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Analysis of the Scale Effect for Microscale Machine Tools

Hyung Wook Park
hyungwook.park@gatech.edu

Steven Y. Liang
Phone: (404) 894-8164
steven.liang@me.gatech.edu

George W. Woodruff School of
Mechanical Engineering
Georgia Institute of Technology
Atlanta, GA 30332-0405, U.S.A.
Fax: (404) 894-4133

ABSTRACT

Miniaturization of conventional machine tools has been initiated due to inherent technical and economical advantages. To support further development in this area, a systematic design scheme must be developed on a quantitative basis to reduce the subjective design of miniaturized machine tools. The present work is the size optimization of the miniaturized milling machine on the basis of the proposed design strategy. This design strategy includes individual mathematical computations of key parameters such as volumetric error, machine working space, static, thermal, and dynamic stiffness. This mathematical modeling is established by using analytical methods followed by experimental validation. Individual computations based on the mathematical modeling are carried out to produce the penalty function of the miniaturized milling machine which is then used to find out the optimal dimension. In addition to this, the sensitivity of weighing factors is discussed to find out which weighing factor is more effective to an optimal solution. This study will eventually contribute to the development of more precise miniaturized machine tools.

INTRODUCTION

The demand for manufacturing of microscale parts and components has initiated much of the earlier work in microscale machining throughout industrial and academic sectors. These micromachining technologies are a relatively early research area compared to other manufacturing processes, and they will eventually emerge as the deciding factors to the survival of many industries in rapidly changing and highly competitive market sectors over the next two to three decades. Therefore, new design concepts, procedures, and machine configurations are needed to meet increasingly stringent requirements and expectations. In this area, one of the research directions is the minimization, on a theoretical and quantitative basis, of traditional machine tools to support micro-manufacturing.

Inherent technical and economical advantages of the above research direction motivated the development of various prototypes of miniaturized machine tools[1-6]. In this area, most research works focus on the development of suitable miniaturized machine tools corresponding to target microscale parts. These prototypes of miniaturized machine tools show good machining capability to produce microscale parts. However, there have not been enough studies for design of miniaturized machine tools and as a result, little design knowledge and experience base toward miniaturization of these systems has been accumulated. In spite of several attempts, optimization of miniaturized machine tools has not been systematically attained. In order to provide guidance for further development, the studies of a systematic design of these systems are necessary.

At the beginning of the research for miniaturized machine tool design, Mishima *et al.* [7] presented a design evaluation method for miniaturized machine tools. This method adopted a kinematic representation of machine structures with the Taguchi method to estimate the contribution of each local error component, which serves as proper background for this study. Chen *et al.* [8] developed a novel virtual machine tool (VMT) integrated design environment in which kinematic functionality was embedded in the description of the sub-components. The results of the VMT configuration analysis for miniaturized machine tools are similar with configuration candidates of traditional machine tools. In these studies, volumetric error for various machine configurations is used as a key criterion to estimate the proper configuration for miniaturized machine tools. Recent work by Lee *et al.* [9] studies the dynamic behavior characterization of a mesoscale machine tool (mMT). In this study, the size effect in the dynamic behavior of the mMT was investigated experimentally and numerically. The results show that the characterization of the dynamic properties of the joints of mMTs is an important factor in determining the dynamic behavior of mMTs. However, previous works don't provide the information of optimal dimensions per each design configuration.

In design of traditional and miniaturized machine tools, a conceptual design stage of all major elements is important. This step defines basic features and capabilities of traditional and miniaturized machine tools. In the case of miniaturized machine tools, this design stage acquires more importance because the ratio of an overall size of miniaturized machine tools to target products significantly decreases compared to traditional machine tools. The conceptual design for miniaturized machine tools has to be accompanied with comprehensive analyses of possible effective factors such as static, dynamic, and thermal stiffness, machine accuracy, and machine working volume.

This present work is an extension of the previous work for optimization of a machine structure size according to different machine configurations [10]. In this study, the theoretical modeling and experimental analysis of possible design parameters of miniaturized machine tools is pursued with the supplement of experimental analysis based on a hammer impact test. Three different sizes of machine tool structures are engaged to identify the variation trend of design parameters as a function of a miniaturized machine tool size. As an indicator of the dynamic characteristics, first natural frequency and damping ratio are experimentally measured to model the dynamic properties of the joints within the miniaturized milling tool. As a result, the trend of the dynamic behavior of the miniaturized milling tool can be theoretically modeled, which provides the basis for optimal dimensioning of minimized manufacturing equipments at the design stage. Finally, the proposed design model in Figure 1 is applied to obtain the optimal size of the miniaturized milling machine.

