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Improved coastal boundary condition for surface water waves

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Abstract

Surface water waves in coastal waters are commonly modeled using the mild slope equation. One of the parameters in the coastal boundary condition for this equation is the direction at which waves approach a coast. Three published methods of estimating this direction are examined, and it is demonstrated that the wave fields obtained using these estimates deviate significantly from the corresponding analytic solution. A new method of estimating the direction of approaching waves is presented and it is shown that this method correctly reproduces the analytic solution. The ability of these methods to simulate waves in a rectangular harbor is examined. © 2000 Elsevier Science Ltd. All rights reserved.

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1. Introduction

Mathematical models of surface water waves are often used in coastal engineering to predict the complex interaction of wave refraction, diffraction, and reflection that occurs near coasts. The mild-slope equation developed by Berkhoff (1976) has become the standard approach to modeling these waves. A coastal boundary condition for this elliptic partial differential equation was presented by Berkhoff (1976) that contains two parameters, a reflection coefficient and a phase shift. This boundary

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condition was modified by Isaacson and Qu (1990) to also incorporate the direction of approaching waves. Accurate estimation of these three parameters is necessary to properly predict wave fields in coastal waters.

The reflection coefficient represents a ratio of the amplitude of waves that approach a coast to the amplitude of waves reflected away from a coast. The reflection coefficient has a large effect on the wave field at locations near a coast (Thompson et al., 1996) and it has a significant effect on the amplitude of waves in a harbor at resonant frequencies (Chen, 1986; Kostense et al., 1986). For these reasons, a large amount of research has been performed on estimating reflection coefficients. Thompson et al. (1996) presented a summary of published values of the reflection coefficient for various types of coastal boundaries. Numerous expressions have been presented that allow the reflection coefficient to be estimated using parameters such as the slope and roughness of a beach, the period of waves, and the height of waves (Shore Protection Manual, 1977; Dickson et al., 1995; Dingemans, 1997). Methods have also been presented to estimate the reflection coefficient using information obtained from wave probes (Isaacson, 1991; Cotter and Chakrabarti, 1992; Isaacson et al., 1996).

The second parameter in the coastal boundary condition is the phase shift that occurs between approaching and reflected waves at the coastal boundary. The phase shift is typically assumed to be zero (e.g., Pos, 1985; Isaacson, 1991). The phase shift was estimated by Dickson et al. (1995) as that produced by a wave traveling between the coastal boundary in the model and the physical boundary of the sea in the direction normal to the coast and in water of constant depth. Two methods of estimating the phase shift were presented by Sutherland and O'Donoghue (1998) for waves that intersect a coast obliquely.

The third parameter in the coastal boundary condition, the direction that waves approach a coast, is the subject of this paper. Three published methods exist for estimating this direction; these will be identified as methods A, B, and C. The most common method (method A) uses the assumption that waves approach a coast in the direction normal to the coast (e.g., Berkhoff, 1976; Chen, 1986; Tsay et al., 1989; Pos et al., 1989; Xu and Panchang, 1993; Thompson et al., 1996; Xu et al., 1996). Since this assumption is not satisfied in general, Isaacson and Qu (1990) assumed that waves approach a coast in the direction of the gradient of the phase (method B). As Isaacson and Qu (1990) point out, this definition is only meaningful 'for portions of a wave field which have readily identifiable directions'. Another method of estimating the direction of approaching wave was presented by Isaacson et al. (1993) using information obtained from the component of the wave field in the direction tangent to the coast (method C).

In this paper, estimates of the direction of approaching waves are obtained using each of these published methods and the resulting wave fields are examined. It is demonstrated that the predicted wave fields deviate significantly from the corresponding analytic solution. A new method of estimating the direction of approaching waves (denoted method D) is presented and the applicability of this method is examined.

2. Model of surface water waves

It is assumed that surface water waves satisfy the mild-slope equation (Berkhoff, 1976),

$$\nabla\cdot(CC_g\nabla\eta)+\omega^2\frac{C_g}{C}\eta=0 \quad (1)$$

where C is the celerity, C_g is the group velocity, ω is the wave frequency, and η is a complex surface elevation function. The magnitude $|\eta|$ is the amplitude of the wave (1/2 of the wave height) and the argument $arg(\eta)$ is the phase of the wave. The velocity potential for surface water waves, Φ, is related to η via

$$\Phi(x_1, x_2, x_3, t)=\mathrm{Re}\left[\left(-\mathrm{i}\frac{g}{\omega}\right)\eta(x_1,x_2)e^{-\mathrm{i}\omega t}\right]\frac{\cosh[k(x_3+h)]}{\cosh(kh)} \quad (2)$$

where g is the acceleration due to gravity, h is the water depth, and the wave number, k, is obtained from

$$\omega^2=kg\tanh(kh) \quad (3)$$

The model domain is bounded by a coastal boundary and an open boundary, shown in Fig. 1 as a semi-circle that separates the model domain from the external sea. The boundary condition along the coast is obtained using the assumption that the

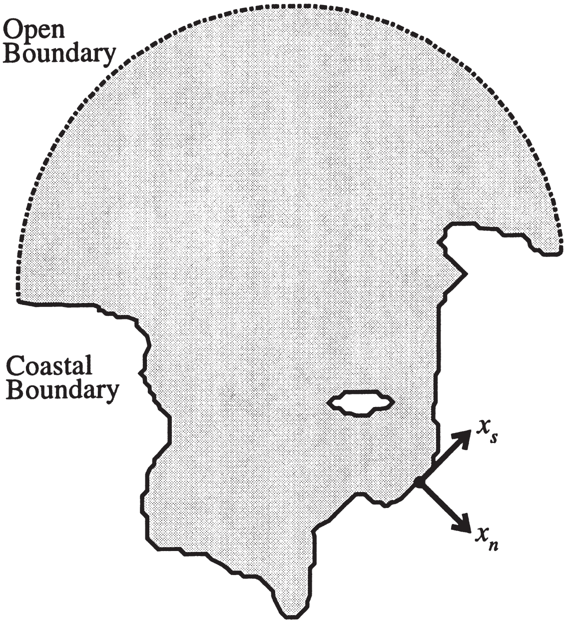

Fig. 1. Model of harbor, Toothacher Bay, Maine, USA.

