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Analysis of Dynamic MRTS Traction Power Supply System Based on Dependent Train Movement Simulation

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ABSTRACT

As the motivation in developing modern MRTS and the upgrading of old systems in many metropolitans all over the world, especially for public transportation in developing countries, the demand to simulate the dynamic traction power supply system (TPSS) more effectively and practically has increased. This paper describes the work of simulating and analysing TPSS. It is based on dependent train movement in conjunction with TPSS simulation to establish a panorama view of the features. The free time scheduled strategy for train movement is applied which is more flexible and effective to reflect operation strategies. To set up the dynamic TPSS structure with respect to the moving train status for the system solution, the schematic arrangement is split into four parts: establishment of the elementary electric network framework; establishment of the dynamic electrical network structure which includes both the elementary power network and the moving trains statuses; dynamic electric network simulation; and system analyses based on the panoramic features drawn from the simulation. The simulation results are based on the Shanghai Metro Line One (excluding the extension section). Simulation and analysis on more lines including Shanghai Line Two and Guangzhou Line One MTRS have also been implemented using similar approaches. The complete profiles of the TPSS provide a practical means in analysing the system configuration as well as organizing the system operation.

NOMENCLATURE

Mass Rapid Transit System, Traction power supply system simulation, Train movement simulation

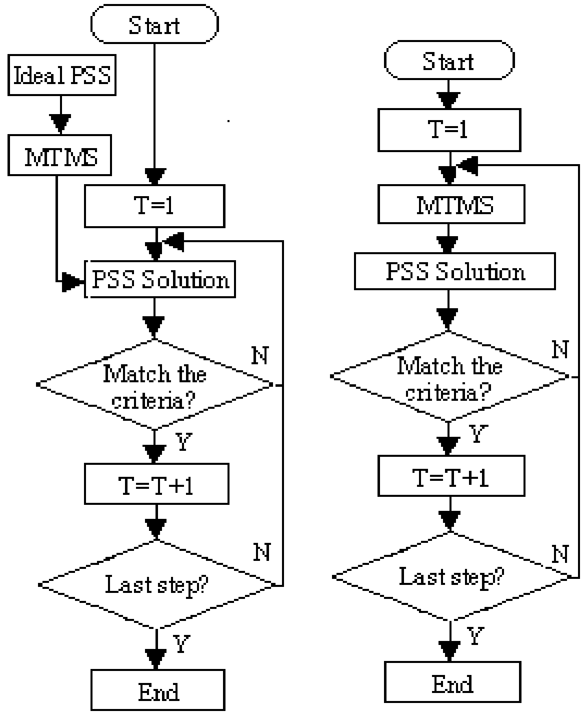
1. INTRODUCTION

In traction power supply system (TPSS), the system's fluctuations caused by the train movements will influence the traction effort and the power efficiency of the train. The TPSS simulation becomes a combination of the power network solution and the Multiple Train Movement Simulation (MTMS).

In the past, approximate calculations are done using independent iterative scheme between the electric network solution (DC system are normally used) and the Multiple Train Movement Simulation (MTMS). That is, the train movement performance simulation is done separately and the results are then utilised within the DC network calculations (see Fig 1(a)) to obtain the whole TPSS simulation [1,2,3]. These methods are called here an independent scheme. Such a scheme leads to unavoidable errors because of neglecting the influences of the varying contact wire voltages as a result of train movements.

In some published literature more precise representation combines the MTMS and DC network calculation [4,5,6,7]. A typical flow diagram can be seen in Fig 1(b), and it is called here a dependent MTMS scheme. In paper [4,7], Mellitt et al established energy-consumption figures to provide a general representation of a 2-road rapid transit railway. It includes modelling the running of the trains and establishing the consequential states of the DC traction power supply. In paper [6], Toffolo et al established an improved train network simulator model taking into account more propulsion conditions of trains and AC and DC combined network. In paper [5], Cai et al accomplished the TPSS solution with an effective algorithm and included the interactions between the multiple train movement features and the fluctuating system features.

However, train movement is a continuous process from the starting up at one station to the stopping at the next one. When the simulation is done on each time interval's states, the



(a) (b)

Fig 1: TPSS simulation process

flexibility of the train's continuous process is somewhat restricted. One notable example is that the flexible braking position of train movement is somehow restricted. The specific braking positions have to be predetermined [4,7]. In this sense, the flexibility of the train movement simulation is sacrificed to accommodate the whole TPSS system simulation process. With the time step procedure used, every train movement in the MTMS is expected to abide by the scheduled time or some prerequisite outline. This means that train status is modified/updated corresponding to each individual time step.

This restriction in train movement simulation is somehow due to the trade-off between the complexity of the computational process and the accuracy of the computational results. Therefore, the motivation of this project is to simulate the whole integrated system and to analyze the character of the system more effectively. The significance of this contribution is that the train movement simulation in each step is being accomplished as a complete process and the panorama views in analyzing the electric configurations and features of the system are established.

2. STRUCTURE OF DYNAMIC TPSS AND SCHEMATIC ARRANGEMENT

Figure 2 shows a sectional structure that is used in the dynamic system solution. This comprises of both the electric network as well as the moving trains. In [4,7,9] the feedback buses are omitted, the DC sources and the running tracks are connected directly to the grounding system. In [5], a multi-ladder network is adopted to represent the structure that involves the feedback busbar. This is an adequate representation of a real system that includes stray currents and track potential voltages.