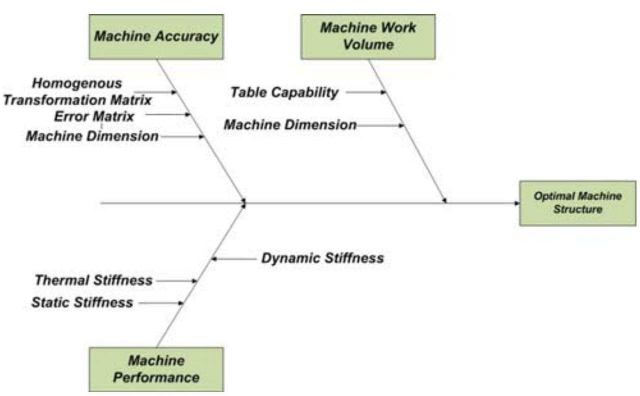


Figure 1: Proposed design strategy

The following section provides evaluation of miniaturized machine tool's structural performance in terms of static, thermal, and dynamic stiffness. This is followed by the computations of the computation of volumetric error and machine work volume according to different machine sizes. Finally, optimization for the developed miniaturized milling machine is carried out.

2. STRUCTURAL PERFORMANCE EVALUATION

The design of a machine structure is an important decision step considering many aspects of various products. It is challenging because existing sources of uncertainty, such as errors due to geometric configuration, thermally induced errors, and load induced error affect machine's final performance. In order to provide more liable machine performance and better product quality, some key factors such as static, thermal, and dynamic stiffness, machine accuracy, and machine working volume have to be considered at a conceptual design stage. In the following sections the analytical modeling is presented and experimental verification using various machine tool sizes is discussed in verification of the models.

2.1 STATIC STIFFNESS EVALUATION

The structure of miniaturized machine tools experiences external forces during machine operation, which can create the deflection of the machine structure. This deformation affects the machine accuracy. In this study, the complex miniaturized machine tool is simplified into a cantilever beam with an intermediate load and moment. The model studied in this paper includes a sliding constraint in the joint of the computed model, which induces the loss of moment transmission at the joint.

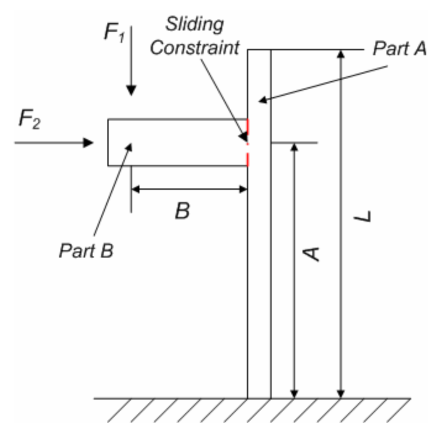


Figure 2: Simplified machine structure model

Assuming that F_1 and F_2 are equal, the static stiffness of the miniaturized machine can be expressed as:

$$K_{static\_stiff} = \frac{6EI}{A^2(2A - 3C_{loss}B)} \tag{1}$$

where A and B are machine dimensions, C_{loss} the moment loss, E the modulus of elasticity, and I the moment of inertia.

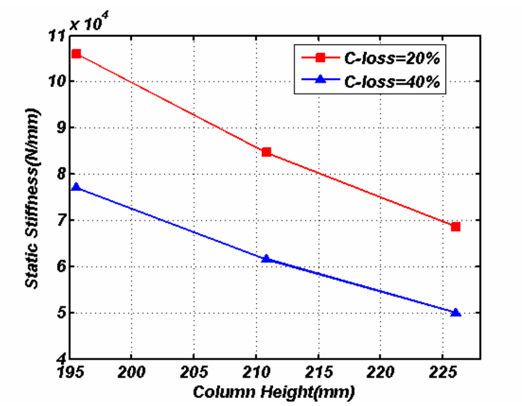
The computed results of static stiffness for this miniaturized machine tool are shown in Figure 3(a).