wave field in a small neighborhood along the coast may be decomposed into one set of plane waves that approach the coast and one set that is reflected away from the coast (Berkhoff, 1976; Isaacson and Qu, 1990). The function η for these waves may be specified in terms of a local x_n-x_s coordinate system with x_n normal to and x_s tangential to this boundary as

$$\eta=Ae^{ik(x_n\cos\gamma+x_s\sin\gamma)}+ARe^{ik(-x_n\cos\gamma+x_s\sin\gamma+\beta)} \tag{4}$$

where A is the amplitude of the approaching waves, R is the reflection coefficient, β is the phase shift, and γ is the angle at which the approaching waves intersect the coast (γ=0° for normally approaching waves). The partial derivative in the x_n-direction is evaluated to obtain

$$\frac{\partial\eta}{\partial x_n}={}^{\mathrm{i}}k\cos\gamma\,(Ae^{ikx_s\sin\gamma})\left[e^{ik(x_n\cos gamma)}-Re^{ik(-x_n\cos\gamma+\beta)}\right] \tag{5}$$

The right-hand side of this expression is multiplied by η and divided by the expression for η in Eq. (4), terms are canceled, and the resulting expression is evaluated along the coast to obtain the boundary condition

$$\frac{\partial\eta}{\partial x_n}={}^{\mathrm{i}}k\cos\gamma\,\frac{1-Re^{ik\beta}}{1+Re^{ik\beta}}\,\eta \qquad (x_n=0) \tag{6}$$

The boundary condition at the open sea is specified in terms of an incident wave field, a reflected wave field that would exist in the absence of a harbor, and a scattered wave field generated by the harbor (Mei, 1989). Implementation of a parabolic boundary condition along this interface is described in detail by Xu and Panchang (1993) for finite difference models and by Xu et al. (1996) for finite element models; this derivation is not reproduced here.

A solution to the mild-slope equation with the specified boundary conditions is obtained using the finite element method. Zienkiewicz et al. (1978) presented the functional

$$\Pi=\iint_{\Omega}\frac{1}{2}\left[CC_g(\nabla\eta)^2-\omega^2\frac{C_g}{C}\eta^2\right]dA-\int_C\eta\frac{\partial\eta}{\partial x_n}\,ds-\int_\Gamma\eta\frac{\partial\eta}{\partial x_n}\,ds \tag{7}$$

where the integration in these three terms is performed over the model domain, along the coastal boundary, and along the open boundary. The boundary conditions along the coast, Eq. (6), and along the open sea (Xu et al., 1996) are substituted into this expression and the resulting functional is minimized by setting the first variation equal to zero. A solution is obtained using the Galerkin method with triangular elements and linear shape functions. The Surface water Modeling System (SMS) described by Zundell et al. (1998) is adopted for mesh generation and graphics. The discretized system of linear equations is solved via the conjugate gradient method.

3. Published methods of estimating the direction of approaching waves

The three published methods of estimating the direction of approaching waves are evaluated using the model domain shown in Fig. 2. This model simulates waves reflecting off a straight coast in water of constant depth. The analytic solution for this wave field is the superposition of incident plane waves traveling in the θ_I-direction (where θ_I is zero for incident waves normal to the coast) and reflected plane waves traveling in the (180°-θ_I)-direction.

The incident and reflected waves that correspond to the analytic solution were specified along the open boundary. The resulting model reproduces the analytic solution when γ in the coastal boundary condition, Eq. (6), is set equal to θ_I. In general, however, the direction γ is not known along a coast; it must be estimated using one of the methods presented in this paper. Each method is used to estimate the direction of approaching waves for θ_I between 0° (incident waves normal to the coast) and 90° (incident waves parallel to the coast). The estimated direction is displayed along the coast and the resulting wave field is displayed.

The dimensions of the finite element model used in this investigation result in a grid with a resolution of 20 nodes per wavelength. This corresponds to a nodal spacing of 6.25 m for waves with a period of 10 s traveling in water of 20 m depth. Consistency of the model results was verified using a finite element model with a different grid resolution.

3.1. Method A: Direction of approaching waves assumed normal to the coast

The coastal boundary condition presented by Berkhoff (1976) assumed that waves approach the coast in the normal direction (i.e., γ=0°); this assumption is commonly used as noted earlier. A visual comparison of the discrepancies between the wave field obtained using this assumption and the analytic solution is clearest when the coast is fully absorbing (R=0). The analytic solution for this case consists of plane waves traveling in the incident direction, and contours of equal phase form straight lines.

The phase that is predicted using the assumption of normally incident waves is

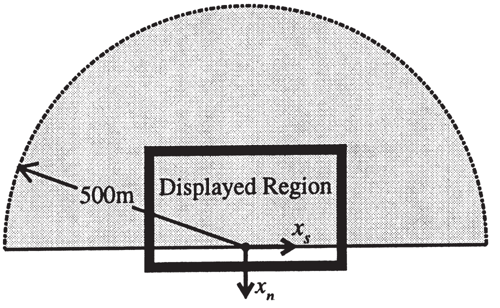

Fig. 2. Model domain used to evaluate methods of estimating the direction of approaching waves along a coast.

shown in Fig. 3; the arrows along the coastal boundary indicate that γ=0° in Eq. (6). The phase varies between −1 (the black regions) and +1 (the white regions). The contours of equal phase in this figure form straight lines when the incident direction is equal to 0°. As the incident direction increases these contours deviate from the analytic solution.

