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Nonlinear Analysis of Dynamic Stability and the Prediction of Wing Rock

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Nonlinear analysis of dynamic stability for a delta wing in rolling motion at high angles of attack is presented based on a modeled differential equation for wing rolling motion. A method for determining the aerodynamic coefficients up to third-order approximation in the modeled equation, which are functions of the amplitude of wing rolling oscillation at a fixed high angle of attack, is proposed by use of the Fourier expansion approach. Using the modeled equations of motion combined with the aerodynamic coefficients determined by the conical Eulerian computations of supersonic flow past a forced rolling delta wing, we predicted the rock motion of a delta wing that was set into a free-to-roll motion. The results were compared with those obtained by direct coupling calculations based on solving the unsteady flow equations and the wing motion equations simultaneously, which proved to be in fairly good agreement with each other. A numerical investigation of active control technique of the wing rock was also performed by use of the present method.

Introduction

In the past two decades, great attention has been paid to understanding and predicting of vortical flows about a highly swept delta wing at high angles of attack. The strengthened vortices over the leeward side of the delta wing can result in augmentation of the lift force. At certain conditions, however, the delta wing may be brought into a self-induced rolling oscillation, known as wing rock, which exerts adverse effects on the stability and control of air vehicles.[1–5] Recently, a comprehensive review of this issue was given by Katz.[6] In classical aircraft dynamics, the dynamic stability analysis is performed by using a set of linear ordinary differential equations, which are obtained by the linearization of the equations of vehicle motions.[7] All aerodynamic coefficients in these linear equations are constants and can be determined either by the theory of unsteady aerodynamics or by aerodynamic measurements. However, this is not the case for the rock-motion prediction, where the fully developed vortical flow is dominant and the dynamic equation of wing motion is highly nonlinear mathematically. To deal with such nonlinear problems, Schiff and Katz[8] have suggested a method that utilizes computational fluid dynamics (CFD) techniques to determine the regime of applicability of the nonlinear models describing the unsteady aerodynamic response to aircraft flight motions.

In the present paper, a series of nonlinear differential equations at different orders of approximation are derived and used to predict the rock motion of a delta wing, in which the unsteady aerodynamic forces are approximated by a finite Fourier series. The aerodynamic coefficients appearing in this series are calculated by solving the flow equations for a delta wing in a forced oscillating motion and then fitted into some polynomial functions of the amplitude of rolling oscillation. When this method is used, the rock motion analysis proves to be much simplified, compared with the coupling analysis by solving the flow equations and the wing rolling equation simultaneously. Nevertheless, the results obtained by both mentioned methods are still in fairly good agreement with each other.

Aerodynamic Function and Nonlinear Equation of Rolling Motion

To start the analysis, we assume that the delta wing undergoes a forced sinusoidal rolling motion at a fixed angle of attack,

$$\phi(t) = \phi_0 \sin(kt) \tag{1}$$

where $k \ll 1$ is the reduced frequency and t is the nondimensional time. At a sufficiently large time, the aerodynamic forces become periodic and can be expressed in Fourier series. In particular, the rolling moment coefficient can be written as

$$C_L(t) = \sum_{n=0}^{\infty} [b_n \cos(nkt) + c_n \sin(nkt)] \tag{2}$$

where b_n and c_n are generally the functions of ϕ_0 with $b_0 = 0$. According to the perturbation method, b_n and c_n can be expressed as

$$b_n = \sum_{i=n}^{\infty} b_n^{(i)} \phi_0^i, \qquad c_n = \sum_{i=n}^{\infty} c_n^{(i)} \phi_0^i \tag{3}$$

Based on the assumption of small ϕ_0, the first-order approximation for b_n and c_n are written as

$$b_1 = b_1^{(1)}\phi_0, \qquad c_1 = c_1^{(1)}\phi_0, \qquad b_2 = b_2^{(2)}\phi_0^2$$
$$c_2 = c_2^{(2)}\phi_0^2, \ldots, \qquad b_n = b_n^{(n)}\phi_0^n, \qquad c_n = c_n^{(n)}\phi_0^n \tag{4}$$

where $b_1^{(1)}, c_1^{(1)}, \ldots, b_n^{(n)}, c_n^{(n)}$ are constants.

