rm GUT} = SU(5)$). The GUT gauge symmetry is broken by a Higgs mechanism to the gauge group of the Standard Model and one has the following pattern of symmetry breaking

$$G_{\rm GUT} \stackrel{M_{\rm GUT}}{\rightarrow} SU(3) \times SU(2) \times U(1) \stackrel{M_Z}{\rightarrow} SU(3) \times U(1)_{\rm em} \ , \tag{53}$$

where $M_{\rm GUT} \approx 10^{15}$ GeV and thus $M_{\rm GUT} \gg M_Z$.

There are basically two different suggestions for 'solving' this problem. The first class of models assume that the Higgs boson of the Standard Model is not an elementary scalar, but rather a condensate of strongly interacting 'techni'-fermions [3, 4]. These theories are called "technicolor" theories but in all such theories it is difficult to arrange agreement with the electroweak precision measurements of this decade [19]. The second class of models are supersymmetric theories where the Higgs boson is elementary but the quadratic divergence in (52) exactly cancels due to the supersymmetric relation $Y^2 = \lambda$.

The cancellation of quadratic divergences is a general feature of supersymmetric quantum field theories and a consequence of a more general non-renormalization theorem: The superpotential W of a supersymmetric quantum field theory is not renormalized in perturbation theory [32] and all quantum corrections solely arise from the gauge coupling and wavefunction renormalization.*

*This non-renormalization theorem only holds in perturbation theory but non-perturbative corrections do appear [33].

The non-renormalization theorem or in other words the ‘taming’ of the quantum corrections is one of the attractive features of supersymmetric quantum field theories. It leads (among other things) to the possibility of stabelizing the weak scale M_Z.

In that sense supersymmetry solves the naturalness problem in that it allows for a small and stable weak scale without fine-tuning. However, supersymmetry does not solve the hierarchy problem in that it does not explain why the weak scale is small in the first place.

4.2 Soft Breaking of Supersymmetry

As we have seen in section 3.3 models with spontaneously broken supersymmetry are phenomenologically not acceptable. For example the mass formula (47), generally valid in such cases, forbids that all supersymmetric particles acquire masses large enough to make them invisible in present experiments. One way to overcome those difficulties is to allow explicit supersymmetry breaking.

In the last section we observed that the absence of quadratic divergences in supersymmetric theories stabilizes the Higgs mass and thus the weak scale. This ‘attractive’ feature of supersymmetric field theories can be maintained in theories with explicitly broken supersymmetry if the supersymmetry breaking terms are of a particular form. Such terms which break supersymmetry explicitely and generate no quadratic divergences are called ‘soft breaking terms’.

One possibility to identify the soft breaking terms is to investigate the divergence structure of the effective potential [34]. Consider a quantum field theory of a scalar field ϕ in the presence of an external source J. The generating functional for the Green’s functions is given by

$$e^{-iE[J]} \;=\; \int \mathcal{D}\phi \exp\left[i\int d^4x(\mathcal{L}[\phi(x)] + J(x)\phi(x))\right] \ . \tag{54}$$

The effective action $\Gamma(\phi_{cl})$ is defined by the Legendre transformation

$$\Gamma(\phi_{cl}) = -E[J] - \int d^4x J(x)\phi_{cl}(x) \ , \tag{55}$$

where $\phi_{cl} = -\frac{\delta E[J]}{\delta J(x)}$. $\Gamma(\phi_{cl})$ can be expanded in powers of momentum; in position space this expansion takes the form

$$\Gamma(\phi_{cl}) = \int d^4x[-V_{eff}(\phi_{cl}) - \frac{1}{2}(\partial_m\phi_{cl})(\partial^m\phi_{cl})Z(\phi_{cl}) + \dots\ ] \ . \tag{56}$$

The term without derivatives is called the effective potential $V_{eff}(\phi_{cl})$. It can be calculated in a perturbation theory of $\hbar$:

$$V_{eff}(\phi_{cl}) = V^{(0)}(\phi_{cl}) + \hbar V^{(1)}(\phi_{cl}) + \dots \tag{57}$$

where $V^{(0)}(\phi_{cl})$ is the tree level and $V^{(1)}(\phi_{cl})$ the one-loop contribution. In a theory with scalars, fermions and vector bosons the one-loop contribution takes the form [35]

$$V^{(1)} \sim \int d^4k \, \mathrm{Str} \ln(k^2 + M^2) = \sum_J (-1)^{2J}(2J+1) \, \mathrm{Tr} \int d^4k \, \ln(k^2 + M_J^2) \quad (58)$$

where M_J^2 is the matrix of second derivatives of $\mathcal{L}|_{k=0}$ at zero momentum for scalars ($J = 0$), fermions ($J = 1/2$) and vector bosons ($J = 1$).† The UV divergences of (58) can be displayed by expanding the integrand in powers of large k. This leads to

$$V^{(1)} \sim \mathrm{Str}\mathbf{1} \int \frac{d^4k}{(2\pi)^4} \ln k^2 + \mathrm{Str}M^2 \int \frac{d^4k}{(2\pi)^4} k^{-2} + \ldots \, . \quad (59)$$

If a UV-cutoff Λ is introduced the first term in (59) is $\mathcal{O}(\Lambda^4 \ln \Lambda)$. Its coefficient $\mathrm{Str}\mathbf{1} = n_B - n_F$ vanishes in theories with a supersymmetric spectrum of particles (cf. (2)). The second term in (59) is $\mathcal{O}(\Lambda^2)$ and determines the presence of quadratic divergences at one-loop level. Therefore quadratic divergences are absent if

$$\mathrm{Str}M^2 = 0 \, . \quad (60)$$

More precisely, one can also tolerate $\mathrm{Str}M^2 = const.$ since this would correspond to a shift of the zero point energy which without coupling to gravity is undetermined. In theories with exact or spontaneously broken supersymmetry (60) is fulfilled whenever the trace-anomaly vanishes as we learned in (47).‡

The soft supersymmetry breaking terms are defined as those non-supersymmetric terms that can be added to a supersymmetric Lagrangian without spoiling $\mathrm{Str}M^2 = const.$. One finds the following possibilities [34]

- Holomorphic terms of the scalars proportional to A^2, A^3 and the corresponding complex conjugates.§

- Mass terms for the scalars proportional to $\bar{A}A$.
  (They only contribute a constant, field independent piece in $\mathrm{Str}M^2$).

†M_J^2 is not necessarily evaluated at the minimum of V_{eff}. Rather it is a function of the scalar fields in the theory. The mass matrix is obtained from M_J^2 by inserting the vacuum expectation values of the scalar fields.