This deviation can be understood by examining the coastal boundary condition, Eq. (6). An expression for the partial derivative of the phase in the normal direction may be obtained by separating the real and imaginary parts of Eq. (6) using

$$\frac{1}{\eta}\frac{\partial\eta}{\partial x}=\frac{1}{|\eta|}\frac{\partial|\eta|}{\partial x}+\mathrm{i}\frac{\partial\arg(\eta)}{\partial x} \tag{8}$$

When the phase shift is equal to zero, the imaginary part of the resulting expression is

$$\frac{\partial\arg(\eta)}{\partial x_n}=k\cos\gamma\frac{1-R}{1+R} \qquad (x_n=0) \tag{9}$$

This expression is satisfied exactly by the analytic solution. As the incident direction varies from 0° to 90° the value of cosγ in the analytic solution varies from 1 to 0. By assuming that γ=0° in the simulations, the term cosγ is equal to 1 and the derivative of the phase in the normal direction is larger in Fig. 3b–d than in the analytic solution.

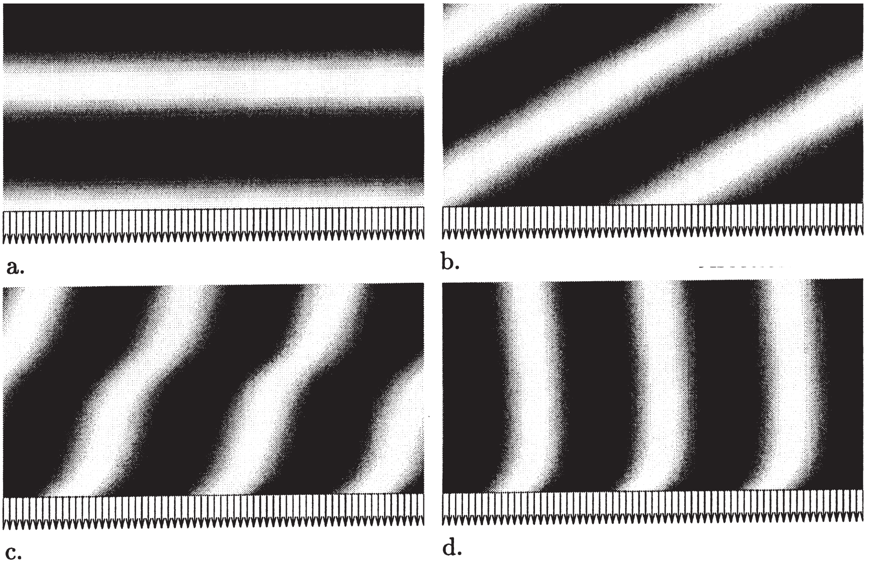

Fig. 3. Method A, estimated direction of approaching waves and phase of waves, R=0.0 (a) 0° Incident Direction; (b) 30° Incident Direction; (c) 60° Incident Direction; (d) 90° Incident Direction.

3.2. Method B: Direction of approaching waves obtained from the gradient of the phase

Isaacson and Qu (1990) obtained estimates for the direction that waves approach a coast assuming that waves travel in the direction of the gradient of the phase for the entire wave field. Thus, the approaching wave direction is obtained using

$$\tan\gamma = \frac{\dfrac{\partial \arg(\eta)}{\partial x_s}}{\dfrac{\partial \arg(\eta)}{\partial x_n}} \tag{10}$$

As Isaacson and Qu (1990) point out this definition is meaningful only for a single set of plane waves. It may be noted that the wave direction obtained using Eq. (10) satisfies the assumption used to obtain the coastal boundary condition, Eq. (4), only when R=0; this direction is not particularly useful when R is non-zero.

A non-linear boundary condition occurs when γ from the last equation is substituted into Eq. (6). The standard approach to determining the solution to such a problem is to linearize the equations and then iterate (Press et al., 1992). Isaacson and Qu (1990) did this by first solving for η assuming that γ=0° along the coast. Iteration was performed by substituting this value for η into Eq. (10) to obtain a new estimate for γ along the coast, and then using this new γ in the coastal boundary condition, Eq. (6), to obtain a new estimate for η throughout the model domain. Iteration continues until convergence occurs. Convergence is determined in this paper by computing the tolerance

$$\varepsilon = \frac{\dfrac{1}{N}\sum_{i=1}^{N}\left| \underset{\text{new}}{A_i} - \underset{\text{old}}{A_i} \right|}{\dfrac{1}{N}\sum_{i=1}^{N}\underset{\text{old}}{A_i}} \tag{11}$$

where $\underset{\text{old}}{A_i}$ and $\underset{\text{new}}{A_i}$ are the old and new values of the wave amplitude at nodes in the finite element mesh and summation occurs over all nodes. Convergence is assumed to occur when $\varepsilon<0.001$.

The results indicate that this iterative process always converges; two to six iterations were required to obtain convergence with more iterations required as θ_I increases. The phase that is obtained using this method is shown in Fig. 4 for a fully absorbing coast. This figure illustrates that the analytic solution is accurately reproduced when R=0 regardless of the incident wave direction; contours of equal phase form straight lines and the arrows along the coast (obtained at convergence) indicate that the estimated direction of approaching waves points in the incident wave direction. Computer simulations (not shown) indicate that this method does not reproduce the analytic solution when the reflection coefficient is non-zero, a fact later acknowledged by Isaacson et al. (1993).