To set up the dynamic TPSS structure with respect to the moving train status for the system solution, the schematic arrangement is split into four parts:

(1) Establishment of the elementary electric network framework

The elementary electric network framework structure is regarded as a fixed network that involves the substation configurations, the parameters of the contact lines (either the third rail or the overhead catenary), the tracks and their corresponding connections.

(2) Establishment of the dynamic electrical network structure which includes both the elementary power network and the moving trains statuses.

When electric trains are running along the track, the running trains statuses are added to modify the fixed network to establish a complete electric network profile.

(3) Dynamic electric network simulation

The nodal current iteration process is used throughout the main network solution. That is, Eq. (1) is the main primary iteration process, where $\boldsymbol{Y}$ represents the nodal conductance matrix, $\boldsymbol{I}$ represents the current vector injected into the system and $\boldsymbol{U}$ is the voltage vector to be solved.

$$\boldsymbol{YU}=\boldsymbol{I} \tag{1}$$

(4) System analyses based on the panoramic features drawn from the simulation.

The key step in the solution process is the incorporation of the moving train statuses into the overall structure. As stated in part one, the success is depending on how much the incorporation is achieved and it is clear that an effective way to involve the train's movement is of priority.

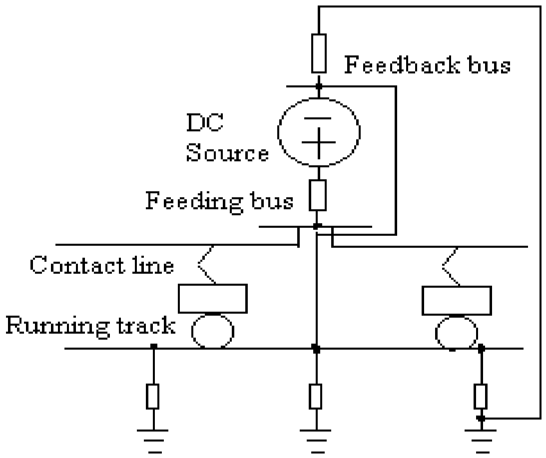


Fig 2: Sectional schematic structure of DC tractive power supply system

3. APPROACHES TO INVOLVE AN EFFECTIVE TRAIN MOVEMENT SIMULATION

The different kinds of train movement simulation can be classified into two catalogues: time scheduled scheme and free time scheduled scheme. The former is based on the assumption that the objective profile of the train movement (expected states of the train movement) is evaluated in advance according to a certain time schedule such as the average velocity. This means that the running time of a train is predetermined. The train movement simulation process is to modify the pre-determined ideal profile with the actual states of the power supply system. At each time interval it is updated to a new modified state in accordance with the track details and specified power condition which is also calculated at each time interval.

In a free time scheduled scheme, where the running time of the train is uncertain before the simulation, it is implemented in accordance with certain controlling strategy for the train movement. The running time is then updated after the completion of each whole process of the train movement on a station-to-station basis.

The simulation of free time scheduled single train movement is not unusual especially in Automatic Train Operation (ATO) [8,9,10,11]. But it is not the case in the multiple train movement simulation in conjunction with the TPSS solution. In general, methods used in TPSS simulation are of a mixed nature. For example in [12,13] a predetermined running time is given whilst in [4], the running time is not given in advance of the simulation, but a series of control points such as "first brake point", "second brake point", "start to coast point" and "brake point for station stop" are specified in advance. In these approaches a list of pre-specified conditions cannot be avoided. Their specific positions have to be predetermined.

The use of a free time scheduled scheme in the TPSS solution is different, although the result for each time interval simulation is still a new set of train state information that provides the condition of the train at each step, a continuous processing of the train movement is still needed. Factors that would influence each step are not confined to the past and present status of the train but also the continuous processing of the train movement, and this includes all the way from one 'start' station to the next 'stop' station. In this sense, each set of information of train movement at every time step is determined by the whole continuous train movement status. This is the fundamental feature for the real free time scheduled simulation, and the computational burden and probability of success in applying a whole consecutive process simulation is therefore much more complex than the time scheduled scheme where the process is to upgrade/modify each pre-determined set of states at every time interval.

The approach to involve an effective train movement simulation in the TPSS is to focus on a free-time scheduled scheme and is due to the following two advantages: firstly, new train controlling strategies could be obtained based on a single train movement. The experience obtained from these strategies could be applied to the whole traction power supply system when multiple train movement conditions are simulated. Secondly, under some controlling strategies variable mode of power supply plans could be better and more flexibly assessed.

4. STRATEGY AND METHODOLOGY

The methodology and strategy to implement the dependent MTMS scheme in the TPSS is to adopt the real free time scheduled train movement simulation to obtain the panoramic feature of the system. That is, the train movement statuses are composed of a series of individual train movement simulation units that are independent of each other. Each train is treated as an independent and active unit that dynamically simulates the free time scheduled scheme strategy. In this way, the multiple train movements are dependently and correspondingly embedded into the TPSS to achieve a complete integrated system solution.

In other words, the active train's simulation is done on a station-to-station basis and it is done within every primary iteration step of the TPSS process. By this means, the fluctuating system voltages from the TPSS network simulation are dynamically reflected onto the train movement simulation. However, as the primary iteration for TPSS is on a time step basis whilst the active unit simulation for individual train within the primary iteration is established from the train's starting up from one station and stopping at the next one instead of considering every individual train state at a single time step interval, the obtained fluctuating system voltages can only be applied on parts of the train's active unit until the simulation results synchronize with the electric network requirement. And the ideal states (nominal system voltage) have to be applied before the electric system iteration approaches or when the mentioned synchronization is reached.