‡Indeed, theories with a non-vanishing D-term have been shown to produce a quadratic divergence at one-loop [36].

§Higher powers of A are forbidden since they generate quadratic divergences at the 2-loop level [34].

- Gaugino mass terms.
(A generic mass matrix of the fermions takes the form

$$M_{1/2} = \begin{pmatrix} W_{ij} + \delta W_{ij} & i\sqrt{2}D_i^b + \delta D_i^b \\ i\sqrt{2}D_j^a + \delta D_j^a & \delta\tilde{m}_{ab} \end{pmatrix} , \qquad (61)$$

where according to (43)

$$\begin{pmatrix} W_{ij} & i\sqrt{2}D_i^b \\ i\sqrt{2}D_j^a & 0 \end{pmatrix}$$

is the supersymmetric part of $M_{1/2}$. Computing the supertrace of (61) reveals that $\mathrm{Str}M^2 = const.$ requires $\delta W = 0 = \delta D$ while $\delta\tilde{m}$ can be arbitrary.)

Thus the most general Lagrangian with softly broken supersymmetry takes the form

$$\mathcal{L} = \mathcal{L}_{\mathrm{susy}} + \mathcal{L}_{\mathrm{soft}} , \qquad (62)$$

where $\mathcal{L}_{\mathrm{susy}}$ is of the form (21) and

$$\begin{aligned} \mathcal{L}_{\mathrm{soft}} &= -m_{ij}^2 A^i \bar{A}^j - (b_{ij} A^i A^j + a_{ijk} A^i A^j A^k + \text{ h.c. }) \\ &\quad - \frac{1}{2}\,\tilde{m}_{ab}\lambda^a\lambda^b + \text{ h.c. } . \end{aligned} \qquad (63)$$

m_{ij}^2 and b_{ij} are mass matrices for the scalars, a_{ijk} are trilinear couplings (often called 'A-terms') and $\tilde{m}_{ab}$ is a mass matrix for the gauginos.

The next step will be to investigate if the more general Lagrangian (62) can be used to construct viable phenomenology. Before we do so let us mention that there is an alternative way to motivate the relevance of softly broken supersymmetric theories. Ultimately one has to couple the supersymmetric Standard Model to gravity. This requires the promotion of global supersymmetry to a local symmetry, that is the parameter of the supersymmetry transformation $\xi_\alpha = \xi_\alpha(x)$ is no longer constant but depends on the space-time coordinates x [37, 8]. This demands the presence of an additional massless fermionic gauge field (the gravitino) $\Psi_{m\alpha}$ with spin 3/2 and an inhomogeneous transformation law

$$\delta_\xi \Psi_{m\alpha} = -\partial_m \xi_\alpha + \dots \ . \qquad (64)$$

(The necessity of this transformation law can be seen for example from the supersymmetry transformation of $\partial_m A$ which now has an extra contribution $\partial_m \delta_\xi A \ \propto \ \partial_m \xi\psi \ = \ \xi\partial_m\psi + (\partial_m\xi)\psi$.) Together with the metric g_{mn} and 6 auxiliary fields $b_m, M, \bar{M}$ the gravitino $\Psi_{m\alpha}$ forms the supergravity multiplet $(g_{mn}, \Psi_{m\alpha}, b_m, M, \bar{M})$.

The potential for the scalar fields is modified in the presence of supergravity and found to be [38]

$$V(A,\bar{A}) = e^{\kappa^2 A\bar{A}}\Big[(D_i W)(\bar{D}_{\bar{i}}\bar{W}) - 3\kappa^2|W|^2\Big] + \frac{1}{2}D^a D^a \ , \tag{65}$$

where

$$\kappa^2 = \frac{8\pi}{M_{Pl}^2} \ , \qquad D_i W = \frac{\partial W}{\partial A^i} + \kappa^2 \bar{A}^i W \ . \tag{66}$$

The limit $\kappa^2 \to 0$ corresponds to turning off gravity and in this limit one obtains indeed $V \to \frac{\partial W}{\partial A^i}\frac{\partial \bar{W}}{\partial \bar{A}^i} + \frac{1}{2}D^a D^a$ in accord with (22). Local supersymmetry is spontaneously broken if $D_i W|_{min} \neq 0$ for some i. This can be achieved by introducing a hidden sector which only couples via non-renormalizable interactions to the observable sector of the supersymmetric Standard Model and which has a superpotential $W_{hid}(\phi)$ suitably chosen to ensure $D_\phi W|_{min} \neq 0$ [39, 10]. In this case the gravitino becomes massive through a supersymmetric Higgs effect [38].

In the limit $\kappa^2 \to 0$ with the gravitino mass $m_{3/2}$ kept fixed the Lagrangian for the fields in the observable sector looks precisely like eqs. (62), (63) [39, 10]. Thus the spontaneous breakdown of supergravity in a hidden sector manifests itself as explicit but soft breakdown of global supersymmetry in the low energy limit of the observable sector.

Finally, a variant of this mechanism is to break supersymmetry dynamically (ie. non-perturbatively) in an additional gauge sector with some asymptotically free gauge theory [33, 40]. In this case the supersymmetry breaking is communicated to the observable sector by renormalizable interactions but as in the previous case the breaking appears in the observable sector as explicit but soft [17, 41].

4.3 The Supersymmetric Standard Model with Softly Broken Supersymmetry

In the previous section we recalled the most general Lagrangian of a softly broken supersymmetric gauge theory in eqs. (62) and (63). For $\mathcal{L}_{\rm susy}$ we continue to take (21) together with the superpotential specified in (25). For $\mathcal{L}_{\rm soft}$ only gauge invariance and R-parity is imposed. This leads to the following possible soft terms [10, 11, 13, 14, 16, 17]

$$\begin{aligned}\mathcal{L}_{\rm soft} \ = \ & -\Big((a_u)_{IJ} h_u \tilde{q}_L^I \tilde{u}_R^J + (a_d)_{IJ} h_d \tilde{q}_L^I \tilde{d}_R^J + (a_e)_{IJ} h_d \tilde{l}_L^I \tilde{e}_R^J + b h_u h_d + \text{ h.c. }\Big) \\ & - \sum_{\text{all scalars}} m_{ij}^2 A^i \bar{A}^j - (\frac{1}{2}\sum_{(a)=1}^{3} \tilde{m}_{(a)} (\lambda\lambda)_{(a)} + \text{ h.c. }) \ . \end{aligned} \tag{67}$$

Obviously a huge number of new parameters is introduced via $\mathcal{L}_{\rm soft}$. The parameters of $\mathcal{L}_{\rm susy}$ are the Yukawa couplings Y and the parameter μ in the Higgs potential. The Yukawa couplings are determined experimentally already in the non-supersymmetric Standard Model. In the softly broken supersymmetric Standard Model the parameter space is enlarged by

$$\Big(\mu, (a_u)_{IJ}, (a_d)_{IJ}, (a_e)_{IJ}, b, m^2_{ij}, \tilde{m}_{(a)}\Big) \ . \tag{68}$$

Not all of these parameters can be arbitrary but quite a number of them are experimentally constrained. Some of these constraints we will see in the following sections.