2.2 THERMAL STIFFNESS EVALUATION

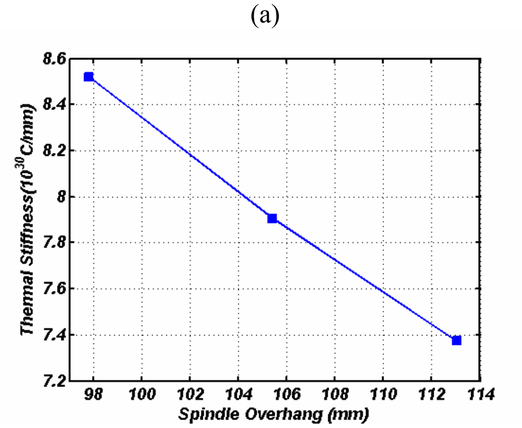
In miniaturized machine tools, if an ambient temperature is stable, the main heat source is the operating spindle with a high RPM. The temperature variation of the miniaturized milling machine is observed around 10º due to this heat source. The spindle is assembled at the part B in Figure 2. So, thermal stiffness that is related to the change in a machine linear dimension can be obtained as:

$$K_{thermal} = \frac{1}{\alpha B} \tag{2}$$

where α is the thermal expansion coefficient.



(a)



(b)

Figure 3: (a) Static and (b) Thermal Stiffness of the miniaturized milling machine

The material selected in this study is Invar 36 steel alloy with the thermal expansion coefficient of $1.2\times10^{-6}/^{o}C$. On the basis of above equation, the results of thermal stiffness according to a machine size are shown above in Figure 3(b).

2.3 DYNAMIC CHARACTERISTIC EVALUATION

In the case of miniaturized machine tools, the spindle rotation as high speeds causes severe vibrations that decrease machine accuracy and affect product quality. The computations of the structural dynamic behavior of miniaturized machine tools are necessary to ensure their stable operation with proper relative displacement between the tool and the workpiece. General methods to characterize the dynamic behavior of machine tools are a lumped parameter method and a finite element analysis of modules. In view of the enumeration of various machine sizes, a lumped parameter model is preferred, which is relatively simple with respect to a finite element (FE) model.

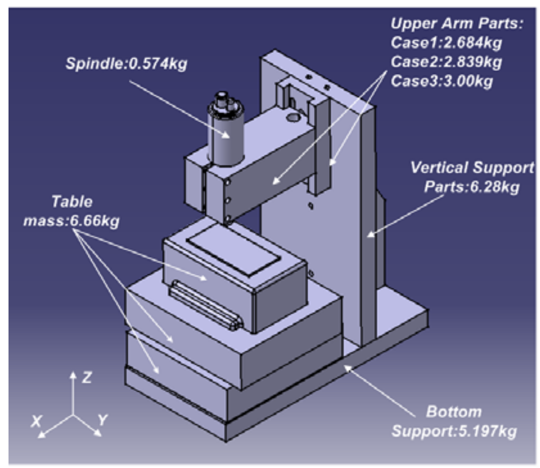


Figure 4: Machine configuration and masses of machine components

In a lumped parameter model, a miniaturized machine is assumed as an assembly of modules connected by joints which are modeled as springs and dampers. In this study, although the miniaturized milling machine consists of many components, it is assumed as a three degree of freedom mass system for the purpose of simple modeling in Figure 5. The masses of main machine components used in this model are shown in Figure 4. The governing equation describing the motion of a multi-degree of freedom system with viscous damping is given by:

$$[M]\{\ddot{x}\} + [C]\{\dot{x}\} + [K]\{x\} = \{f\} \tag{3}$$

where $[M]$ is the mass matrix, $[C]$ is the damping matrix, and $[K]$ is the joint stiffness matrix.

Assuming a solution of the displacement is the harmonic motion, the receptance matrix with substituting this assumed displacement into the above equation will be:

$$[G(w)] = \frac{1}{[K] - \omega^2[M] - j\omega[C]} \tag{4}$$

where $[G(w)]$ is the receptance matrix and ω is frequency.