When each individual train's active unit has completed simulation from one station to the next one, it is fully updated to match the actual circumstances of the TPSS system. Then a successive sector is started and the train's previous station-to-station information becomes inactive and is stored in a corresponding non-active unit. Under this method, a series of real free time scheduled dependent train moving statuses are individually integrated in the whole network simulation and makes it permissible to apply flexible train control strategy.

The individual train's controlling strategy adopted in this project is based on acceleration control for an energy-efficient drive consisting of three control phases: maximum acceleration, coast and maximum possible brake. The detailed solution is described in [16].

With the strategy of embedding the active moving train simulation, the dynamic simulation for the TPSS is therefore applicable. As the iteration method is applied to the dynamic electric network, the iteration process is thus to solve nodal current matrix equations to a certain tolerance limitation of the electric parameters as stated in Eq. (1).

Researches have done considerable work on iteration methods to solve the ordinary electric network problems. The mature methodologies include Gauss-Sediual method, Newton Raphson method and decomposition method. Other revised ones are also applicable and we can therefore focus solutions on the following aspects:

1. To update the fixed network structure to involve the moving trains' statues
2. To account for the moving train's electric characters properly
3. To select an adequate model to represent the DC source in the iteration.

These aspects reflect the updating process in reforming the conductance matrix and the current vector at every iteration. The updating of the fixed network structure depends on the dynamic moving train's position. The modification of the current vector comprises of two elements, one is the moving train's electric condition and the other is the substation power source.

As for the fixed network structure, to identify network configuration, a classification for different kinds of node types for the power network is used in the project. These node types include positive and negative source nodes, equivalent feeding contact line nodes and equivalent rail nodes (the concentrated representation of distributed parameters of the contact line and the rail track) and the electric connection nodes of rail. As some nodes could be a combined node type featuring more than one role in the system, say, a node of an electric rail cross connection can also be a equivalent rail segment node and so forth. To account for the combined feature of a node, a compound node types classification is further applied. These classifications of the node types are tracing clues to identify different kind of nodes and are thus useful in the analysis of their characteristics.

Although the train's movement itself combines the kinetic, dynamic and electric elements, its exhibition in the electric network completely depends on its electric characteristic, the characteristic of the power demanding status of the train and the train's current flow in the network. This electric characteristic exhibits the train as a non-linear load (the equivalent resistance/impedance is non-linear) regardless of the power demanding state. An ideal current source is applied in this model to represent the non-linear feature as follows:

$$I_{ij,t}^{k+1} = -P_{i,t}^{k+1} * (U_{i,t}^{k} - U_{j,t}^{k}) \qquad (2)$$

where P and U refer to the power demanding of the train and the voltage the system offers respectively; t is the time and k is the iteration count at time t; i, j are nodes of the moving train links to the contact line and the track.

When electric trains run along the track, it could behave as either an electric load or an electric source (if regeneration mode is involved) as in Eq. (2). Irrespective of whether the possible regeneration energy is fed back to the contact line or not, two specific node types are added to update the fixed network. These two kinds of dynamic nodes are, in conjunction with the contact line and with the running rail respectively, may overlap with the existing node type to distinguish the moving train status. In this way, the conductance matrix $\boldsymbol{Y}$ is updated to involve all electric nodes in the system to establish a complete electric network profile.

The non-zero elements in the current vector $\boldsymbol{I}$ in Equation (1) reflect not only the features of the moving trains but also of the DC source in the substations. For the DC source in the substation, the current source is determined by

$$I_i = - Y_i * U_{sys} \qquad (3)$$

where $\boldsymbol{Y_i}$ and $\boldsymbol{U_{sys}}$ refer to internal resistance of the substation and system voltage respectively.

The above aspects are successfully applied to the advanced Gaussian Variable Decomposition Elimination algorithm, which is an equivalent method by using LU triangular factorization algorithm [14,15]. It is demonstrated that the number of iteration is less than 5 when the initial value of $\boldsymbol{U}$ at each node is set as either the rated system voltage or the ground voltage depending on the node type, the maximum error matches the expected tolerance of 0.5% or 2 volts in a 1500 volts nominal system voltage.

5. SIMULATION RESULTS

Simulation results are done on several Chinese Mass Rapid Transit Systems. The results shown in this paper are based on one of the systems tested -- the Shanghai Line One MRTS (excluding the extension line). It is 13450m long with 11 passenger stations and 6 substations. Due to page limitation of the paper, only selected results are presented.

The simulation starts with the first train leaving the initial passenger station and stopping at the terminal station and successive trains leave one after another along the track ensuring that the given headway and passenger station stopping intervals are satisfied. The headway is the time interval between the starting up time of two adjacent trains. The passenger station stopping intervals can be varied depending on the passenger flows at a station. The maximum number of trains running along the track at a time largely depends on the headway and the stopping time assigned for each passenger station. A high density of trains along the track heavily influences the TPSS features.