Within this much larger parameter space it is possible to overcome several of the problems encountered in the supersymmetric Standard Model. For example, the supersymmetric particles can now easily be heavy (due to the arbitrariness of the mass terms m^2_{ij}) and therefore out of reach of present experiments. Furthermore, the Higgs potential is changed and vacua with spontaneous electroweak symmetry breaking can be arranged.

However, the soft breaking terms introduce their own set of difficulties. For generic values of the parameters (68) the contribution to flavor-changing neutral currents is unacceptably large [42, 43], additional (and forbidden) sources of CP-violation occur [44, 45] and finally the absence of vacua which break the $U(1)_{\rm em}$ and/or $SU(3)$ is no longer automatic [46]. It is beyond the scope of these lectures to review all of these aspects in detail. Let us therefore focus on a few selected topics and refer the reader to the literature for further details and discussions.

4.4 Electroweak Symmetry Breaking

In section 3.2 we noticed that for unbroken or spontaneously broken supersymmetry the electroweak symmetry remains intact in the supersymmetric version of the Standard Model. Let us now review the situation in the presence of soft breaking terms [47]. The Higgs sector of the supersymmetric Standard Model consists of two $SU(2)$-doublets

$$h_u = \begin{pmatrix} h_u^+ \\ h_u^0 \end{pmatrix} \quad , \qquad h_d = \begin{pmatrix} h_d^0 \\ h_d^- \end{pmatrix} \quad ,$$

which carry eight real degrees of freedom, four of them neutral and four charged. Like in the Standard Model $SU(2)_L \times U(1)_Y$ will be broken to $U(1)_{\rm em}$ by non-vanishing VEVs of the neutral Higgs bosons h_u^0 and h_d^0. For that purpose consider their potential which can be derived from eqs. (26), (67)

$$V(h_u^0, h_d^0) = \hat{m}_u^2 |h_u^0|^2 + \hat{m}_d^2 |h_d^0|^2 - b(h_u^0 h_d^0 + \bar{h}_u^0 \bar{h}_d^0) + \frac{g_1^2 + g_2^2}{8}\Big(|h_u^0|^2 - |h_d^0|^2\Big)^2, \tag{69}$$

where

$$\begin{aligned}\hat{m}_u^2 &= m_u^2 + |\mu|^2 \ , \\ \hat{m}_d^2 &= m_d^2 + |\mu|^2 \ . \end{aligned} \tag{70}$$

Notice that the coefficient of the $|h^0_{u,d}|^4$-term is exactly as in (26) determined by the gauge couplings and not changed by soft breaking terms. This term is positive so that the potential is bounded from below for large values of h_u^0, h_d^0 as long as $|h_u^0| \neq |h_d^0|$. To secure this bound also in the direction $|h_u^0| = |h_d^0|$ one has to impose the following constraint on the parameter space

$$\hat{m}_u^2 + \hat{m}_d^2 \geq 2|b| \ . \tag{71}$$

Exercise: *Verify formula (69) using eqs. (26), (67).*
Exercise: *Verify the condition (71).*

The existence of a minimum of V with broken gauge symmetry requires that the Hessian of $V(h_u^0, h_d^0)|_{h_u^0=h_d^0=0}$

$$\begin{pmatrix} \hat{m}_u^2 & -b \\ -b & \hat{m}_d^2 \end{pmatrix} \tag{72}$$

has at least one negative eigenvalue. Together with (71) this implies¶

$$\hat{m}_u^2 \hat{m}_d^2 < b^2 \ . \tag{73}$$

So the soft terms have to satisfy (71), (73) in order to induce electroweak symmetry breaking but in addition also the masses of the Z- and W-bosons have to come out correctly. These masses are given by

$$\begin{aligned} M_Z^2 &= \frac{1}{4}(g_1^2 + g_2^2)(v_u^2 + v_d^2) = \frac{1}{2}(g_1^2 + g_2^2) v^2 \\ M_W^2 &= \frac{1}{4} g_2^2 (v_u^2 + v_d^2) = \frac{1}{2} g_2^2 v^2 \ , \end{aligned} \tag{74}$$

where

$$\langle h_u^0 \rangle = \frac{1}{\sqrt{2}}\, v_u = v \sin\beta, \qquad \langle h_d^0 \rangle = \frac{1}{\sqrt{2}}\, v_d = v \cos\beta \ . \tag{75}$$

The electroweak symmetry breaking is parameterized by the two Higgs vacuum expectation values v_u, v_d (which can be chosen real) or equivalently v and β. As in

¶This is the generalization of the condition $\mu^2 < 0$ in the Standard Model to a Higgs sector with two Higgs doublets.

the Standard Model v has to be chosen such that it reproduces the experimentally measured Z- and W-masses. β on the other hand is an additional parameter which arises as a consequence of the enlarged (2 doublet) Higgs sector.

The Higgs expectation values are directly related to the parameters of the Higgs potential via the minimization conditions

$$\begin{aligned}
\frac{\partial V}{\partial h_u^0} &= 2\hat{m}_u^2 v_u - 2b v_d + \frac{g_1^2+g_2^2}{2}(v_u^2 - v_d^2)v_u = 0 \ , \\
\frac{\partial V}{\partial h_d^0} &= 2\hat{m}_d^2 v_d - 2b v_u - \frac{g_1^2+g_2^2}{2}(v_u^2 - v_d^2)v_d = 0 \ . \qquad (76)
\end{aligned}$$

This in turn can be used to derive a more physical relationship among the parameters. Using (74) and (75) the minimization conditions (76) can be rewritten as

$$\begin{aligned}
b &= \frac{1}{2}\sin 2\beta\,(\hat{m}_u^2 + \hat{m}_d^2) \\
M_Z^2 &= -2|\mu|^2 + \frac{2}{1-\tan^2\beta}(m_u^2\tan^2\beta - m_d^2) \ . \qquad (77)
\end{aligned}$$

Finally, the full Higgs potential including all eight real degrees of freedom can be used to compute the 8×8 mass matrix of all Higgs bosons. After a somewhat lengthy calculation [47] one finds that this mass matrix has three eigenvalues zero corresponding to the three Goldstone modes 'eaten' by the $W^\pm$ and the Z. The remaining five degrees of freedom yield the physical Higgs bosons of the model:

$H^\pm$	charged Higgs boson pair
A^0	CP-odd neutral Higgs boson
H^0, h^0	CP-even neutral Higgs bosons .