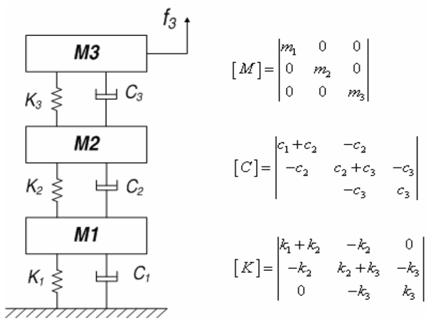


Figure 5: Three degree of freedom lumped parameter model

A classic approach for solving the above equation is the modal superposition. The natural frequencies of this system can be computed in this way. At the design stage, the development of damping matrix is not straightforward because it is hard to assume certain values. In this case, Rayleigh damping theory [11] is used, in which it is assumed that the damping matrix is proportional to mass and stiffness matrices as the following:

$$[C]=\alpha[M]+\beta[K] \tag{5}$$

where α and β are coefficients.

Based on this theory, the damping coefficient in mode i can be expressed as:

$$\xi_i=\frac{\alpha}{2\omega_{ni}}+\frac{\beta\omega_{ni}}{2} \tag{6}$$

where ξ_i is the damping ratio and ω_{ni} is the natural frequency in mode i.

However, there is the need for identifying the parameters of springs and the coefficients of this damping model. Therefore, the analytical computation of a lumped parameter model has to be supplemented by the experimental analysis such as a hammer impact test.

2.3.1 EXPERIMENTAL ANALYSIS

The machine configuration studied here is the knee and column type vertical milling machine in Figure 6 where A is the column height, B the spindle overhang, C the spindle protrusion. In order to identify the above described parameters of this machine configuration, a classic hammer impact test is engaged to investigate its structural dynamic behavior experimentally for three different dimensions that are listed in Table 1.

Table 1: Dimensions of the miniaturized milling machine

	A(inch(mm))	B(inch(mm))	C(inch(mm))
Case 1	7.7(195.8)	3.85(97.91)	3(76.2)
Case 2	8.3(211.1)	4.15(105.5)	3(76.2)
Case 3	8.9(226.3)	4.45(113.2)	3(76.2)

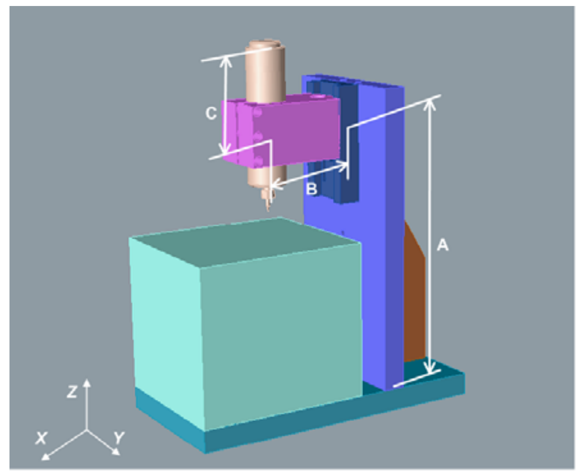
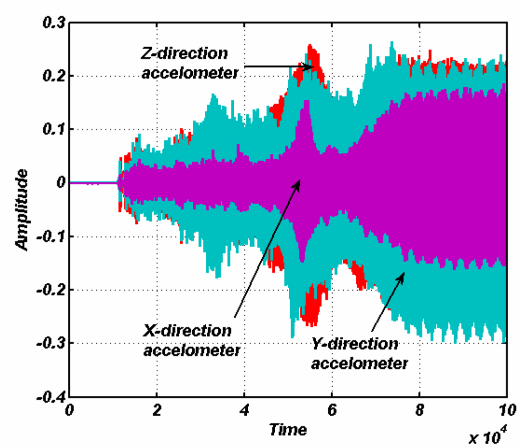
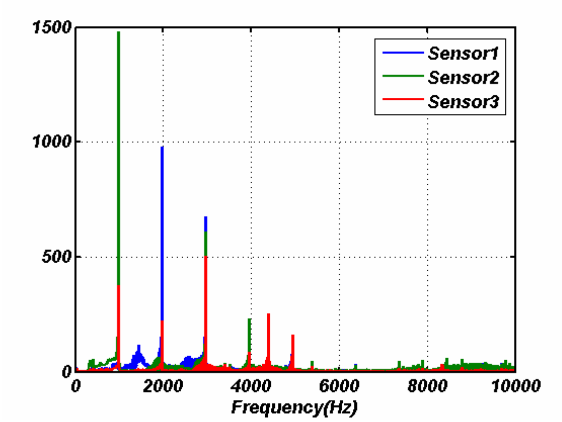