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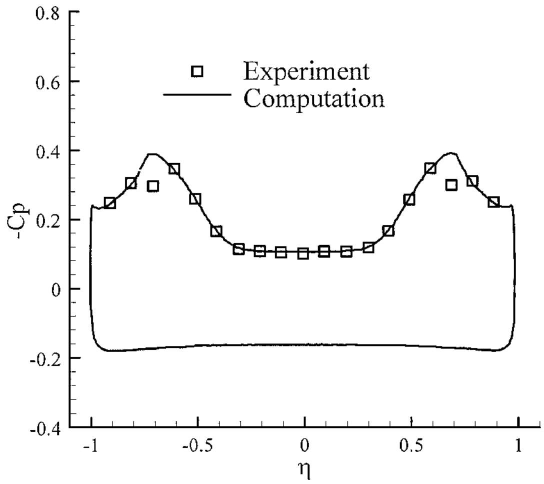
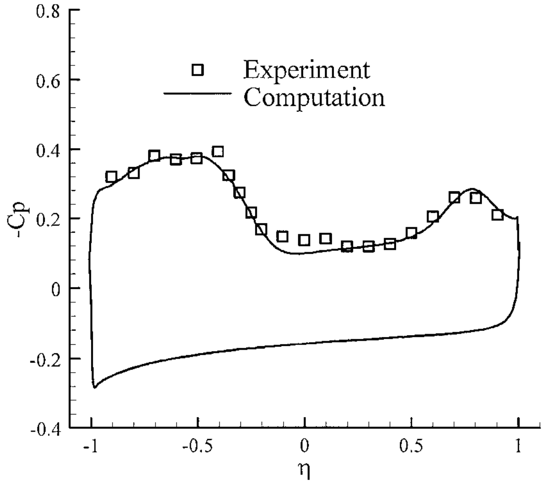


$\beta = 0$ deg



$\beta = 8$ deg

Fig. 1 Pressure coefficient comparison for a delta wing with a swept angle of 75 deg at M_∞ = 1.7 and $\alpha = 12$ deg.

Now, we can write the equation of roll motion at different order approximations. First, we take

$$C_L(t) = b_1 \cos(kt) + c_1 \sin(kt) \tag{5}$$

Because $\sin(kt) = \phi/\phi_0$ and $\cos(kt) = \dot{\phi}/(k\phi)$, where the overdot indicates time differentiation, $C_L(t)$ can be expressed as

$$C_L(t) = c_1^{(1)}\phi + \left(b_1^{(1)}/k\right)\dot{\phi} = C_{L\phi}\phi + C_{L\dot{\phi}}\dot{\phi} \tag{6}$$

where $C_{L\phi}$ and $C_{L\dot{\phi}}$ are the stiffness and damping derivatives, respectively. Then, the first approximation of the equation of rolling motion is

$$I_{xx}\ddot{\phi} - C_{L\dot{\phi}}\dot{\phi} - C_{L\phi}\phi = 0 \tag{7}$$

where I_{xx} is the mass moment of inertia about the longitudinal axis.

The aerodynamic coefficients in Eq. (7) can be obtained from the calculated or measured time trajectory $C_L(t)$ and expressed as

$$C_{L\phi} = \frac{k}{\pi\phi_0}\int_{t_0}^{t_0+2\pi/k} C_L(t)\sin(kt)\,\mathrm{d}t$$

$$C_{L\dot{\phi}} = \frac{1}{\pi\phi_0}\int_{t_0}^{t_0+2\pi/k} C_L(t)\cos(kt)\,\mathrm{d}t \tag{8}$$

When the second-order approximation is considered, we can write

$$C_L(t) = b_1\cos(kt) + c_1\sin(kt) + b_2\cos(2kt) + c_2\sin(2kt) \tag{9}$$

By the use of

$$\cos(2kt) = 1 - 2\phi^2/\phi_0^2, \qquad \sin(2kt) = 2\phi\dot{\phi}/\left(k\phi_0^2\right)$$

we have

$$C_L(t) = C_{L\phi}\phi + C_{L\dot{\phi}}\dot{\phi} + C_{L\phi\phi}\phi^2 + C_{L\phi\dot{\phi}}\phi\dot{\phi} + \Delta C_L \tag{10}$$

where $C_{L\phi\phi} = -2b_2^{(2)}$, $C_{L\phi\dot{\phi}} = 2c_2^{(2)}/k$, and $\Delta C_L = b_2^{(2)}\phi_0^2$, where ΔC_L represents the steady streaming effect and has nothing to do with the rock motion. When the orthogonality of trigonometrical