Their tree-level masses are given by

$$\begin{aligned}
m_A^2 &= \hat{m}_u^2 + \hat{m}_d^2 \\
m_{H_\pm}^2 &= m_A^2 + M_W^2 \\
m_{h^0}^2 &= \frac{1}{2}\Big[m_A^2 + M_Z^2 - \sqrt{(m_A^2+M_Z^2)^2 - 4m_A^2 M_Z^2\cos^2 2\beta}\ \Big] \\
m_{H^0}^2 &= \frac{1}{2}\Big[m_A^2 + M_Z^2 + \sqrt{(m_A^2+M_Z^2)^2 - 4m_A^2 M_Z^2\cos^2 2\beta}\ \Big] \ . \qquad (78)
\end{aligned}$$

The Higgs masses obey physically interesting mass relations. From (78) we learn

$$m_{H^\pm} \geq M_W \ , \qquad m_{H^0} \geq M_Z \ , \qquad m_{h^0} \leq M_Z \ . \qquad (79)$$

Exercise: *Derive the mass relations (79) from (78).*

Physically the most interesting is the last inequality since it predicts the existence of a light Higgs boson. This 'prediction' can be directly traced to the fact the quartic couplings in the Higgs potential are fixed by the (measured) gauge couplings and are not free parameters as in the Standard Model. However, radiative corrections for this lightest Higgs boson mass can be large and after taking into account quantum corrections the upper bound on m_{h^0} is pushed up to about 150 GeV [48]. However, the prediction of one light neutral Higgs boson remains and is one of the characteristic features of the supersymmetric two-doublet Higgs sector. It even holds in the limit that all masses of the supersymmetric particles are sent to infinity. In this limit one recovers the non-supersymmetric Standard Model – albeit with a light Higgs.

Finally, a proper minimization of the scalar potential requires to take into account all scalar fields and not truncate to the neutral Higgs bosons. It is possible (and does occur in certain regions of the supersymmetric parameter space) that there exist minima which not only break the electroweak symmetry but also $SU(3)$ and/or $U(1)_{\rm em}$. An extended analysis of this aspect can be found, for example, in ref. [46].

4.5 Weak-Scale Supersymmetry

Let us briefly come back to the hierarchy and naturalness problem. In section 4.1 we learned that supersymmetry sheds no light on the hierarchy problem, i.e. the question why M_Z is so much smaller than M_{Pl}. However, because of the absence of quadratic divergences it does solve the naturalness problem, that is it provides a stable hierarchy in the presence of a light Higgs boson. To only add soft supersymmetry breaking terms into the supersymmetric Standard Model was precisely motivated by this feature. However, from eq. (77) we learn an important additional constraint on the soft breaking parameters. In order to introduce no new fine-tuning all soft terms in eq. (77) should be of the same order of magnitude, i.e. of order $\mathcal{O}(M_Z)$ or at most in the TeV range [49]. If this were not the case the soft parameters would have to be delicately tuned in order to add up to M_Z. This in turn implies the breaking of supersymmetry should occur at the weak scale and that most likely all supersymmetric particles have masses in that range.‖

‖This argument is somewhat imprecise since not only the soft breaking terms (m_u, m_d) but also μ have to be $\mathcal{O}(M_Z)$. However μ is not related to supersymmetry breaking in any obvious way but rather a parameter in the superpotential. Thus one needs a mechanism which also explains the approximate equality of μ with (m_u, m_d). This is known as the μ-problem [50, 10].

This line of argument is used to motivate what is called "weak-scale supersymmetry" and indeed the current LEP II experiments actively look for supersymmetric particles with masses slightly above the weak scale.

4.6 Further Constraints on the Supersymmetric Parameter Space

The experimental searches for supersymmetric particles impose additional constraints on the supersymmetric parameter space. First and foremost the direct lower bounds on the masses of the supersymmetric particles [1] exclude certain regions of the parameter space. The translations of experimental bounds into the supersymmetric parameter space is complicated by the fact that the states which are listed in table 2 are interaction eigenstates, but not necessarily mass eigenstates. The only exception are the gluinos $\tilde{G}$ and the mass bounds directly translate into bounds on $\tilde{m}_3$. On the other hand the three Winos $\tilde{W}^{\pm}, \tilde{W}^3$, the $\tilde{B}$ and the four Higgsinos $\tilde{h}^0_{u,d}, \tilde{h}^-_d, \tilde{h}^+_u$ combine into a four-vector of neutral Weyl fermions consisting of $\mathbf{N} \equiv (\tilde{B}, \tilde{W}^3, \tilde{h}^0_u, \tilde{h}^0_d)$ and two pairs of charged Weyl fermions $\mathbf{C}^- \equiv (\tilde{W}^-, \tilde{h}^-_d)$, $\mathbf{C}^+ \equiv (\tilde{W}^+, \tilde{h}^+_u)$ with the following set of mass matrices

$$\mathcal{L}_{\rm fmass} = -\frac{1}{2}\,\tilde{m}_3\,\tilde{G}^a\tilde{G}^a \;-\; \mathbf{C}^- M_C (\mathbf{C}^+)^T \;-\; \frac{1}{2}\,\mathbf{N} M_N \mathbf{N}^T + h.c.\ , \qquad (80)$$

where

$$M_C = \begin{pmatrix} \tilde{m}_2 & i\sqrt{2} g_2 v_u \\ i\sqrt{2} g_2 v_d & \mu \end{pmatrix}\ , \qquad (81)$$

and

$$M_N = \begin{pmatrix} \tilde{m}_1 & 0 & \frac{i}{2} g_1 v_u & -\frac{i}{2} g_1 v_d \\ 0 & \tilde{m}_2 & -\frac{i}{2} g_2 v_u & \frac{i}{2} g_2 v_d \\ \frac{i}{2} g_1 v_u & -\frac{i}{2} g_2 v_u & 0 & \mu \\ -\frac{i}{2} g_1 v_d & \frac{i}{2} g_2 v_d & \mu & 0 \end{pmatrix}\ . \qquad (82)$$

Thus, the physical mass eigenstates of M_C and M_N are parameter dependent linear combinations of the corresponding interaction eigenstates and they are termed *charginos* and *neutralinos*, respectively.