Figure 6: Illustration of the miniaturized milling machine with variable dimensions

At first, when the spindle rotates with a maximum speed (60,000rpm), the dynamic properties of the machine structure are observed without machining. In the experiment, light weight accelerometers (Kistler 8630C50) having a 6 kHz frequency range are attached at the locations shown in Figure 8(a). Since the mass of this accelerometer is much smaller than that of the miniaturized machine tool, the mass effect of these sensors can be ignored in the analysis of experiment data.



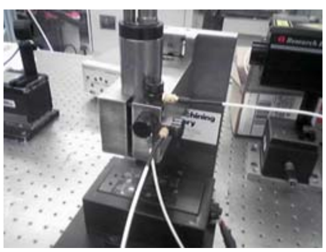
(a)



(b)

Figure 7: (a) Original shape and (b) FFT analysis of output signals from accelerometer sensors in the case 1

It is observed that a magnitude of the output signal in the Y direction is largest among output signals in Figure 7(a). As in Figure 7(b), the major peaks in the fast fourier transform (FFT) analysis of these measured signals are around 1000Hz, 2000Hz, and 3000Hz, which come from the spindle rotation speed. Therefore, the dynamic characteristic of the miniaturized machine tool in the Y direction is more important compared to other directions.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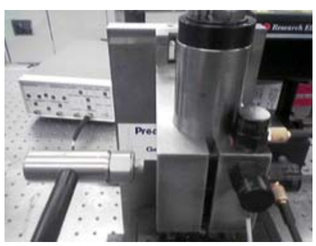
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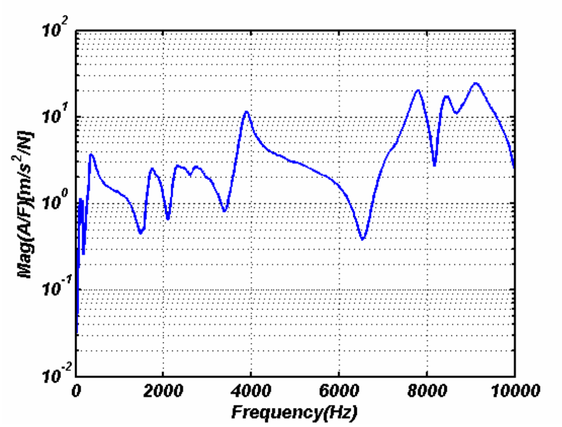

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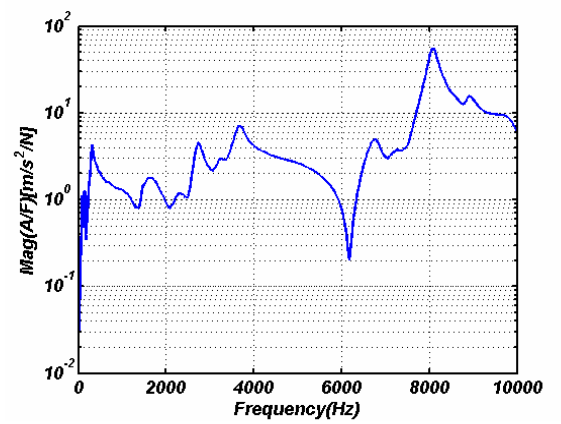


(d)

Figure 8: (a) Illustration of sensor positions, (b) the hammer impact, (c) the FRF of the case 1 and (d) the case 2 in the y direction

In this experiment, a hammer impact test is adopted as a tool for inspection of the dynamic behavior of miniaturized machine tools. Basically, the vibration of a machine tool is excited by an impulse force, referred to as a hammer impact, and a response of excited vibration is recorded by accelerometers. From the time histories recorded during tests, frequency response functions (FRFs) can be obtained as shown in Figure 8(c) and (d).

The impact hammer used in this study is a medium impact hammer with a metal tip (Kistler type 9722A), and 20 impacts were recorded and averaged linearly. The obtained results for three different sizes are listed in Table 3.