Example profiles of a single train movement on a

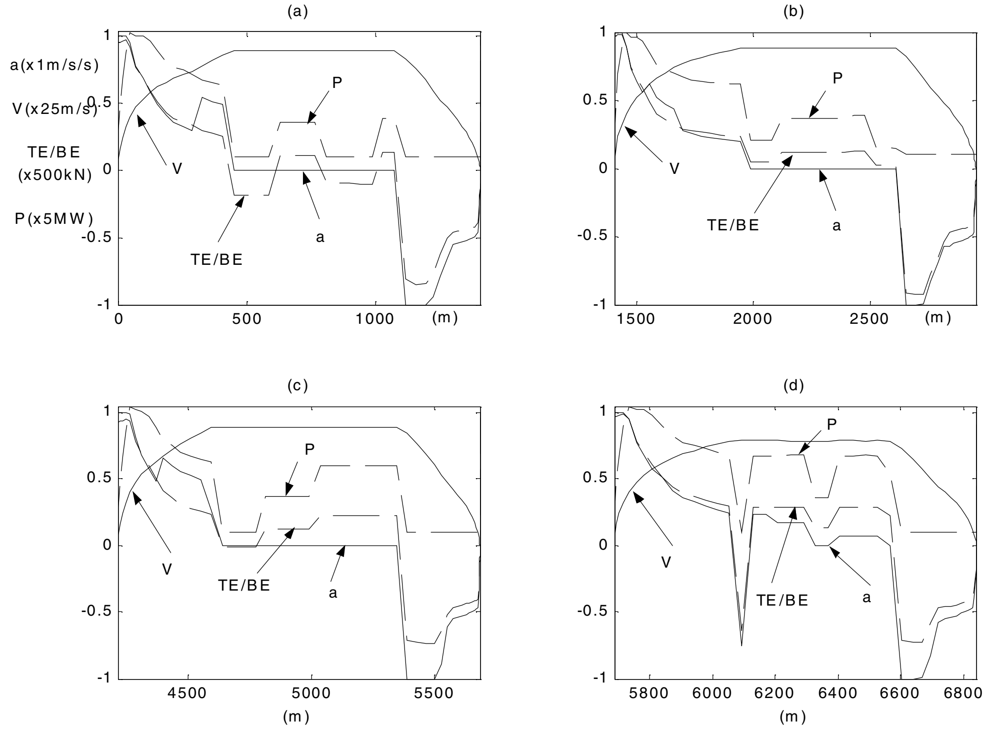


Fig 3: Single train movement profile on a station-to-station basis

station-to-station section are shown in Fig 3. Diagrams (a) to (d) represent four sectional graphs. Each sectional profile is illustrated with 4 characters: the two solid line curves represent acceleration and velocity while the two dotted line curves represent Traction effort/Braking effort (TE/BE) and power consumption (versus time) of the train.

The whole journey of the train movement is shown in Fig 4. They are overall views of an individual train's features on the whole journey of running from the first station to the last one, including train velocity, TE/BE, power consumption and train voltage (from Fig 4(a) to (d)). The diagrams show the 1st train's features. Other physical and electrical features and successive trains profiles are similar.

Figure 5 is the profile of positions of multiple trains along the track corresponding to time. Figure 5(a) shows all the running trains positions on both up and down the rail tracks, while Fig 5(b) illustrate trains in one direction (the up rail track is shown in the diagram). In Fig 5(a), the curves starting from the bottom axis represent trains running on the up rail track while the curves starting from the top axis represent those on the down rail track. In Fig 5(b), the first curve on the left represents the whole journey of the first train and the first vertical straight line represents the time and position it finishes its journey. The second left curve and the second left straight line represent those of the second train, and so forth. From the commencement of the simulation, the total number of trains running on the track increases consistently until the first train reaches its terminal station. After that, it will go to daily routine operation condition and the total number of trains running on the track will then remain almost stable.