Exercise: *Derive the mass matrices (81) and (82).*

The other supersymmetric particles are the spin-0 partners of the quarks and leptons, the *squarks* $\tilde{q}^I_L, \tilde{u}^I_R, \tilde{d}^I_R$ and the *sleptons* $\tilde{l}^I_L, \tilde{e}^I_R$. Their mass eigenstates are derived from the following three 6×6 and one 3×3 mass matrices

$$\mathbf{U} M_U \mathbf{U}^\dagger\ , \qquad \mathbf{D} M_D \mathbf{D}^\dagger\ , \qquad \mathbf{E} M_E \mathbf{E}^\dagger\ , \qquad \tilde{\nu} M_\nu \bar{\tilde{\nu}}\ , \qquad (83)$$

where

$$\mathbf{U} \equiv (\tilde{u}^I_L, \bar{\tilde{u}}^I_R) \ , \qquad \mathbf{D} \equiv (\tilde{d}^I_L, \bar{\tilde{d}}^I_R) \ , \qquad \mathbf{E} \equiv (\tilde{e}^I_L, \bar{\tilde{e}}^I_R) \ . \tag{84}$$

Explicit forms of these mass matrices in terms of the soft parameters can be found e.g. in ref. [13]. Constraints on the soft parameters imposed by the experimental bounds can be found in [1, 13, 16, 17, 18, 19, 51].

4.7 The Minimal Supersymmetric Standard Model (MSSM)

The supersymmetric version of the Standard Model we discussed so far has a huge parameter space and therefore very limited predictive power. A much more constrained version (with less free parameters) became known as the Minimal Supersymmetric Standard Model (MSSM) which is the topic of this section. The MSSM was motivated by the success of Grand Unified Theories combined with a simple, flavor blind mechanism of supersymmetry breaking in a hidden sector [52]. Over the last 15 years this model went through a few alterations but today it is a well defined model with a very particular set of soft supersymmetry breaking terms which are flavor blind and in some sense the minimal choice of free parameters [10, 11, 13, 14]. One imposes that *all* scalar masses are the same $m^2_{ij} = m^2_0 \delta_{ij}$, all gaugino masses are the same $\tilde{m}_1 = \tilde{m}_2 = \tilde{m}_3 = \tilde{m}$, all a-parameters are proportional to the Yukawa couplings with the same universal proportionality constant a_0 and finally that the b-parameter is of a specific form. Altogether one has

$$\begin{aligned} m^2_{ij} &= m^2_0\, \delta_{ij} \ , \qquad \tilde{m}_1 = \tilde{m}_2 = \tilde{m}_3 = \tilde{m} \ , \qquad b = b_0\, m_0\, \mu \ , \\ (a_u)_{IJ} &= a_0\, (Y_u)_{IJ} \ , \qquad (a_d)_{IJ} = a_0\, (Y_d)_{IJ} \ , \qquad (a_l)_{IJ} = a_0\, (Y_l)_{IJ} \ . \end{aligned} \tag{85}$$

Thus, the parameter space of the MSSM is spanned by the 5 parameters

$$(m_0, \tilde{m}, a_0, b_0, \mu) \ ,$$

which are subject to one further non-trivial constraint (77) which ensures the observed electroweak symmetry breaking.

Of course the relations (85) are meant to be tree level relations and they do enjoy quantum corrections which are governed by the appropriate renormalization group equations [53, 54]. The quantum corrections destroy the universality of the soft parameters but the deviation from universality is small and so far in accord with all measurements [16, 18, 55, 43]. In particular, the smallness of flavor-changing neutral currents is ‘naturally explained’ in the MSSM [42, 56]. Thus even without an underlying GUT theory this set of parameters seems to have some phenomenological attraction.

However, one should also stress that supersymmetric GUTs are an extremely viable possibility today. Among other things the LEP precision experiments determined the gauge coupling constant g_2 very precisely. In light of these measurements one can ask to what extend the three gauge couplings g_1, g_2, g_3 do unify at some high energy scale $M_{\rm GUT}$ given the experimental input at M_Z. At one-loop order the energy dependence of the gauge couplings is given by

$$g_{(a)}^{-2}(M_Z) \;=\; g_{(a)}^{-2}(M_{\rm GUT}) \,+\, \frac{b_{(a)}}{8\pi^2}\,\ln\big(\frac{M_{\rm GUT}}{M_Z}\big)\;, \tag{86}$$

where $b_{(a)}$ are the coefficients of the one-loop beta-function which depend on the massless spectrum of the theory. Let us first recall that the index $T_{(a)}(R)$ of a representation R is defined as ${\rm Tr}_R(T^aT^b)_{(a)} = T_{(a)}(R)\delta^{ab}$ where T^b are the generators of the gauge group. In terms of the indices $b_{(a)}$ is given by

$$b_{(a)} = -\frac{11}{3}T_{(a)}({\rm G}) + \frac{2}{3}T_{(a)}({\rm R})\, n_{WF} + \frac{1}{6}T_{(a)}({\rm R})\, n_{RS}\;, \tag{87}$$

where G denotes the adjoint representation and n_{WF} (n_{RS}) counts the number of Weyl fermions (real scalars) in the representation R.

For the non-supersymmetric Standard Model one finds

$$(b_1, b_2, b_3) = (\frac{41}{10}, -\frac{19}{6}, -7)\;,$$

which does not lead to a unification of coupling constants at any scale. That is, one cannot find an $M_{\rm GUT}$ where $g_{(1)}(M_{\rm GUT}) = g_{(2)}(M_{\rm GUT}) = g_{(3)}(M_{\rm GUT})$ holds. However, in the supersymmetric Standard Model one finds

$$(b_1, b_2, b_3) = (11, 1, -3)\;,$$

which does lead to a unification of couplings at $M_{GUT} \simeq 10^{16}$ GeV [57]. This can be taken as a (strong) hint for a supersymmetric GUT.