2.3.2 IDENTIFICATION OF DYNAMIC PARAMETERS

Based on experimental results, the optimization of objective functions is necessary to identify the values of springs and damping coefficients at joints. Objective functions for computations of joint stiffness and damping coefficients are conducted from comparisons between experimental and computed results. The forms used in this work are:

$$f = \min\left(abs\left(\sum_{i=1}^{3} \omega_{ni}\left(k_1, k_2, k_3\right) - \omega_{\exp,ni} \right) \right) \tag{7}$$

and

$$f = \min\left(abs\left(\sum_{i=1}^{3} \left[\xi_i\left(\alpha, \beta\right) - \xi_{\exp,i} \right] \right) \right) \tag{8}$$

The optimizations of these objective functions are performed by using the embedded function ‘fminsearch’ in MATLAB® based on the Simplex method. This method is generally a robust and simple algorithm that is an application to many different problems. However, the solutions of this method depend on initial guesses. Based on the aforementioned method, the following values in Table 2 are selected to model dynamic properties of the joints.

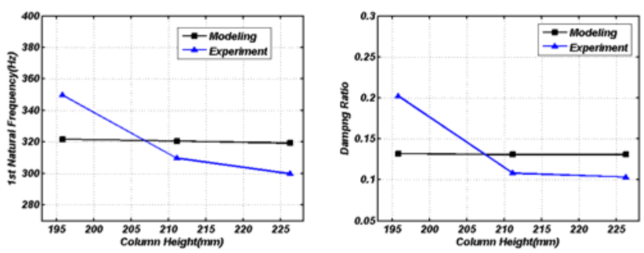
Table 2: Values of spring and damping parameters

Joint Stiffness			Rayleigh damping	
k_1	k_2	K_3	α	B
8.81×10^7	5.22×10^9	5.17×10^8	1×10^{-4}	121.79

The computation results for dynamic properties of this miniaturized machine tool based on above values of spring and damping parameters are compared with experiment data in Figure 9.

Table 3: Natural frequencies and damping ratios of experimental analysis and modeling

	1st Natural frequency		Damping ratio	
	Experiment	Modeling	Experiment	Modeling
Case1	349.8	321.78	0.202	0.1321
Case2	309.8	320.56	0.1081	0.1309
Case3	299.9	319.3	0.1031	0.1307



(a) (b)

Figure 9: Comparisons between experiment data and modeling values in terms of (a) 1st natural frequency and (b) damping ratio

3. VOLUMETRIC ERROR EVALUATION

In this study, a kinematic chain representation model, referred to as the form shaping theory [12] is used to estimate a volumetric error of a miniaturized machine tool according to its size. In this method, coordinate transformation, which uses homogenous transform matrix (HTM), is applied to represent the relative displacement between a workpiece and a machine tool as a chain of directly linked rigid components. The mathematical form for computing the result of transformation matrices between a workpiece and a tool is:

$$r_{ideal} = \left[HTM\right]_{P_z} \times \left[HTM\right]_{P_y} \cdots \left[HTM\right]_{B} \times \left[HTM\right]_{L_t} \qquad (9)$$

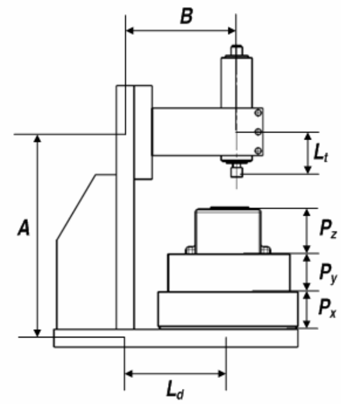


Figure 10: Schematic diagram of the miniaturized machine tool

There are many error sources such as alignment errors and geometric errors in actual machine tools. These errors can be considered as errors in transformations between components with an error HTM. The basic form of this matrix is:

$$\left[HTM\right]_{error_i} = \begin{vmatrix} 1 & -\gamma_i & \beta_i & \delta_{xi} \\ \gamma_i & 1 & -\alpha_i & \delta_{yi} \\ \beta_i & \alpha_i & 1 & \delta_{zi} \\ 0 & 0 & 0 & 1 \end{vmatrix} \qquad (10)$$

where α_i, β_i, and γ_i are rotation errors and δ_{xi}, δ_{yi}, and δ_{zi} are translational errors.