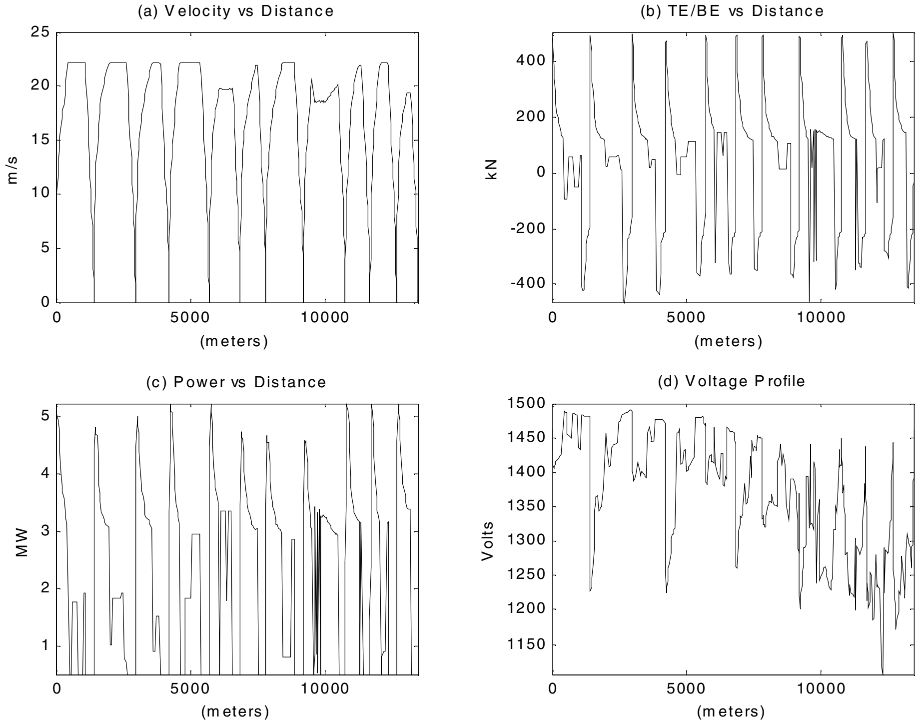


Fig 4: Single train's movement profile for a whole journey

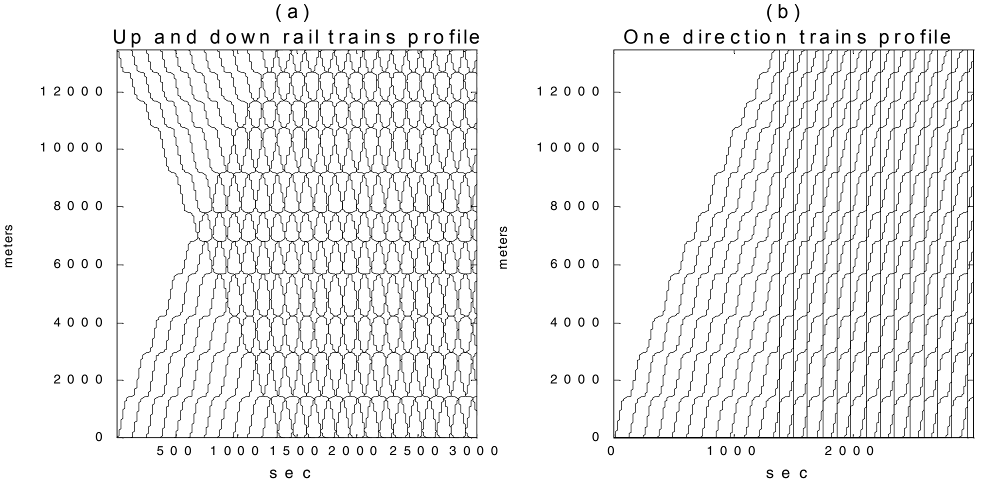


Fig 5: Multiple trains' positions profile verses time

Moreover, the free time scheduled scheme applied makes it possible way to see the influence of the electric system on the individual train's relative position status. It is possible to estimate the position differentials between two individual trains due to the influence of the fluctuating system voltages at their same relative running times. This differential is the relative position difference between trains instead of the absolute ones. Trains' absolute distance differences are kept large at any time and are highly depending on the headways. When subtracting the corresponding headway(s) from the absolute running time of the trains, we can compare the relative position statuses of the individual train. Under constant (consistent) rated voltage conditions, the relative position statuses of successive trains are identical at the same relative running time. This is the case shown in the train operational timetable. However, under fluctuating system voltage, it is impossible to keep the identical relative positions among consecutive running trains and relative position differentials occur. Figure 6 shows examples of the differentials. The dotted line curve is the relative position differential between two individual trains (the 4th train vs. the 1st train) and the solid line is that of one train (the 4th train) vs. a train running under the rated system voltage condition. The

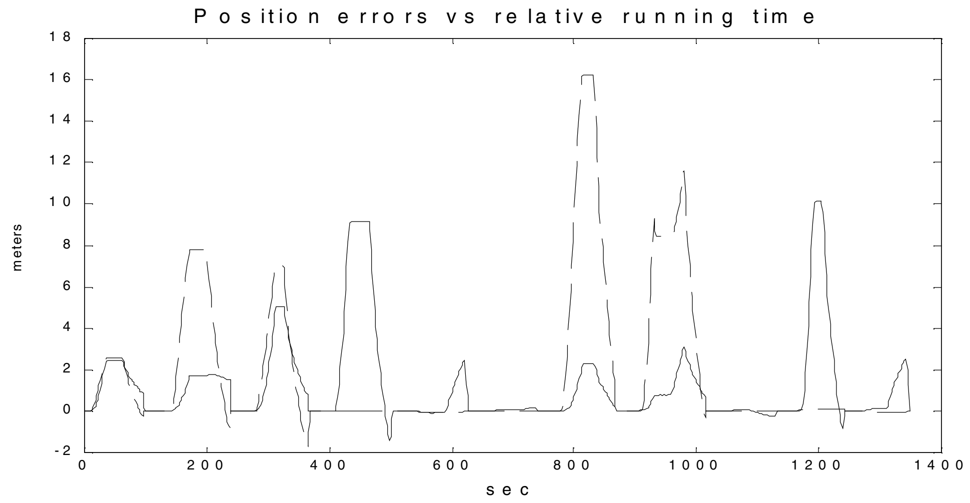


Fig 6: Train's position errors at relative running time

maximum error (differential) could be as tens of meters at the same relative running time among consecutive trains. By this means, we can provide scope of errors for consecutive running trains when compared to the scheduled timetable positions and have an idea on whether or not safety precaution need to be taken.

The above simulation results focus on the individual and consecutive trains and their features. The following results are of the TPSS electric profiles regarding to different aspects of characteristics. They give a panoramic profile of the TPSS system, which includes the dynamic electric statues of the contact lines, the rail tracks and the substations.

Figure 7 shows the 3-D features of line voltages and rail potentials corresponding to time and distance. This is the condition of both the contact line voltage (upper graph) and rail line potential (lower graph) profiles along one direction rail track on the double track system (up rail direction is shown in this diagram). And it refers to a 2-minute headway, a very high running density. Conditions for the diagram along the down rail direction and for those under lower densities, say from 3 to 15-minute headways, can also be obtained.