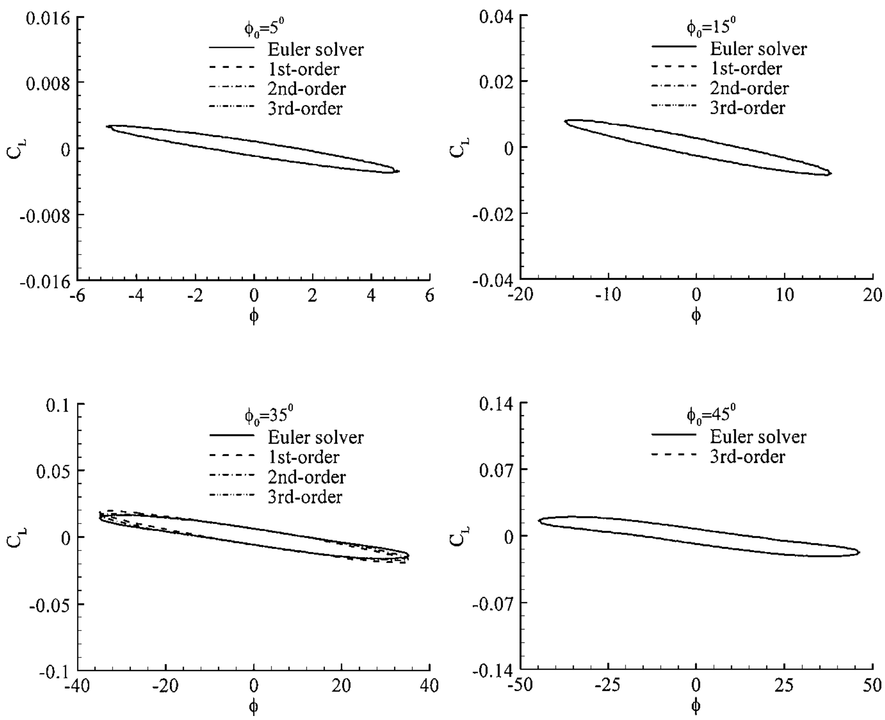


Fig. 2 Rolling moment coefficient vs roll angle at M_∞ = 1.2, $k = 0.15$, and $\alpha = 10$ deg.

functions are used, the second-order dynamic derivatives can be determined by

$$C_{L\phi\phi} = -\frac{2k}{\pi\phi_0^2}\int_{t_0}^{t_0+2\pi/k} C_L(t)\cos(2kt)\,\mathrm{d}t$$

$$C_{L\phi\dot{\phi}} = \frac{2}{\pi\phi_0^2}\int_{t_0}^{t_0+2\pi/k} C_L(t)\sin(2kt)\,\mathrm{d}t \tag{11}$$

Then, the second-order approximation of the equation of rolling motion is

$$I_{xx}\ddot{\phi} - (C_{L\dot{\phi}} + C_{L\phi\dot{\phi}}\phi)\dot{\phi} - (C_{L\phi} + C_{L\phi\phi}\phi)\phi - \Delta C_L = 0 \tag{12}$$

In a similar way, a nonlinear differential equation of rolling motion at the third-order approximation can also be derived and written as

$$I_{xx}\ddot{\phi} - C_{L\phi}\phi - C_{L\dot{\phi}}\dot{\phi} - C_{L\phi\phi}\phi^2 - C_{L\phi\dot{\phi}}\phi\dot{\phi} - C_{L\phi\phi\phi}\phi^3 - C_{L\dot{\phi}\dot{\phi}\dot{\phi}}\dot{\phi}^3 - \Delta C_L = 0 \tag{13}$$

with

$$C_L(t) = C_{L\phi}\phi + C_{L\dot{\phi}}\dot{\phi} + C_{L\phi\phi}\phi^2 + C_{L\phi\dot{\phi}}\phi\dot{\phi} + C_{L\phi\phi\phi}\phi^3 + C_{L\dot{\phi}\dot{\phi}\dot{\phi}}\dot{\phi}^3 + \Delta C_L \tag{14}$$

The expressions for relevant dynamic derivatives will not be listed here for brevity.

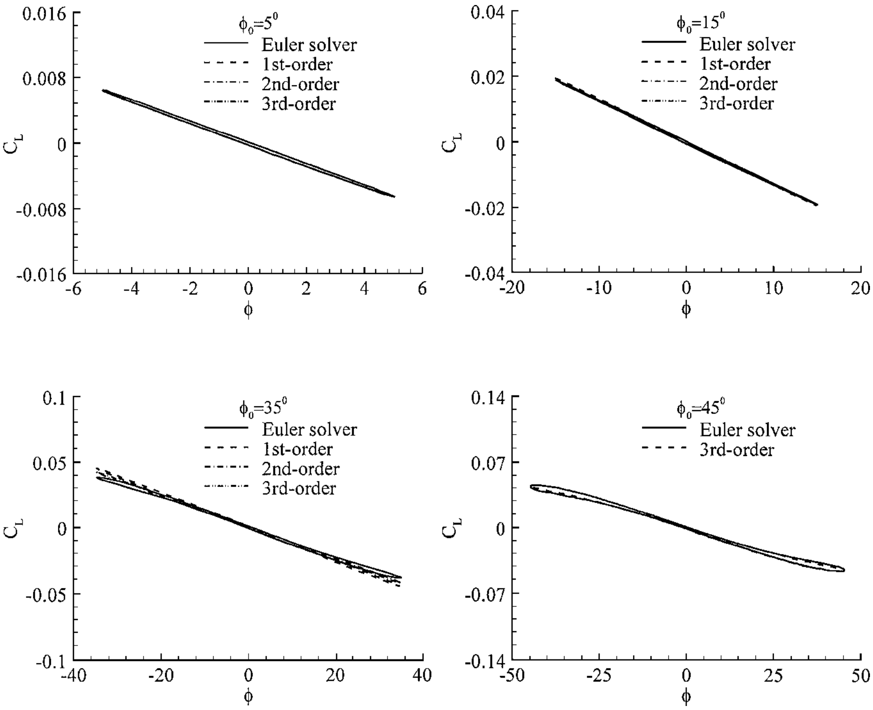


Fig. 3 Rolling moment coefficient vs roll angle at $M_\infty = 1.2$, $k = 0.15$, and $\alpha = 20$ deg.