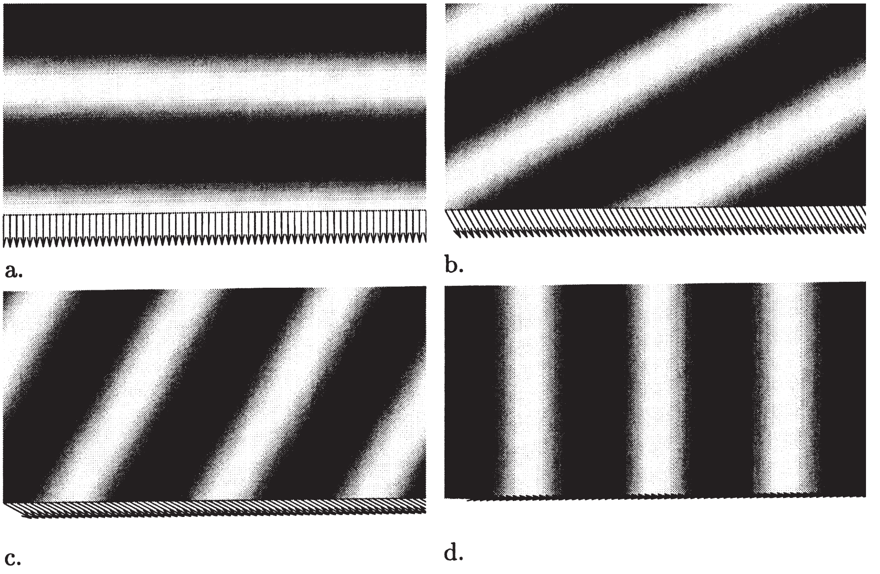

Fig. 4. Method B, estimated direction of approaching waves and phase of waves, R=0.0. (a) 0° Incident Direction; (b) 30° Incident Direction; (c) 60° Incident Direction; (d) 90° Incident Direction.

3.3. Method C: Direction of approaching waves obtained from the tangential component of the gradient of the phase

Isaacson et al. (1993) presented a method of estimating the direction of approaching waves that accounts for non-zero reflection coefficients. This method is based on the assumption used by Berkhoff (1976) to obtain the coastal boundary condition; that the wave field may be decomposed into a single set of approaching and reflected plane waves in a small neighborhood along a coast. Isaacson et al. (1993) took the tangential derivative of η for these waves, Eq. (4), to obtain

$$\frac{\partial\eta}{\partial x_s}=(ik\sin\gamma)\eta \quad (12)$$

The direction γ may be expressed in terms of the partial derivative of the phase in the tangential direction using Eq. (8); this gives

$$\sin\gamma=\frac{1}{k}\frac{\partial\arg(\eta)}{\partial x_s} \quad (13)$$

Isaacson et al. (1993) obtained estimates for the approaching wave direction by approximating γ using Eq. (12) and using this approximation in the same iterative technique as in method B.

This iterative method was found to 'converge rapidly' by Isaacson et al. (1993). Later, Isaacson (1995) stated that this method may not always converge but that successive iterations do not significantly affect the wave field; hence, 'generally only one or two iterations are adopted'. The approaching wave direction and the phases that were obtained using this method are shown in Fig. 5 for waves along a fully absorbing coast. These results indicate that this iterative method converges and that waves are accurately predicted when $\theta_I \leq \sim 40°$ (the convergence criteria of $\varepsilon < 0.001$ was satisfied in two to six iterations). It was also found that this method does not converge when $\theta_I > 40°$ (the results in Fig. 5c and d were obtained when iteration was terminated after eight iterations). Similar results were obtained for coasts with a non-zero reflection coefficient.

An examination of this iterative procedure showed that, in general, successive iterates shifted γ in the correct direction. (Note that the first iterate is obtained using Eq. (6) with $\cos\gamma$ set equal to 1, a value that is too large for obliquely incident waves.) When $\theta_I \leq 40°$ successive iterates lie between the preceding iterate and the value of θ_I; iteration leads to new estimates for γ that approach the correct solution. When $\theta_I > 40°$ successive iterates overshoot the value of θ_I; iteration leads to estimates for γ at locations along the coast in Fig. 5c and d that oscillate between $\gamma = 0°$ to 90° for successive iterations.

4. Method D: New method of estimating the direction of approaching waves

The three published methods of estimating the direction of approaching waves each have limitations; methods A and C are applicable only when waves approach a coast at small values of γ, and method B is applicable only for fully absorbing

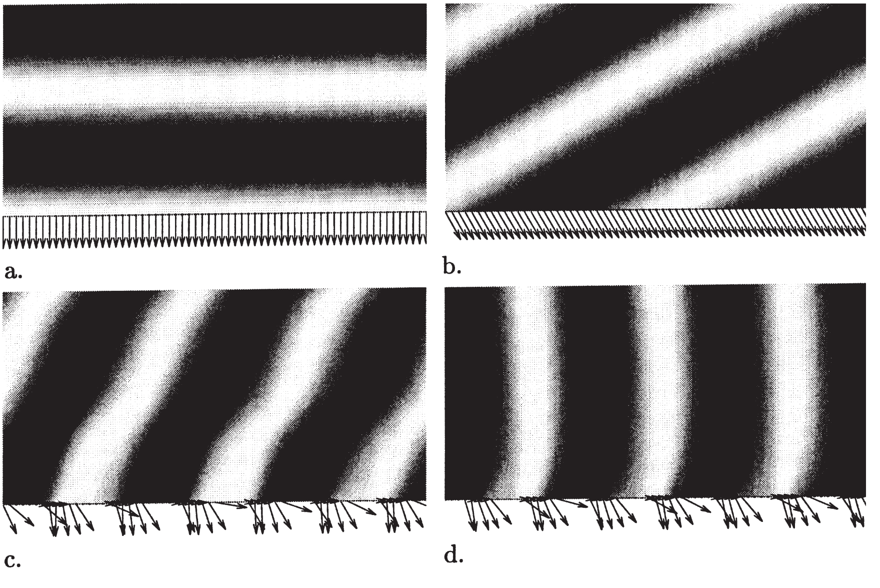

Fig. 5. Method C, estimated direction of approaching waves and phase of waves, R=0.0. (a) 0° Incident Direction; (b) 30° Incident Direction; (c) 60° Incident Direction; (d) 90° Incident Direction.

coasts. A new method (D) of estimating the direction of approaching waves is proposed that reduces to method B for fully absorbing coasts and may be viewed as an extension of it.