In Fig 7, the dark areas at the front parts of the graphs are the states of the commencement of running trains ([0,0] axis reference) and the states of locations where the trains are approaching the terminal passenger station. This is equivalent to the initial part on the left of Fig 5(b). The sparse grey area at the back end parts in the diagram corresponds to the end of the running trains. The graphs gradually turn to complete sparse grey area along the whole distance when the first train is approaching the terminal station. From these, we can see the effects over the system voltages (including over the rail potentials).

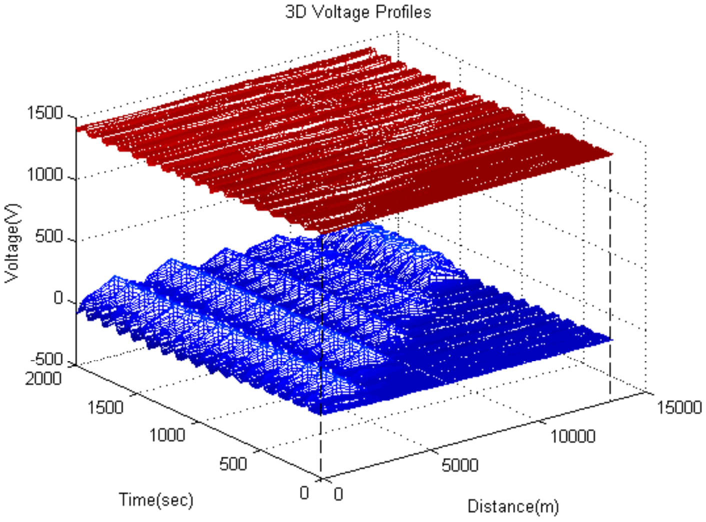
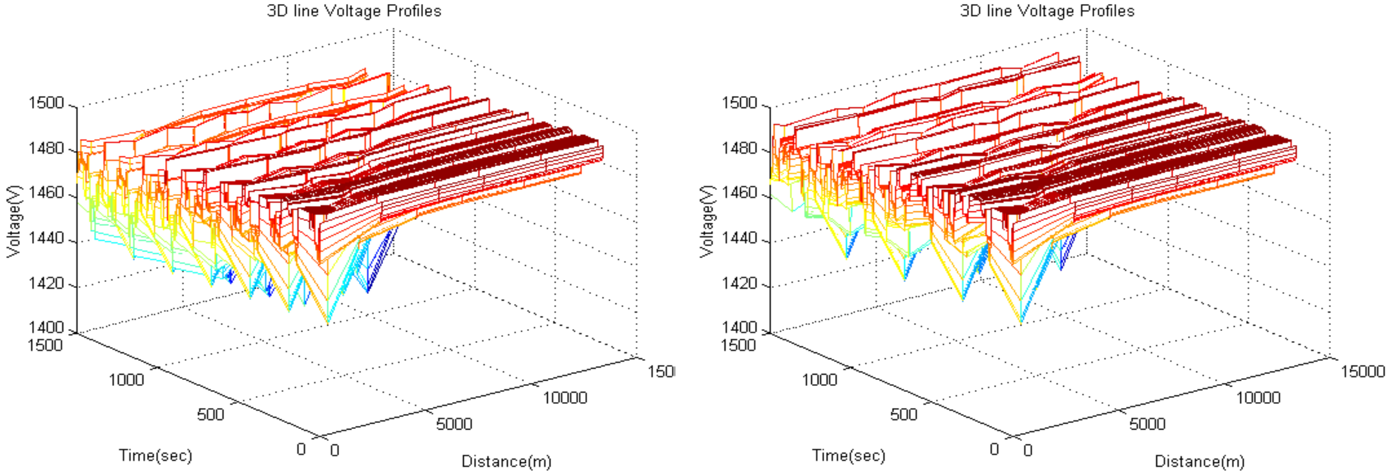


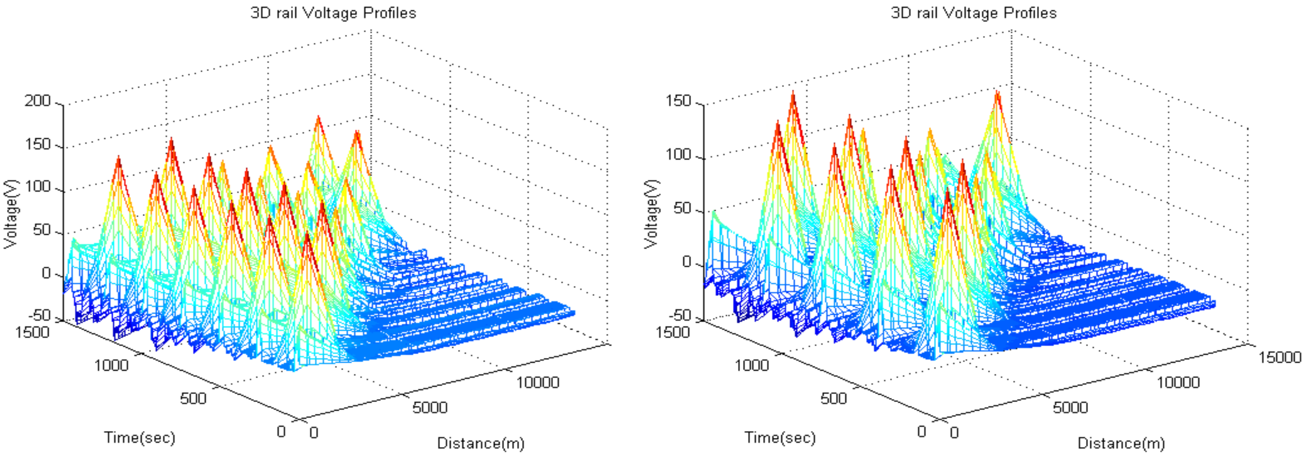
Fig 7: Contact line voltage and rail potential profiles