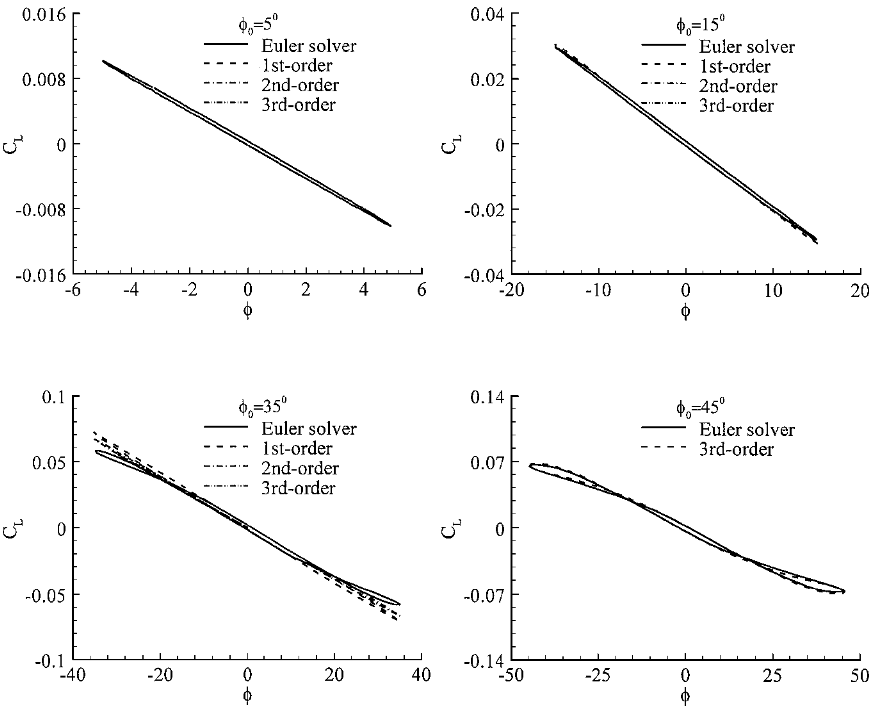


Fig. 4 Rolling moment coefficient vs roll angle at $M_\infty = 1.2$, $k = 0.15$, and $\alpha = 30$ deg.

The work done by the aerodynamic force on the wing during one cycle of harmonic motion, ΔE, can also give some insight into the dynamic stability character. ΔE in the rolling motion is defined as

$$\Delta E = \oint C_L \, \mathrm{d}\phi \tag{15a}$$

If $\Delta E < 0$, the flow will extract energy from the oscillating wing, which would have a stabilizing effect on the rolling motion, and vice versa. Substituting Eq. (2) into Eq. (15a), we get

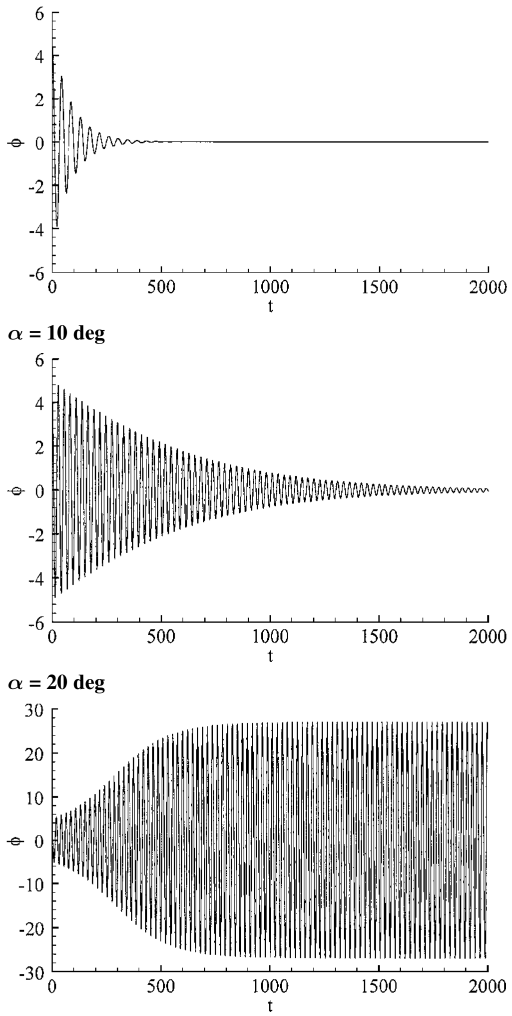


Fig. 5 Time trajectories of ϕ in the case of $\alpha = 10$, 20, and 30 deg.