This method is based on estimating $\tan\gamma$ under the assumption that η can be decomposed into a single set of approaching and reflected plane waves in a small neighborhood along a coast; in contrast, method B was based on approaching waves only. An expression for $\tan\gamma$ may be obtained by evaluating the cross product of a vector pointing in the direction of approaching waves and the gradient of η; this gives

$$\cos\gamma\frac{\partial\eta}{\partial x_s}-\sin\gamma\frac{\partial\eta}{\partial x_n}=\cos\gamma\,[ik\sin\gamma\,\eta]-\sin\gamma\left[ik\cos\gamma\frac{1-Re^{ik\beta}}{1+Re^{ik\beta}}\eta\right]\ (x_n=0) \tag{14}$$

using Eq. (12) for $\partial\eta/\partial x_s$ and Eq. (6) for $\partial\eta/\partial x_n$ at the coast. Note that when R=0, the right-hand side of this expression is equal to zero, and waves travel in the direction of the gradient of η. This expression is factored into real and imaginary parts using Eq. (8); the imaginary part of the resulting expression is

$$\cos\gamma\frac{1}{k}\frac{\partial\arg(\eta)}{\partial x_s}-\sin\gamma\frac{1}{k}\frac{\partial\arg(\eta)}{\partial x_n}=\sin\gamma\cos\gamma\frac{2R[\cos(k\beta)+R]}{1+2R\cos(k\beta)+R^2}\ (x_n=0) \tag{15}$$

The first term in this expression is moved to the right-hand side and the third term is moved to the left-hand side,

$$\sin\gamma\left\{\frac{1}{k}\frac{\partial\arg(\eta)}{\partial x_n}+\frac{2R[\cos(k\beta)+R]}{1+2R\cos(k\beta)+R^2}\cos\gamma\right\}=\cos\gamma\left\{\frac{1}{k}\frac{\partial\arg(\eta)}{\partial x_s}\right\} \tag{16}$$

$$(x_n=0)$$

and terms are factored to obtain

$$\tan\gamma=\frac{\frac{1}{k}\frac{\partial\arg(\eta)}{\partial x_s}}{\frac{1}{k}\frac{\partial\arg(\eta)}{\partial x_n}+\frac{2R[\cos(k\beta)+R]}{1+2R\cos(k\beta)+R^2}\cos\gamma}\qquad(x_n=0) \tag{17}$$

Note that when R=0 this expression reduces to Eq. (10) in method B.

The direction of approaching waves was determined by obtaining estimates for γ using the nonlinear expression

$$f(\gamma)=\tan\gamma-\frac{\frac{1}{k}\frac{\partial\arg(\eta)}{\partial x_s}}{\frac{1}{k}\frac{\partial\arg(\eta)}{\partial x_n}+\frac{2R[\cos(k\beta)+R]}{1+2R\cos(k\beta)+R^2}\cos\gamma} \tag{18}$$

and then using these estimates in the iterative technique presented in method B. There are four possible solutions to $f(\gamma)$=0; this is observed by squaring both sides of Eq. (16) and substituting $\sin^2\gamma=1-\cos^2\gamma$ to obtain a fourth-order polynomial in terms of $\cos\gamma$ with four possible roots. The value of γ at a root of $f(\gamma)$ is obtained using bracketing and bisection (Press et al., 1992). First γ is bracketed between $-90°$ and $0°$ or between $0°$ and $90°$ based on the sign of $\partial \arg(\eta)/\partial x_s$. It was found that for typical values of $\partial \arg(\eta)/\partial x_s$ and $\partial \arg(\eta)/\partial x_n$ the function $f(\gamma)$ is either monotone strictly increasing or decreasing through zero over the bracketed intervals; only one root of $f(\gamma)$ lies in this interval. The location where $f(\gamma)$ is equal to zero is determined by bisection.

The estimated direction of approaching waves obtained via this method is identical to that in Fig. 4 when R=0. This method was used to obtain estimates for γ over a range of reflection coefficients between 0 and 1 and it was found that γ is accurately predicted for all incident wave directions. For example, the approaching wave direction and the corresponding phase of the wave field are illustrated in Fig. 6 for a coast with a reflection coefficient of 0.5. Note that oscillations of the phase in Fig. 6b and c are not spurious; they represent oblique partially standing waves.

5. Application

The published and proposed methods of estimating the direction of approaching waves were used to simulate waves in a rectangular harbor. The geometry of the model domain is shown in Fig. 7; the period of incident waves was chosen such that the ratio of wavelength to the width of the harbor entrance is equal to 1.0. A rectangular harbor with this geometry and wavelength was used by Isaacson and Qu (1990) and Isaacson et al. (1993) to evaluate their methods of estimating the direction of approaching waves. Pos and Kilner (1987) presented laboratory measurements of

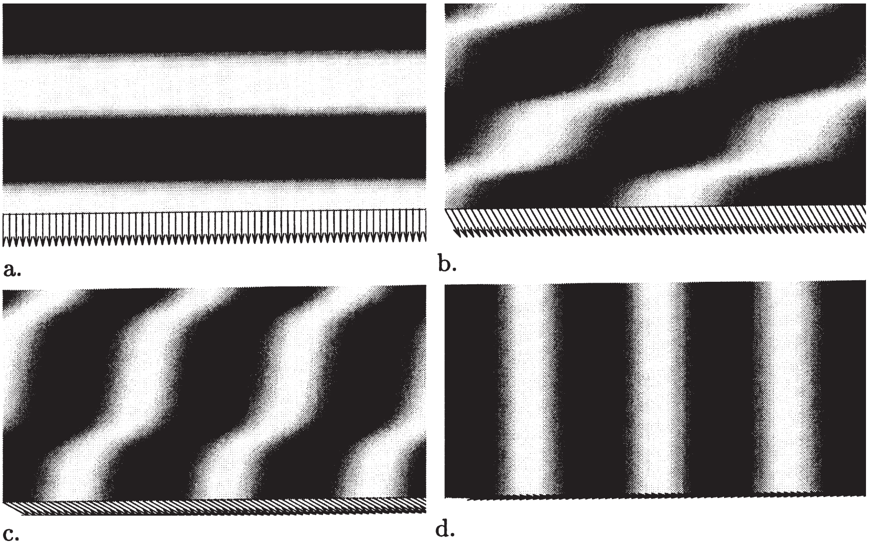

Fig. 6. New method D, estimated direction of approaching waves and phase of waves, R=0.5. (a) 0° Incident Direction; (b) 30° Incident Direction; (c) 60° Incident Direction; (d) 90° Incident Direction.