In routine service condition, the high density of trains results in lower voltage profile of the contact line and the more fluctuating rail potential of the track. We can see the increasingly train density effects in Fig 8 and 9. They are example profiles



(a) On a 4-minute headway (b) On a 6-minute headway

Fig 8: Contact line voltage under different headways showing the effects of two different headways: Fig 8(a) and 9(a) are for a 4-minute headway; Fig 8(b) and 9(b) for a 6-minute headway. It can be seen that the 4-minute headway results in bigger dip in contact line voltage profile and more fluctuating rail potential over the 6-minute headway. From the diagram, it also shows that high potential happens not only in high density but also near the positions where the substations are. This is shown in the contour diagram in Fig 10 too. The blank area refers to potential values in the vicinity of zero (less than 10% of the maximum value range), the high potential happens in the dark areas. These results are obtained on an assumption of evenly grounding resistance. Therefore, to decrease the high fluctuating potentials near substations, parameter settings and corresponding configurations of the nearby sections of substations should be taken into consideration when proposing equivalent grounding parameters.



(a) On a 4-minute headway (b) On a 6-minute headway

Fig 9: Rail potential along the track corresponding to Fig 8

Figure 11 shows an example of the substation voltages under different headways. Figure 11(a) represents the profiles of two substations (the 3rd and the 4th substations) under 2 and 4 minutes headways and Fig 11(b) represents those of 4 and 6 minutes headways. The solid curves correspond to the higher density run while the dotted lines correspond to the lower density run. We can see that the substation voltages go down from 1450 volts to 1430 volts and 1350 volts when the headway decreases from 6 minutes to 4 minutes and 2 minutes respectively in routine daily service condition. This illustrates that the voltage will reach its limiting value when the density of trains increases. Likewise, the currents of the substations increase significantly with decreased headways.

The above are selections of simulation results to show the dynamic characters of the TPSS under fixed parameters and changing headways. Parameters that could influence system performances and the correct choice of their values to optimize overall system configuration and system operation must be carefully analyzed. The following parameters are also important items:

- capacities of the substations
- number and positions of the substations
- electric parameters of the contact line
- electric parameters of the rail track
- electric connections of the rail track
- grounding parameters of the system etc.

These factors and their influences on the TPSS can be obtained using the same method of analysis. For instance, influences of the substation capacities and their positions using the same headway can be seen in Fig 12 which shows the relations of the different substation currents. Figure 12(a) refers to the two end substations and Fig 12(b) refers to two of the mid-position substations. We can see the different peak magnitudes between the currents in the end position and mid-position substations. For the former substation the maximum current is about 3000A but for the latter substation it is about 3500A. For this reason, the end position substation capacity can be chosen smaller than the mid position ones and still meets the requirement. Figure 13 is a demonstration of this point, it is the average current difference of substations under two different configurations: all substations have equally the same capacity and the two end substation capacities are 25% less. The overall influence of the decreased end position substation capacities leads to having lower average currents and acceptable current increase in mid position substations. In this sense, the