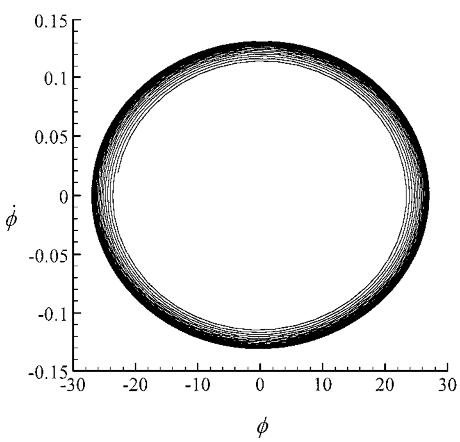


Fig. 6 Phase diagram (limit cycle) in the plane of $(\phi, \dot{\phi})$ at $\alpha = 30$ deg.

$$\Delta E = \oint \sum_{n=1}^{\infty} [b_n \cos(nkt) + c_n \sin(nkt)] \, \mathrm{d}(\phi_0 \sin kt)$$

$$= \pi \phi_0 b_1 = \pi k \phi_0^2 C_{L\dot{\phi}} \tag{15b}$$

Equation (15) is valid for all orders of approximation. Thus, we can see that $C_{L\dot{\phi}}$ always has the same sign as ΔE and can be regarded as a measure of the dynamic stability even in the nonlinear cases.

Validation of Present Aerodynamic Models by CFD Techniques

In this section, we will perform the rolling dynamic stability analysis by using the aerodynamic models suggested in the preceding section. Also a coupling analysis, by solving the flowfield equations and the exact differential equation of rolling motion simutaneously, will be carried out for the case of a free-to-roll motion experienced by a highly swept delta wing flying in supersonic flow at high angles of attack. Then, comparisions between the results obtained in these two different ways will be taken as a validation for our simplified aerodynamic models.

For simplicity, the supersonic flow past a rolling delta wing is assumed to be an unsteady conical inviscid flow, so that the unsteady conical flow Euler equation is taken as the fluid dynamic equation.[9,10] Thus, the numerical problem is reduced to the solution to an unsteady two-dimensional flow equation.

The configuration of the delta wing considered is one of 75-deg sweep angle and 0.025 thickness-to-span ratio plus a 10-deg

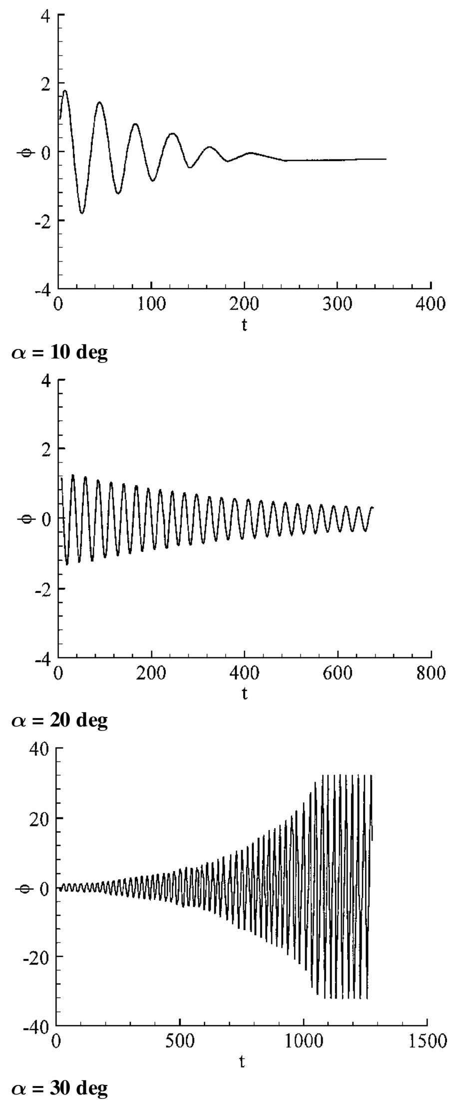


Fig. 7 Influence of the angle of attack on free-to-roll response at $M_\infty = 1.2$.

lower-edge bevel angle, which is coincident with that given in Ref. 10. To perform the numerical computation, a conical body-fitting grid system orthogonal to the wing surface, which has 177 grid points in the circumferential direction and 49 points in the normal direction, respectively, is generated by an algebraic method. The grid is clustered on the leeward side of the wing to improve the resolution of vortical flow there. To validate our computational code, the cases of steady flow past a delta wing at $M_\infty = 1.7$, $\alpha = 12$ deg, and $\beta = 0$ and 8 deg have been calculated, where β is the yaw-angle. The calculated pressure distributions are shown in Fig. 1 together with the corresponding experimental data given in Ref. 11. The agreement is fairly good.