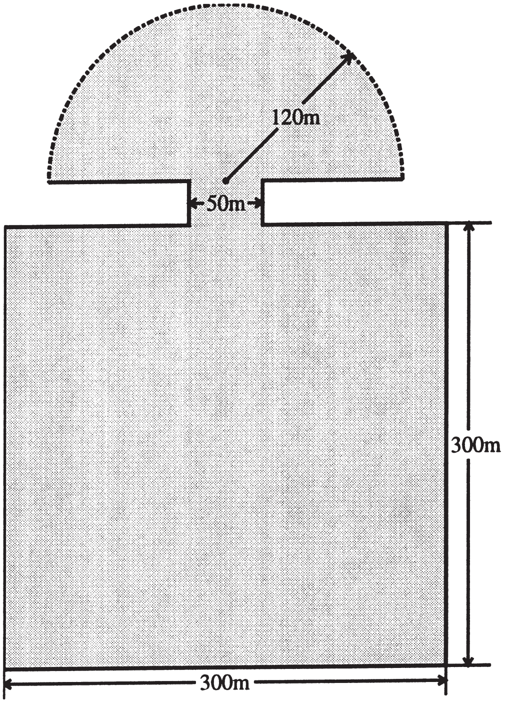

Fig. 7. Model domain used to simulate waves in a rectangular harbor.

waves in a rectangular harbor with a similar geometry and this wavelength. Waves were simulated using a constant reflection coefficient along the portion of the coast inside the harbor; the coast is assumed to be fully absorbing outside the harbor (similar to Pos (1985) and Pos and Kilner (1987)).

First, the published and proposed methods of estimating the direction of approaching waves were evaluated for the case of a fully absorbing coast inside the harbor. This estimated direction and the corresponding phase of the wave field are illustrated in Fig. 8. The predicted wave amplitude is shown in Fig. 9. This amplitude varies from 0.0 (the black regions) to 1.0 (the white regions) with a contour interval of 0.1. It should be noted that although the phase diagrams are very similar for each method, the predicted direction of approaching waves are clearly different. This has important implications to modeling wave-current interaction since the wave direction is required for these calculations (e.g., Kirby, 1984; Kostense et al., 1988).

The model results indicate that the new method provides the most acceptable estimate of the direction of approaching waves. The results obtained using method A (Figs. 8a and 9b) show reflected waves being generated along the coast; spurious oscillations are observed in the phase diagram near the coast and in the amplitude diagram in the harbor. The wave field predicted using method B (Figs. 8b and 9b) compare well to the results presented by Isaacson and Qu (1990) and Pos and Kilner (1987). These researchers noted that along the centerline of the harbor these ampli-

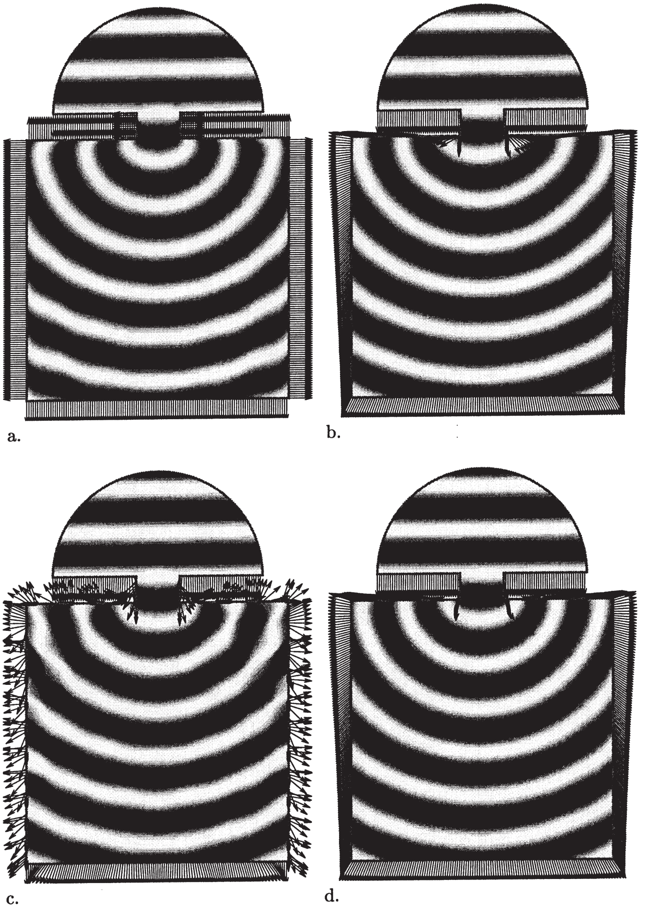

Fig. 8. Estimated direction of approaching waves and phase of waves in a rectangular harbor, R=0.0. (a) Method A; (b) Method B; (c) Method C; (d) New method D.

tudes are larger than those observed in the laboratory. The wave field predicted using method C (Fig. 9c) is similar to that presented by Isaacson et al. (1993). Noise along the lateral boundaries is observed which suggest that waves travel between neighboring locations on the coast (reflected waves being generated along portions of the coast where γ in Fig. 8c is smaller than that in Fig. 8b and being absorbed along the portions of the coast where this γ is larger). Note also that method C did not converge; results are presented for the wave field obtained after eight iterations. The wave field predicted using the new method D (Figs. 8d and 9d) is similar to that obtained using method B. The primary difference is that approaching waves are always directed towards the coast in the new method, while the direction of approaching waves obtained in method B is incorrectly directed into the harbor along portions of the coast. This results in slightly smaller amplitudes being predicted along