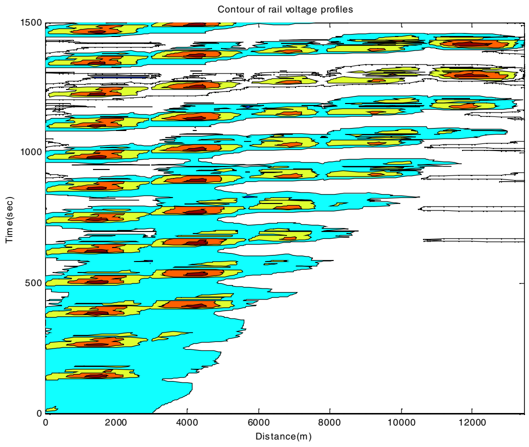


Fig 10: Contour of rail potential profile

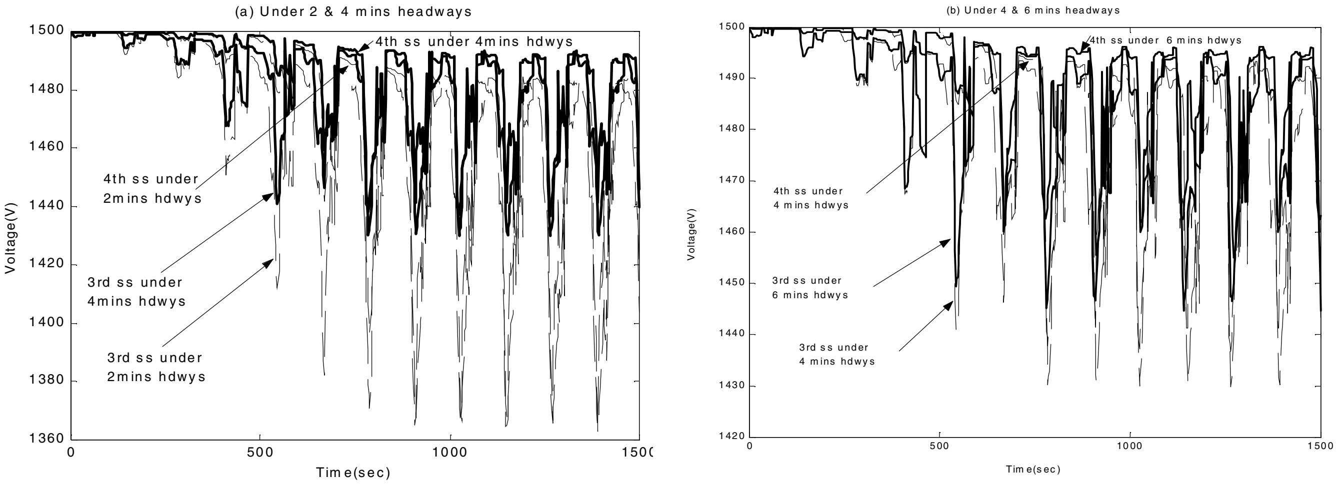


Fig 11: Substations voltage profiles of different headways

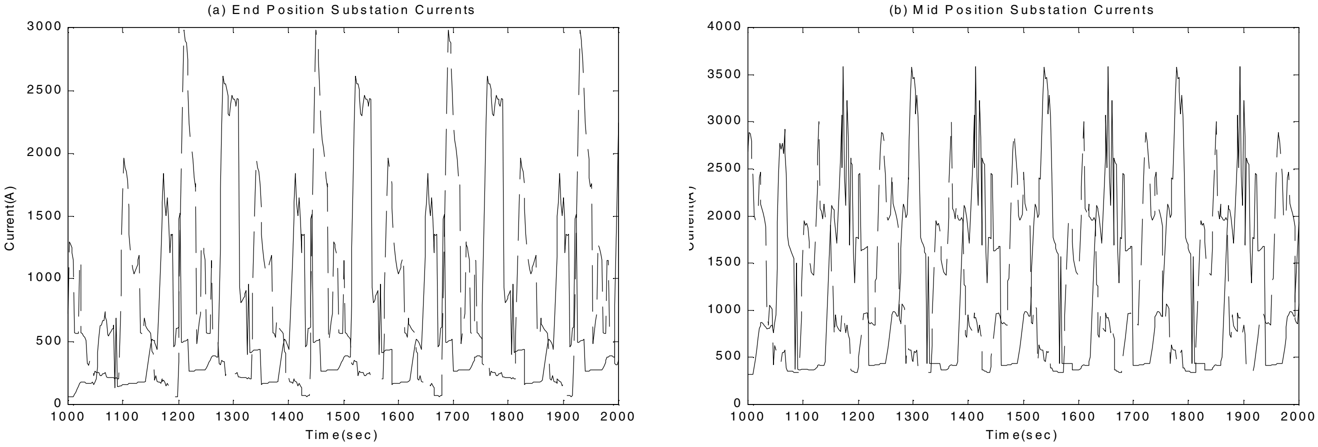


Fig 12: End and mid position substations current profiles

configuration of choosing a smaller end position substation capacity is thus verified.

Therefore, the synthetic influences of the individual factor can further be realized based on the simulation results and the optimisation of the TPSS can thus be implemented.

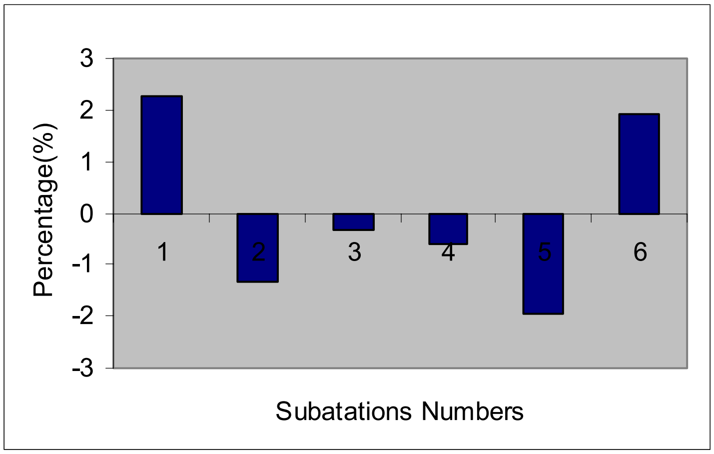


Fig 13: Percentage difference of average currents between two conditions: equal capacity for all substations and smaller end position substation capacity

6. CONCLUSION

Simulation and analysis of dynamic TPSS of dependent multiple train movements is achieved. The free time scheduled strategy for train movement is applied which is more flexible and effective to reflect operation strategies. The significance of this motivation is that the individual active train movement simulation in each TPSS iteration step is being accomplished by considering a complete process of the train's starting up from one station and stopping at the next one instead of considering every individual train state at a single time step interval. In this way the simulation of flexible train movement is achieved. It has the advantages in assessing both the train moving strategies and the configuration of the TPSS. This is an important issue in the simulation of a modern mass transit power supply system that involves a diversity of independent train operation strategies and their influences on the whole network.

The simulation results are based on the Shanghai Metro Line One (excluding the extension section). Simulation and analysis on more lines including Shanghai Line Two and

Guangzhou Line One MTRS have also been implemented using similar approaches. Future works will include a proposal for the solutions of the dynamic system by using modern optimization methods.

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