Then this delta wing is set into a forced sinusoidal rolling motion

$$\phi(t) = \phi_0 \sin(kt) \tag{16}$$

at $\alpha = 10, 20$, and 30 deg, respectively, where the motion parameters are taken to be $\phi_0 = 5$, 15, 35, and 45 deg, $\beta = 0$ deg, and $k = 0.15$. The unsteady conical Euler calculation is performed for each of those cases until the flow approaches a stationary oscillating state. Following the procedure given in the preceding section, we calculate the aerodynamic derivatives of different orders for each case. Then, using the values of a certain aerodynamic derivative, for example, $C_{L\phi\phi}$, at $\phi_0 = 5$, 15, 35, and 45 deg, we can fit a simple polynomial function of ϕ_0 to this aerodynamic coefficient at a given angle of attack. On substitution of all of the fitted functions of aerodynamic derivatives into the wing motion equations (7), (12), and (13), we obtain the dynamic equations of rolling motion at different orders of approximation with their coefficients as some given functions of the variable ϕ. Now it is easy to see that, using the present nonlinear ordinary differential equations, we can predict the behaviors of wing rock numerically with much less computational efforts than a fully coupling numerical analysis by solving the fluid dynamic equation and the flight dynamic equation simultaneously.

To validate the aerodynamic models given in the preceding section, the procedure can be divided into three steps.

The first step is comparison between the curves C_L–ϕ obtained from both the conical Euler calculation of the flow induced by the forced wing rolling and its counterparts given by the aerodynamic models described by Eqs. (6), (10), and (14). The former is denoted as $C_L(\phi)$ and the latter as $C_L^{(p)}(\phi)$. The curves of both $C_L(\phi)$ and $C_L^{(p)}(\phi)$ are loops in the C_L, ϕ plane due to the periodicity of $\phi(t)$ and $C_L(t)$ as $t \to \infty$. In view of Eq. (15a), the shape of loop $C_L(\phi)$ can give some insight into the local character of dynamic stability for the free-to-roll motion of the delta wing. The loops of C_L and $C_L^{(p)}$ vs ϕ in all of the described cases are shown in Figs. 2–4. It can be seen from Figs. 2–4 that the third-order approximations of $C_L^{(p)}(\phi)$ are often in very good agreement with the corresponding $C_L(\phi)$, whereas the first and second approximations are only valid for smaller ϕ_0. The results also indicate that the motion is less stable as the angle of attack gets larger.

The second step is prediction of wing rock by the present aerodynamic models. When the delta wing is set into a free-to-roll motion, the timescale of the wing oscillation is much smaller than that of the variation of the amplitude with time. The local character of dynamic

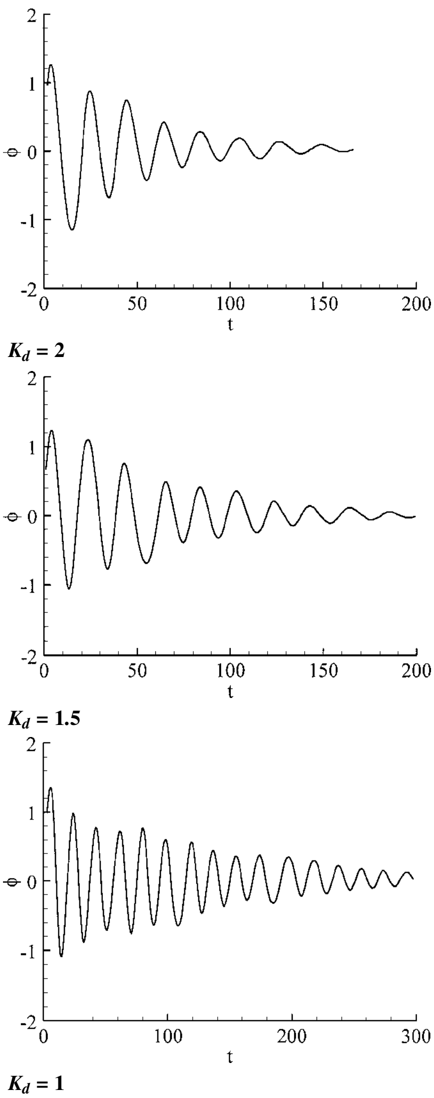


Fig. 8 Roll angle vs time with flap deflection for 75-deg swept delta wing at $M_\infty = 1.2$ and $\alpha = 30$ deg.

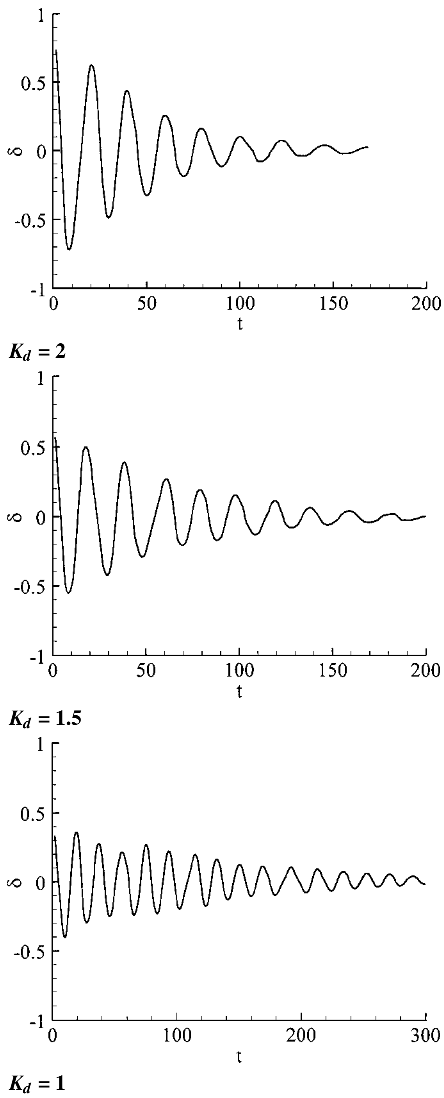


Fig. 9 Flap deflection angle vs time for 75-deg swept delta wing at $M_\infty = 1.2$ and $\alpha = 30$ deg.