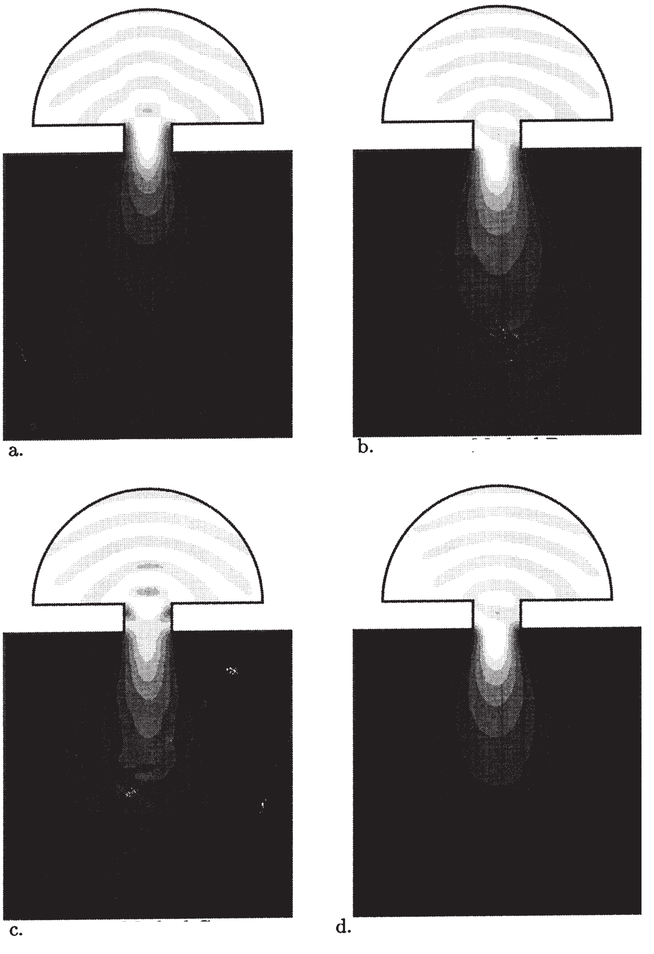

Fig. 9. Amplitude of waves in a rectangular harbor, R=0.0. (a) Method A; (b) Method B; (c) Method C; (d) New method D.

the centerline of the harbor using the new method; these amplitudes are closer to the laboratory results presented by Pos and Kilner (1987).

Next, the direction of approaching waves was estimated for the case of a fully reflecting coast inside the harbor. The method of choosing the wave direction is unimportant for this case since the right-hand side of the coastal boundary condition, Eq. (6), is equal to zero when R=1 and β=0 regardless of the value of γ. The phase and amplitude of the waves that are predicted for a fully reflecting coast are shown in Fig. 10. These results indicate that standing waves are generated between the front and back walls of the tank. These standing waves are distorted by the waves reflected off the lateral boundaries.

The published and proposed methods of estimating the direction of approaching waves were also used to simulate waves in a harbor with a partially reflecting coast.

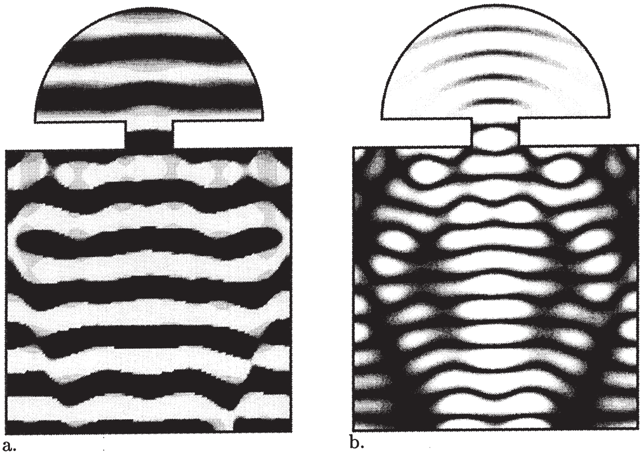

Fig. 10. Phase and amplitude of waves in a rectangular harbor, R=1.0. (a) Phase of waves; (b) Amplitude of waves.

The estimated approaching wave direction and the corresponding phase of the wave field are illustrated in Fig. 11 for a harbor with a reflection coefficient of 0.5. The corresponding wave amplitude is shown in Fig. 12.

These predicted wave fields may be contrasted to obtain an understanding of the limitations of each method for simulating waves in a harbor with a partially reflecting coastline. As before, the results obtained using method A (Figs. 11a and 12a) result in a solution where $\partial \arg(\eta)/\partial x_n$, along the coast is too large. This results in predicted amplitudes that are smaller than those obtained using the other methods for this particular harbor geometry and wavelength. The wave field obtained using method B (Figs. 11b and 12b) has predicted values of the direction of approaching waves with larger $|\gamma|$ than the other methods, particularly along the lateral boundaries. This results in smaller values of $\partial \arg(\eta)/\partial x_n$ and standing waves are generated between the front and back walls with predicted amplitudes that are larger than the other methods. Method B also incorrectly predicts approaching waves that are directed into the harbor along portions of the coast, especially in the shadow zone behind the breakwater. The wave field predicted using method C (Figs. 11c and 12c) and that predicted using the new method D (Figs. 11d and 12d) are very similar. There are, however, two important differences. Firstly, method C did not converge; the new method satisfied the convergence criteria after six iterations. Secondly, the approaching wave directions in Figs. 11c and d are different along portions of the coast where the estimate of $|\gamma|$ obtained using the new method is relatively large (e.g., along the breakwater in the harbor entrance). The estimated directions of approaching waves are similar for methods C and D along the portions of the coast where $|\gamma|$ is relatively small; this includes most of the coast inside the harbor for this example.