Estimation of resultant volumetric error, which includes the errors between moving components, can be obtained by implementing error HTMs between two ideal HTMs. In this study, assumptions used to simplify the numerical calculation in miniaturized machine tool accuracy are the following: 1) rigid body motion, 2) small errors approximation, and 3) negligible assembly errors. With these assumptions, the second, third, and forth orders error terms can be cancelled during the matrix multiplications. After this computation, the relative error between a tool and a workpiece can be obtained as the following:

$$\Delta r = r_{error} - r_{ideal} \qquad (11)$$

On the basis of the miniaturized milling machine structural drawing in Figure 10, the amount of error in each direction which includes error terms and machine's dimensions can be obtained as the following:

$$\begin{vmatrix}\Delta x\\ \Delta y\\ \Delta z\end{vmatrix}=\begin{vmatrix}\delta_{x1}+\delta_{x2}+\delta_{x3}+\delta_{x4}+(\beta_1+\beta_2+\beta_2)A-\beta_1P_y-(\beta_1+\beta_2)P_x\\ +(\gamma_1+\gamma_2+\gamma_3+\gamma_4)B+(\beta_1+\beta_2+\beta_3+\beta_4)L_t-\gamma_1W\\ \delta_{y1}+\delta_{y2}+\delta_{y3}+\delta_{y4}+(\gamma_1+\gamma_2+\gamma_3)L_d-(\alpha_1+\alpha_2+\alpha_3)A\\ +(\gamma_1+\gamma_2)V+(\alpha_1+\alpha_2)P_x+(\alpha_1+\alpha_2+\alpha_3+\alpha_4)L_t+\alpha_1P_y\\ \delta_{z1}+\delta_{z2}+\delta_{z3}+\delta_{z4}-(\alpha_1+\alpha_2+\alpha_3+\alpha_4)B-(\beta_1+\beta_2)V\\ -(\beta_1+\beta_2+\beta_3)L_d+\alpha_1W\end{vmatrix} \tag{12}$$

where V and W are workpiece sizes.

On the basis of equation(11), the amount of error for each direction can be estimated as Δx, Δy, and Δz. For the purpose of a simple modeling, error terms which are used to compute machine accuracy are assumed to have the amplitude of unity (1μm) for the sake of analysis. The variance of the error amounts has been used to estimate the error magnitude according to a machine size because there are different signs within the computation of error amounts in each axis, which is shown in equation (13)

$$\Delta_x=\Delta x-\Delta x_{ref},\Delta_y=\Delta y-\Delta y_{ref},\Delta_z=\Delta z-\Delta z_{ref} \tag{13}$$

Then, the mathematical function to compute machine accuracy of the miniaturized milling machine can be expressed as:

$$f_{error}=\sqrt{\Delta_x^2+\Delta_y^2+\Delta_z^2} \tag{14}$$

The volumetric error in this study is computed at the tip of a microscale tool on the basis of the above equations. The magnitudes of this volumetric error are obtained according to different dimensions. These errors are used in equation(17).

4. FORMULATION OF THE PENALTY FUNCTION

Individual computations for key factors such as volumetric error, machine working space, and static, thermal, and dynamic stiffness yielded the computational penalty function. In this study, discrete computation results are translated into the continuous domain using a non-linear regression or analytical solutions to facilitate the quantitative analysis of the penalty functions. Furthermore, the machine work volume is considered as a function of positioning table capability and machine dimensions in the following:

$$f_{volume}=Travel\_X\times Travel\_Y\times (A-L_t+P_x+P_y+P_z) \tag{15}$$

Finally, the penalty function associated with the miniaturized milling machine can be formulated as the following form:

$$f_{penalty}=c_1f_{error}+c_2f_{volume}+c_3f_{static}+c_4f_{thermal}+c_5f_{natural\ frequency}+c_6f_{damping} \tag{16}$$

where c_i is the weighing factor.

After constructing the penalty function, a normalization routine is implemented to identify the variation of each computing parameter. All parameters are divided into the positive effect and the negative effect parameters. Generally, larger stiffness parameters, larger work volume, and smaller accuracy values are desired. Thus, considering individual effects, the penalty function is normalized as:

$$f=C_1\frac{f_{error}}{f_{error}^{\max}}+C_2\frac{f_{volume}^{\max}}{f_{volume}}+C_3\frac{f_{static}^{\max}}{f_{static}}+C_4\frac{f_{thermal}^{\max}}{f_{thermal}}+C_5\frac{f_{natural\ frequency}^{\max}}{f_{natural\ frequency}}+C_6\frac{f_{damping}^{\max}}{f_{damping}} \tag{17}$$

During the mathematical computation, an equal weighing factor is used to find out optimal the dimensions of the miniaturized machine tools.