stability can be approximately described by the forced oscillation motion with the same amplitude. Then, using the fitted functions of $C_{L\phi}, C_{L\dot{\phi}}, \ldots$, in terms of ϕ_0 given by the calculations of the flow in forced rolling motions (dropping the subscript 0), we can solve the initial value problem of the relevant ordinary differential equations of free-to-roll motion numerically to predict the time trajectories of the wing rolling, where $I_{XX} = 0.1776 \times 10^{-3}$ kgm^2 is taken from Ref. 11. The resulted time trajectories of ϕ in the cases of $\alpha = 10$, 20, and 30 deg are shown in Fig. 5. It can be seen that in the cases of $\alpha = 10$ and 20 deg, the rolling oscillations are damped, and the rate of damping is smaller at $\alpha = 20$ deg than that at $\alpha = 10$ deg. In the case of $\alpha = 30$ deg, however, the initial oscillations will be amplified with time and gradually approach a stationary roll oscillation, that is, a limit cycle. The phase diagram in the plane of $(\phi, \dot{\phi})$ is plotted in Fig. 6. The calculated amplitude and reduced frequency for the rock motion at $\alpha = 30$ deg are $\phi_{\max} = 27.8$ deg and $k = 0.135$, respectively.

The third step is prediction of wing rock by the coupling analysis for the free-to-roll motion of a delta wing. We solve the conical Euler equations and the exact differential equation of rolling motion simultaneously to predict the wing rock at $\alpha = 10$, 20, and 30 deg once again. The resulted time trajectories of roll angle for $\alpha = 10$ and 20 deg are shown in Fig. 7. It is seen that the free-to-roll motions are damped for both $\alpha = 10$ and 20 deg. However, the rate of damping at $\alpha = 20$ deg is obviously smaller than that of $\alpha = 10$ deg, which is consistent with both the aerodynamic model analysis and the energy criteria ΔE given in Figs. 2 and 3. At $\alpha = 30$ deg, the rock motion appears with a amplitude of oscillation $\phi_{\max} = 33$ deg and a reduced frequency about $k = 0.126$, which agrees well with the experimental value of $k = 0.125$ in Ref. 12, indicating the conical flow assumption is valid. On the other hand, agreement between the second step and the third step seems not as satisfactory, but still acceptable, at least for the application to aeronautical design.

Application to Numerical Study of Wing Rock Control

In the past decade, the investigation of control technique of wing rock has received much attention.[13–15] As an application, the numerical analysis of active rate feedback control is performed here on the suppression of wing rock in a supersonic flow for the delta wing considered at $\alpha = 30$ deg. Two means of active control, that is, unsteady leading-edge flap deflection and blowing and suction, are examined.

In the former case, a simple control law of $\delta = K_d\dot{\phi}$ is employed, where K_d is the control gain and δ is the active control parameter, the deflection angle of leading-edge flap.

In the forced oscillation motion $\phi = \phi_0 e^{ikt}$, the linear expression for $C_L(t)$ can be written in a complex form:

$$\begin{aligned} C_L &= C_{L\phi}\phi + C_{L\delta}\delta = C_{L\phi}\phi + C_{L\delta}K_d\dot{\phi} \\ &= \{[\mathrm{Re}(C_{L\phi}) - kK_d\,\mathrm{Im}(C_{L\delta})] + i[\mathrm{Im}(C_{L\phi}) + kK_d\,\mathrm{Re}(C_{L\delta})]\}\phi \end{aligned} \tag{17}$$