Let us close this section with a discussion of electroweak symmetry breaking in the MSSM. At the tree level one now has $\hat{m}_d^2 = \hat{m}_u^2$ and as a consequence the conditions for electroweak symmetry breaking (71) and (73) cannot be satisfied simultaneously

$$\begin{aligned} \hat{m}_u^2 + \hat{m}_d^2 &= 2(m_0^2 + \mu^2) \geq 2|b| \\ \hat{m}_u^2\hat{m}_d^2 &= (m_0^2 + \mu^2)^2 < |b|^2\;. \end{aligned}$$

However, quantum corrections alter this situation and naturally induce electroweak symmetry breaking [58]. Thus, the MSSM naturally displays a supersymmetric version of the Coleman-Weinberg mechanism [35] where quantum corrections generate a non-trivial minimum in the Higgs potential. The electroweak symmetry breaking scale is not put in hand, but related to the scale of the supersymmetry breakdown.

5 Summary

Supersymmetry is a generalization of the space-time symmetries of quantum field theories and it transforms fermions into bosons and vice versa. In the supersymmetric Standard Model all particles of the Standard Model are accompanied by superpartners with opposite statistics. Moreover, it is necessary to enlarge the Higgs sector and add a second Higgs doublet to the spectrum. Supersymmetry cannot be an exact symmetry of nature and has to be realized in its broken phase (if at all). However, spontaneously broken global supersymmetry is phenomenologically ruled out. Spontaneously broken local supersymmetry on the other hand leads to models which are (still) in agreement with all present observations. These models do provide a solution to the naturalness problem as long as the supersymmetric partners have masses not much bigger than $1TeV$. The supersymmetric Standard Model has one solid prediction: a light neutral Higgs with a mass smaller than 150 GeV (as well as additional charged but not necessarily light Higgs bosons). The gauge couplings of the supersymmetric Standard Model do unify at a scale $M_{GUT} \simeq 10^{16}$ GeV, which may indicate a supersymmetric GUT. The MSSM – motivated by GUTs – contains only four new parameters and is consistent with observations in large regions of this parameter space. In this model the electroweak symmetry breaking is driven by radiative corrections realizing a supersymmetric version of the Coleman-Weinberg mechanism. Finally, the concept of supersymmetry also arises naturally in string theories which might be another hint towards its realization in nature.

Exercise: *How can supersymmetry be verified or falsified? Distinguish between necessary and sufficient conditions.*

6 Appendix-Conventions and Notation

In these lectures the notation and conventions of ref. [8] are used. The four-dimensional Lorentz metric is chosen as

$$\eta_{mn} = \mathrm{diag}(-1,1,1,1)\ . \tag{88}$$

Lorentz indices are labeled by Latin indices $m, n, ...$ which run from 0 to 3. Greek indices are used to denote spinors. A two-component Weyl spinor can transform under the $(\frac{1}{2},0)$ or the complex conjugate $(0,\frac{1}{2})$–representation of the Lorentz group and dotted or undotted indices are used to distinguish between these representations. ψ_α denotes a spinor transforming under the $(\frac{1}{2},0)$ representation while $\bar{\chi}_{\dot{\alpha}}$ transforms under the $(0,\frac{1}{2})$ representation of the Lorentz group. The spinor indices α and $\dot{\alpha}$ can take the values 1 and 2. These indices can be raised and lowered using the skew-symmetric $SU(2)$– invariant tensor $\epsilon^{\alpha\beta}$ or $\epsilon_{\alpha\beta}$.

$$\psi^\alpha = \epsilon^{\alpha\beta}\psi_\beta\ , \qquad \psi_\alpha = \epsilon_{\alpha\beta}\psi^\beta\ , \tag{89}$$

where

$$\epsilon_{21} = -\epsilon_{12} = 1, \quad \epsilon_{11} = \epsilon_{22} = 0, \quad \epsilon_{\alpha\gamma}\epsilon^{\gamma\beta} = \delta_\alpha^\beta\ .$$

For dotted indices the analogous equations hold. The product $\epsilon^{\beta\alpha}\psi_\alpha\chi_\beta = \psi^\beta\chi_\beta$ is a Lorentz scalar. Spinors are anticommuting objects and one has the following summation convention:

$$\begin{aligned}\psi\chi &= \psi^\alpha\chi_\alpha = -\psi_\alpha\chi^\alpha = \chi^\alpha\psi_\alpha = \chi\psi\ ,\\ \bar{\psi}\bar{\chi} &= \bar{\psi}_{\dot{\alpha}}\bar{\chi}^{\dot{\alpha}} = -\bar{\psi}^{\dot{\alpha}}\bar{\chi}_{\dot{\alpha}} = \bar{\chi}_{\dot{\alpha}}\bar{\psi}^{\dot{\alpha}} = \bar{\chi}\bar{\psi}\ .\end{aligned} \tag{90}$$

The convention for the conjugate spinors are chosen such that it is consistent with the conjugation of scalars:

$$(\psi\chi)^\dagger = (\psi^\alpha\chi_\alpha)^\dagger = \bar{\psi}_{\dot{\alpha}}\bar{\chi}^{\dot{\alpha}} = \bar{\psi}\bar{\chi} = \bar{\chi}\bar{\psi}\ . \tag{91}$$

The σ-matrices $\sigma^m_{\alpha\dot{\alpha}}$ are given by:

$$\begin{aligned}\sigma^0 &= \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}, \qquad \sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix},\\ \sigma^2 &= \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \qquad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.\end{aligned} \tag{92}$$

The invariant ϵ–tensor raises and lowers their indices:

$$\bar{\sigma}^{m\dot{\alpha}\alpha} = \epsilon^{\dot{\alpha}\dot{\beta}}\epsilon^{\alpha\beta}\sigma^m_{\beta\dot{\beta}} \tag{93}$$

and we have:

$$\bar{\sigma}^0 = \sigma^0 , \qquad \bar{\sigma}^{1,2,3} = -\sigma^{1,2,3} . \tag{94}$$

The generators of the Lorentz group in the spinor representation are given by

$$\sigma^{nm} = \frac{1}{4}(\sigma^n \bar{\sigma}^m - \sigma^m \bar{\sigma}^n) , \qquad \bar{\sigma}^{nm} = \frac{1}{4}(\bar{\sigma}^n \sigma^m - \bar{\sigma}^m \sigma^n) . \tag{95}$$

The Dirac-γ–matrices can be written in terms of Weyl matrices:

$$\gamma^m = \begin{pmatrix} 0 & \sigma^m \\ \bar{\sigma}^m & 0 \end{pmatrix} \tag{96}$$

which fulfill

$$\{\gamma^m, \gamma^n\} = -2\,\eta^{mn} . \tag{97}$$

A four component Dirac spinor contains two Weyl spinors

$$\Psi_D = \begin{pmatrix} \psi \\ \bar{\chi} \end{pmatrix} = \begin{pmatrix} \psi_\alpha \\ \bar{\chi}^{\dot{\alpha}} \end{pmatrix} . \tag{98}$$