It should be noted that a non-linear boundary condition is obtained when γ from the new method, Eq. (17), is substituted into the coastal boundary condition, Eq.

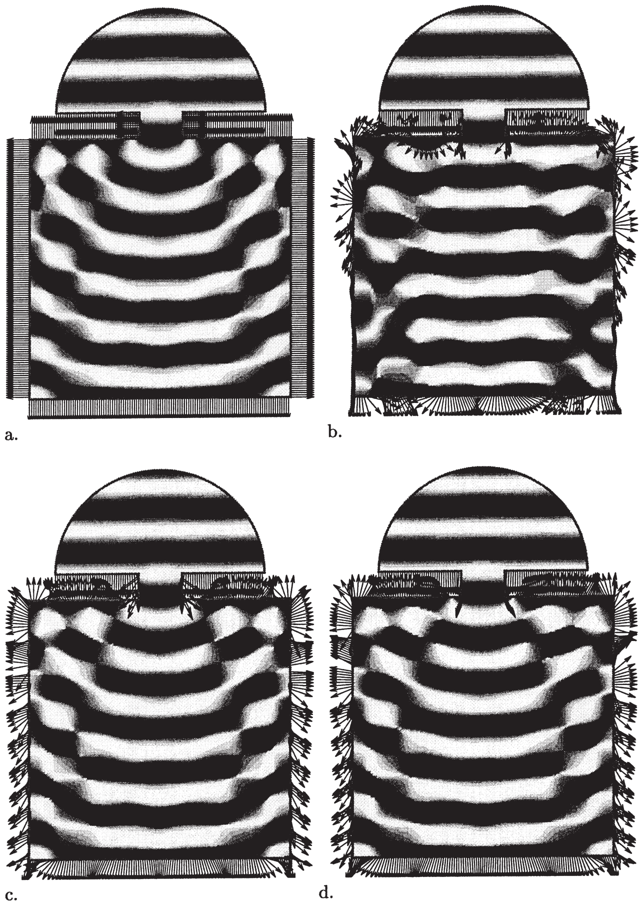

Fig. 11. Estimated direction of approaching waves and phase of waves in a rectangular harbor, R=0.5. (a) Method A; (b) Method B; (c) Method C; (d) New method D.

(6). Although the method used to linearize this boundary condition converged for all cases in this paper, it is difficult in general to establish convergence properties.

6. Conclusions

The three previously published methods of estimating the direction of approaching waves, γ, were examined and it was found that each method has shortcomings that limit their ability to accurately reproduce wave fields:

- The first method (A) is accurate only when γ=0°.
- The second method (B) is accurate only when the reflection coefficient, R, is equal to zero.

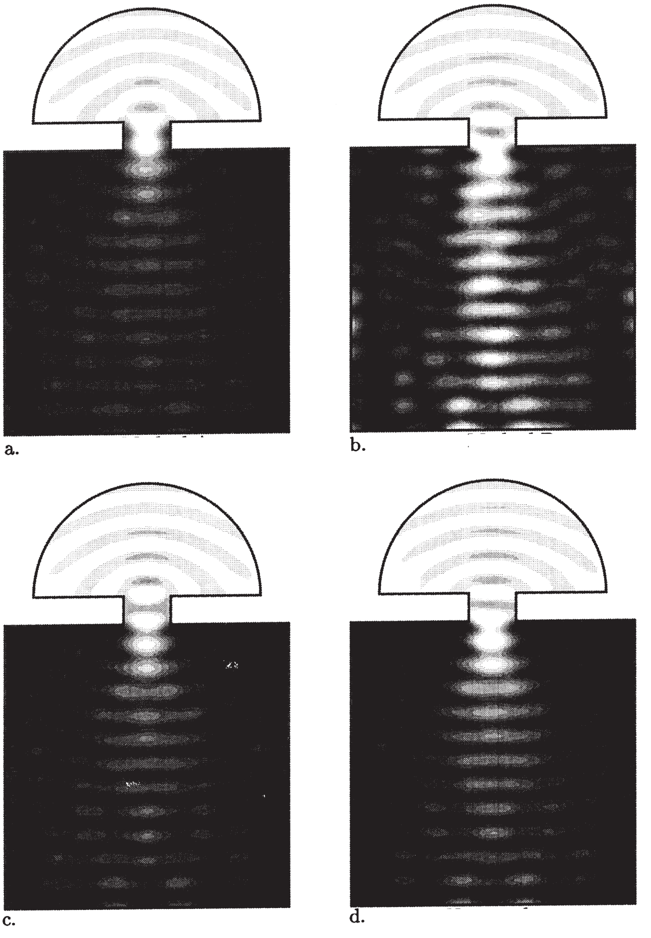

Fig. 12. Amplitude of waves in a rectangular harbor, R=0.5. (a) Method A; (b) Method B; (c) Method C; (d) New method D.

- The third method (C) is accurate only when $|\gamma|\leq\sim 40°$; this method does not converge when $|\gamma|>40°$.

Thus, method B is the only method that can accurately estimate the approaching wave direction for all incident wave directions, however this method is valid only for fully absorbing coasts.

A new method (D) of estimating the direction of approaching waves was presented that is equivalent to method B when R=0. This method is based on an expression obtained from the cross product of a vector pointing in the direction of approaching waves and the gradient of the free surface elevation function, η. This new method correctly reproduces the analytic solution for waves approaching a coast at any direction and for any value of R between 0 and 1.

The applicability of these published and proposed methods was evaluated for waves in a rectangular harbor. These results indicate that problems associated with the published methods (i.e., over and underestimating $\partial \arg(\eta)/\partial x_n$, approaching wave directions directed away from the coast, and iterative methods that do not converge) do not occur with the new method. Thus, the new method provides the most reliable predictions of the direction of approaching waves in a harbor. It is expected that the new method will allow more accurate simulation of wave-current interaction since the wave direction is required for these calculations.

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