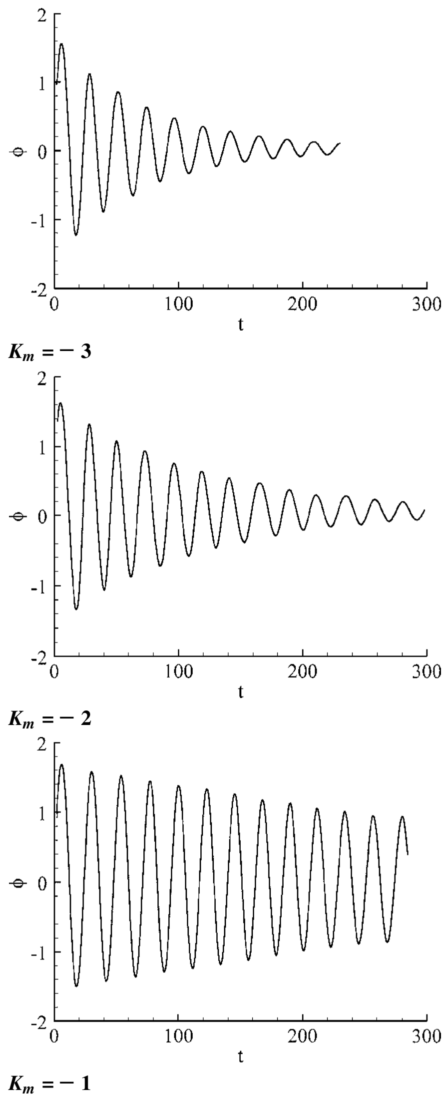


Fig. 10 Roll angle vs time with blowing and suction for 75-deg swept delta wing at $M_\infty = 1.2$ and $\alpha = 30$ deg.

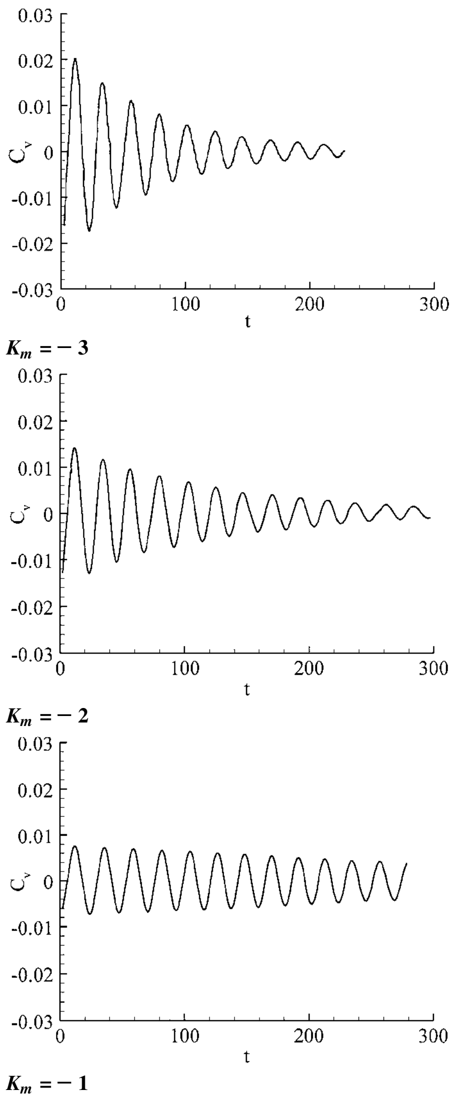


Fig. 11 Blowing and suction strength vs time for 75-deg swept delta wing at $M_\infty = 1.2$ and $\alpha = 30$ deg.

To damp the rock motion, we need

$$\mathrm{Im}(C_{L\phi}) + kK_d\,\mathrm{Re}(C_{L\delta}) < 0$$

That is,

$$K_d > -\frac{1}{k}\frac{\mathrm{Im}(C_{L\phi})}{\mathrm{Re}(C_{L\delta})} \tag{18}$$

where $\mathrm{Re}(C_{L\delta}) < 0$ is the aerodynamic stiffness of the leading edge flap. The analysis for the forced-rolling motion at $\alpha = 30$ deg shows that $\mathrm{Im}(C_{L\phi})$ is positive motion for small ϕ_0, implying the aerodynamic system is unstable. So, tentatively we set $K_d = 2$, 1.5 and 1. Then, using the aerodynamic models given earlier, the calculated $\phi(t)$ and $\delta(t)$, which exhibit a damped response as the active control of flap deflection is imposed, are shown in Figs. 8 and 9.

Instead of deflection of leading-edge flap, we can also use antisymmetric blowing–suction, which is expressed as

$$q = K_m\dot{\phi} \tag{19}$$

where q is the mass flux of blowing–suction on the wing upper surface near the leading edge. The virtual mass flux is numerically imposed by adding a velocity normal to the wing surface in the blowing–suction area. The positive and negative values of q are defined for blowing and suction, respectively. Thus, here we choose $K_m = -1, -2$, and -3. The time history of $\phi(t)$ and $q(t)$ are shown in Figs. 10 and 11 and indicate that the approach of blowing–suction is effective as well.

Conclusions

A simplified method of nonlinear analysis for the dynamic stability of a delta wing at high angles of attack is suggested. The model consists of a series of nonlinear ordinary differential equations, which describe the vehicle's motion approximately. The aerodynamic coefficients appearing in the ordinary differential equations of wing motion are then determined by solving the fluid dynamic equations numerically for the cases of forced-oscillating wing motion and fitted into simple algebraic functions. Dynamic stability analysis, in particular, the prediction of rock motion, follows by using, respectively, the simplified model and the fully coupling analysis, which solves the fluid dynamic equations and the wing motion equation simultaneously. The results obtained from these two different approaches proved to be in fairly good agreement.

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