Its conjugate is

$$\bar{\Psi}_D = \Psi_D^\dagger \gamma^0 = (\chi^\alpha, \bar{\psi}_{\dot{\alpha}}) . \tag{99}$$

The Dirac equation describing relativistic spin-$\frac{1}{2}$ particles reads:

$$(i\,\gamma^n \partial_n + m)\,\Psi_D = 0. \tag{100}$$

It can be decomposed into two Weyl equations

$$\begin{aligned} i\sigma^n \partial_n \,\bar{\chi} \,+\, m\psi &= 0 , \\ i\bar{\sigma}^n \partial_n \psi \,+\, m\bar{\chi} &= 0 . \end{aligned} \tag{101}$$

Exercise: *Show the validity of the following identities:*

$$\psi^\alpha \chi_\alpha = \chi^\alpha \psi_\alpha, \quad \chi^\alpha \sigma^n_{\alpha\dot{\alpha}} \bar{\psi}^{\dot{\alpha}} = -\bar{\psi}_{\dot{\alpha}} \bar{\sigma}^{n\dot{\alpha}\beta} \chi_\beta, \quad (\psi^\alpha \phi_\alpha)\,\bar{\chi}_{\dot{\beta}} = -\frac{1}{2}\left(\phi^\alpha \sigma^m_{\alpha\dot{\delta}} \bar{\chi}^{\dot{\delta}}\right) \psi^\gamma \sigma^m_{\gamma\dot{\beta}}$$

Exercise: *Compute in terms of σ-matrices* the following matrices

$$\gamma_5 = -i\gamma_0\gamma_1\gamma_2\gamma_3 \qquad P_L = \frac{1}{2}(1-\gamma_5), \qquad P_R = \frac{1}{2}(1+\gamma_5)$$

Exercise: *Express the following Lorentz-invariants in terms of Weyl-spinors:*

$$\bar{\Psi}_D \Psi_D,\ \bar{\Psi}_D \gamma^5 \Psi_D,\ \bar{\Psi}_D \gamma^m \Psi_D,\ \bar{\Psi}_D \gamma^5 \gamma^m \Psi_D,\ \bar{\Psi}_D [\gamma^m, \gamma^n] \Psi_D$$

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References

[1] C. Caso et al. , Particle Data Group, *Eur. Phys. J.* **C3** (1998) 1.

[2] For a recent review see, for example, J. Erler and P. Langacker, hep-ph/9809352 and references therein.

[3] L. Susskind, *Phys. Rev.* **D20** (1979) 2619;
S. Weinberg, *Phys. Rev.* **D19** (1979) 1277;
S. Dimopoulos and L. Susskind, *Nucl. Phys.* **B155** (1979) 337.

[4] E. Fahri and L. Susskind, *Phys. Rep.* **74C** (1981) 277;
I.A. D'Souza and C.S. Kalman, *Preons*, World Scientific, Singapore, 1992.

[5] H. Georgi and S. Glashow, *Phys. Rev. Lett.* **32** (1974) 438.

[6] G. Ross, *Grand Unified Theories*, Benjamin/Cummings, 1985.

[7] J. Wess and B. Zumino, *Nucl. Phys.* **B70** (1974) 39.

[8] J. Wess and J. Bagger, *Supersymmetry and Supergravity*, Princeton University Press, 1992.

[9] M.B. Green, J.S. Schwarz and E. Witten, *Superstring Theory*, Cambridge University Press, 1987;
D. Lüst and S. Theisen, *Lectures on String Theory*, Springer Verlag, 1989;
J. Polchinski, *String Theory*, Cambridge University Press, 1998.

[10] H.P. Nilles, *Phys. Rep.* **110** (1984) 1.

[11] H.E. Haber and G.L. Kane, *Phys. Rep.* **117** (1985) 75.

[12] M.F. Sohnius, *Phys. Rep.* **128** (1985) 39.

[13] H.E. Haber, in *Recent Directions in Particle Theory: From Superstrings and Blach Holes to the Standard Model*, eds. J. Harvey and J. Polchinski (World Scientific, Singapore, 1993).

[14] F. Zwirner, Trieste 1991, Proceedings, *High Energy Physics and Cosmology*, vol 1, 193.

[15] W. Hollik, R. Rückl and J. Wess (eds.), *Phenomenological Aspects of Supersymmetry*, Springer Lecture Notes, 1992.

[16] J. Bagger, in *Boulder 1995, QCD and beyond*, 109, hep-ph/9604232.

[17] M. Dine, hep-ph/9612389.

[18] J.F. Gunion, hep-ph/9704349.

[19] M. Peskin, hep-ph/9705479.

[20] S. Martin, hep-ph/9709356.

[21] S. Ferrara, *Supersymmetry*, World Scientific, Singapore, 1987.

[22] R. Haag, J. Lopuszanski and M. Sohnius, *Nucl. Phys.* **B88** (1975) 257.

[23] S. Coleman and J. Mandula, *Phys. Rev.* **159** (1967) 1251.

[24] S. Weinberg, *Phys. Rev.* **D26** (1982) 287;
N. Sakai and T. Yanagida, *Nucl. Phys.* **B197** (1982) 533.

[25] P. Fayet, *Nucl. Phys.* **B90** (1975) 104;
G.R. Farrar and P. Fayet, *Phys. Lett.* **76B** (1978) 575.

[26] For a recent review see, for example, H. Dreiner, hep-ph/9707435, and references therein.

[27] P. Fayet and J. Iliopoulos, *Phys. Lett.* **51B** (1974) 461.

[28] L. O'Raifeartaigh, *Nucl. Phys.* **B96** (1975) 331.

[29] S. Ferrara, L. Girardello and F. Palumbo, *Phys. Rev.* **D20** (1979) 403.

[30] G. 't Hooft, in *Recent Developments in Gauge Theories*, eds. G. 't Hooft et al. (Penum, New York, 1980).

[31] E. Gildner, *Phys. Rev.* **D14** (1976) 1667.

[32] M.T. Grisaru, W. Siegel and M. Rocek, *Nucl. Phys.* **B159** (1979) 429.

[33] I. Affleck, M. Dine and N. Seiberg, *Phys. Rev. Lett.* **52** (1983) 1026; *Nucl. Phys.* **B 241** (1984) 493.