5. RESULTS

Figure 11 shows that the optimal value of the mathematical calculation is around the column height of 220mm for an equal weighing factor. In that case, summaries of other results such as working volume, volumetric error amount, static, thermal, and dynamic stiffness are given in Table 4.

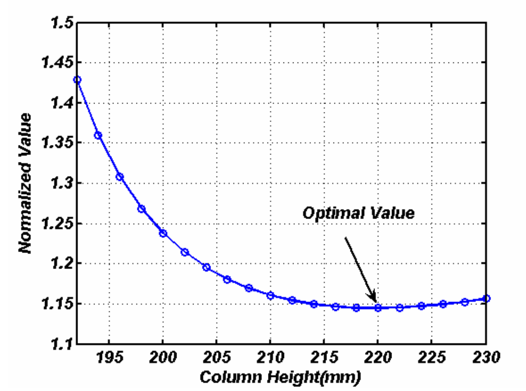


Figure 11: Assessment of optimal dimensions of the miniaturized milling machine

Table 4: Magnitudes of other parameters at the optimal point

	Values
Volumetric error[mm]	1.55
Working volume[mm^3]	25×25×42
Static stiffness[$\times10^4$]	7.448
Thermal stiffness[$\times10^3$]	7.576
1st Natural frequency[Hz]	319.85
Damping ratio	0.1308

The results of this computation will facilitate the objective decision of design of miniaturized machine tools because this computation can provide cursory technical information according to machine frame sizes. On the other hand, in order to identify the effect of weighing factors such as C_1, C_2, C_5, and C, computations based on the L9 matrix of the Taguchi method are carried out. In these computations, C_3 and C_4 are held as constants. The results of this sensitivity analysis of weighing factors in Figure 12 show that the working volume weighing factor, C_2, has the dominant effect to the optimal solution of the penalty function.

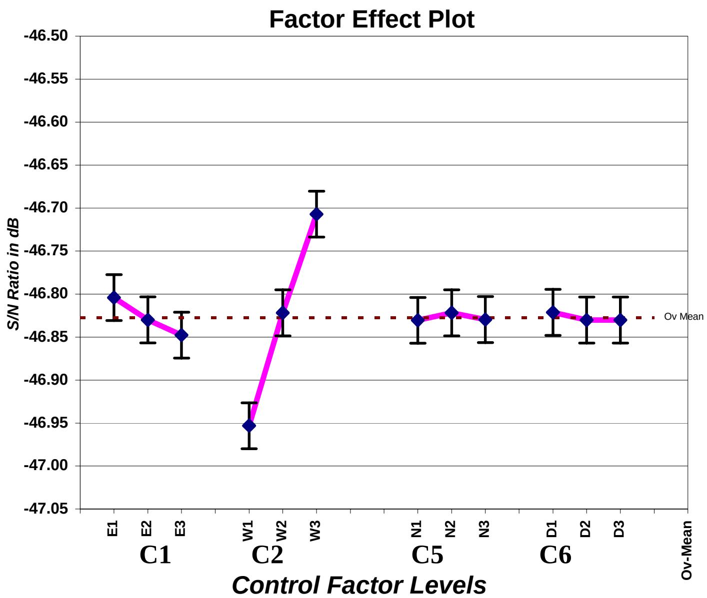


Figure 12: The sensitivity analysis of weighing factors

6. CONCLUSIONS

In this study, the optimization of the miniaturized machine tool is performed on the basis of the proposed design strategy. Individual mathematical modeling of key parameters such as volumetric error, machine working space, and static, thermal, and dynamic stiffness are conducted and supplemented with experimental analysis using a hammer impact test. These computations yield the optimal size of the miniaturized milling tool with the technical information of other parameters. In addition to this, the sensitivity of optimal machine size with respect to weighing factor of the penalty function is also discussed on the basis of the Taguchi method. It is observed that the working volume weighing factor plays an important role in determining an optimal size of miniaturized machine tool.

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