[34] L. Girardello and M.T. Grisaru, *Nucl. Phys.* **B194** (1982) 65.

[35] S. Coleman and E. Weinberg, *Phys. Rev.* **D7** (1973) 1888;
S. Weinberg, *Phys. Rev.* **D7** (1973) 2887;
J. Iliopoulos, C. Itzykson and A. Martin, *Rev. Mod. Phys.* **47** (1975) 165.

[36] W. Fischler, H.P. Nilles, J. Polchinski, S. Raby and L. Susskind, *Phys. Rev. Lett.* **47** (1981) 757.

[37] For a review see, P. van Nieuwenhuizen, *Phys. Rep.* **68** (1981) 189.

[38] E. Cremmer, B. Julia, J. Scherk, S. Ferrara, L. Girardello and P. van Nieuwenhuizen, *Nucl. Phys.* **B147** (1979) 105;
E. Cremmer, S. Ferrara, L. Girardello and A. Van Proyen, *Nucl. Phys.* **B212** (1983) 413;
E. Witten and J. Bagger, *Phys. Lett.* **115B** (1983) 202.

[39] R. Barbieri, S. Ferrara and C. Savoy, *Phys. Lett.* **119B** (1982) 343;
L. Hall, J. Lykken and S. Weinberg, *Phys. Rev.* **D27** (1983) 2359;
S.K. Soni and H. A. Weldon, *Phys. Lett.* **126B** (1983) 215;
G.F. Giudice and A. Masiero, *Phys. Lett.* **206B** (1988) 480.

[40] M. Dine, A. Nelson and Y. Shirman, *Phys. Rev.* **D51** (1995) 1362;
S. Dimopoulos, G. Dvali G. Giudice and R. Rattazzi, *Nucl. Phys.* **B510** (1997) 12.

[41] For a recent review on dynamical supersymmetry breaking see, for example, E. Poppitz and S. Trivedi, hep-th/9803107, and references therein.

[42] J. Ellis and D.V. Nanopoulos, *Phys. Lett.* **110B** (1982) 44;
R. Barbieri and R. Gatto, *Phys. Lett.* **110B** (1983) 211;
J. Hagelin, S. Kelley and T. Kanaka, *Nucl. Phys.* **B415** (1994) 293.

[43] For a review see, for example,
A. Masiero and L. Silvestrini, hep-ph/9711401; M. Misiak, S. Pokorski and J. Rosiek, hep-ph/9703442 and references therein.

[44] J. Ellis, S. Ferrara and D.V. Nanopoulos, *Phys. Lett.* **114B** (1982) 231;
W. Buchmüller and D. Wyler, *Phys. Lett.* **121B** (1983) 321;
J. Polchinski and M.B. Wise, *Phys. Lett.* **125B** (1983) 393.

[45] For a review see, for example,
Y. Grossman, Y. Nir and R. Rattazzi, hep-ph/9701231;
A. Masiero and L. Silvestrini, hep-ph/9711401 and references therein.

[46] See, for example,
J.P. Derendinger and C. Savoy, *Nucl. Phys.* **B237** (1984) 307;
G. Gamberini, G. Ridolfi and F. Zwirner, *Nucl. Phys.* **B331** (1990) 331;
J.A. Casas, A. Lleyda and C. Munoz *Nucl. Phys.* **B471** (1996) 3,
and references therein.

[47] J. F. Gunion, H. E. Haber, G. L. Kane and S. Dawson, *The Higgs Hunters Guide*, Univ. of California, June 1989.

[48] H.E. Haber and R. Hempfling, *Phys. Rev. Lett.* **66** (1991) 1815;
J. Ellis, G. Ridolfi and F. Zwirner, *Phys. Lett.* **262B** (1991) 477;
M. Carena, J.R. Espinosa, M. Quiros and C.E.M. Wagner, *Phys. Lett.* **335** (1995) 209;
P.H. Chankowski and S. Pokorski, hep-ph/9702431, to appear in *Perspectives on Higgs Physics II*, ed. G.L. Kane, (World Scientific, Singapore, 1997);
H.E. Haber, hep-ph/9707213, to appear in *Perspectives on Higgs Physics II*, ed. G.L. Kane, World Scientific, Singapore, 1997;
J.R. Espinosa, in *Surveys High Energ. Phys.* **10** (1997), 279, hep-ph/9606316.

[49] E. Witten, *Nucl. Phys.* **B188** (1981) 513;
M. Veltman, *Acta Phys. Pol.* **B12** (1981) 437;
J. Ellis, K. Enquist, D.V. Nanopoulos and F. Zwirner, *Mod. Phys. Lett.* **A1** (1986) 57;
R. Barbieri and G. Giudice, *Nucl. Phys.* **B306** (1988) 63.

[50] J. Kim and H.-P. Nilles, *Phys. Lett.* **138B** (1984) 150.

[51] The latest constraints can often be found on the CERN home-page (`http://www.cern.ch/CERN/Experiments.html`) at the four experiments ALEPH, DELPHI, L3, OPAL.

[52] S. Dimopoulos and H. Georgi, *Nucl. Phys.* **B193** (1981) 150.

[53] L. Alvarez-Gaume, J. Polchinski and M. Wise, *Nucl. Phys.* **B221** (1983) 495.

[54] L. Ibanez and C. Lopez, *Nucl. Phys.* **B233** (1984) 511;
L. Ibanez, C. Lopez and C. Munoz, *Nucl. Phys.* **B256** (1985) 218;
S.P. Martin and M.T. Vaughn, *Phys. Rev.* **D50** (1994) 2282;
for a review see, for example, L. Ibanez and G.G. Ross, in *Perspectives on Higgs Physics*, 229.

[55] P.H. Chankowski and S. Pokorski, *Phys. Lett.* **B356** (1995) 307, hep-ph/9505308.

[56] S. Bertolini, F. Borzumati and A. Masiero, *Nucl. Phys.* **B294** (1987) 321.

[57] P. Langacker and M. Luo, *Phys. Rev.* **D44** (1991) 817;
J. Ellis, S. Kelley and D.V. Nanopoulos, *Phys. Lett.* **B249** (1990) 441;
U. Amaldi, W. de Boer and H. Fürstenau, *Phys. Lett.* **B290** (1991) 447;
P. Langacker and N. Polonsky, *Phys. Rev.* **D47** (1993) 4028, **D52** (1995) 3081.

[58] L. Ibáñez and G. Ross, *Phys. Lett.* **110B** (1982) 215.