Exergy Analysis for the Performance of Solar Collectors

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The optimum control and performance evaluation of solar collectors are analyzed from the standpoint of exergy. The pressure drop inside the collector is introduced to the analysis using the Hottel-Whillier model. By treating the friction process as exergy loss, the optimum operating conditions are presented in a simple statement. The maximum capability of collectors is determined and expressed by a relationship among the collector parameters and the environment in which they operates.

Introduction

The optimum operating conditions of solar collectors have so far been investigated on the basis of collected thermal energy. The criterion generally adopted is to maximize the difference between the collected thermal energy and the required pumping power [1-3]. However, this criterion equalizes the value of mechanical energy and thermal energy. The required pumping power is converted to thermal energy by friction. This process reduces the quality of energy, but not the quantity. The quality of thermal energy can be treated by means of exergy [4], which is an equivalent concept to availability or available work. Recently, Bejan, Kearney, and Kreith [5] obtained an optimum flow rate on the basis of the second law of thermodynamics (mainly entropy) and showed one example where the inlet temperature was equal to the ambient temperature. But it can be shown in the present study that the optimum flow rate becomes infinite in some region of the inlet temperature if the same criterion is used. In order to avoid this difficulty, the pressure drop inside the collector is introduced in this study. In the references [5] and [6], the optimum conditions of general heat exchangers were treated on the basis of entropy generation, which was also including pressure drop. The criterion of these references, however, compares the entropy generation caused by the pressure drop with that caused by the temperature difference between the wall and the fluid. Since the temperature difference doesn't appear explicitly in the collector property equation commonly used, the criterion is not applicable directly to such a collector model. As exergy has the same dimension as energy, it can be compared directly to pumping power, and a new criterion can be established.

This paper analyzes the performance of solar collectors on the basis of exergy and suggests a criterion for optimum control and performance evaluation.

First, exergy analysis using the Hottel-Whillier model [7] will be presented and the need to introduce the exergy loss caused by friction process will be noted. Next, by introducing the pressure drop into the analysis, the optimum operating conditions will be expressed by a simple statement. Also, the expression of the maximum capability of solar collectors will be shown in a simple form.

Exergy Analysis Using the Hottel-Whillier Model

The properties of flat-plate solar collectors are, in general, given by the relationship among the outlet and inlet temperatures T_o, T_i, the mass flow rate $\dot{m}$, and insolation I, as [7]

$$T_o = T_a + \tau\alpha I/U_L + (T_i - T_a - \tau\alpha I/U_L)\exp(-F' U_L A_c/\dot{m}C_P) \tag{1}$$

From equation (1), instantaneous thermal efficiency, η_t, defined by

$$\eta_t \equiv \dot{m}C_P(T_o - T_i)/IA_c \tag{2}$$

is obtained as follows

$$\eta_t = F_R\{\tau\alpha - (T_i - T_a)U_L/I\} \tag{3}$$

where

$$F_R = (\dot{m}C_P/U_LA_c)\{1 - \exp(-F'U_LA_c/\dot{m}C_P)\} \tag{4}$$

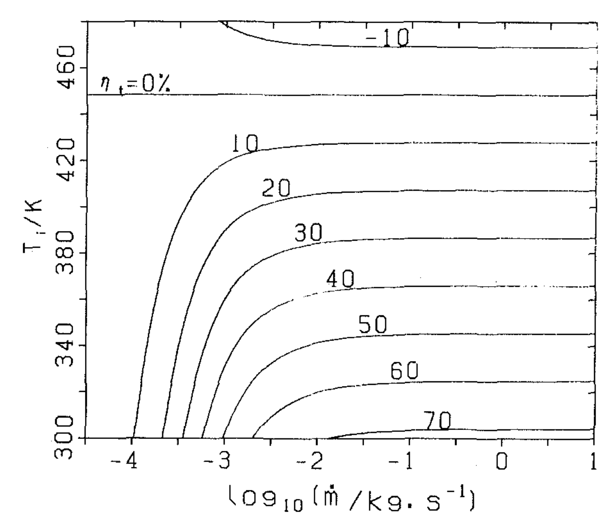

Fig. 1 Contour map of thermal efficiency η_t

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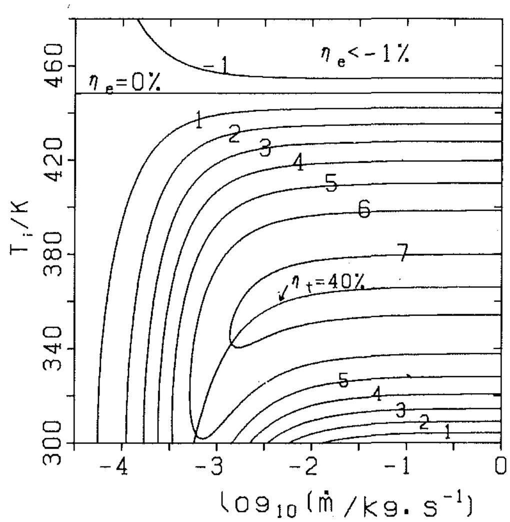

Fig. 2 Contour map of exergy efficiency η_e

which is the well-known Hottel-Whillier equation. According to equations (3) and (4), the thermal efficiency, η_t, is expressed as a function of T_i and $\dot{m}$. In the present study, one model collector having the following parameters is treated: $U_L=3.5$ W m^{-2}K^{-1}, $F'=0.9$, $\tau\alpha=0.8$, $A_c=1.0$ m^2, $C_P=4190$ J kg^{-1} K^{-1}. These are typical values for a solar water heater.

For the investigation of the optimum control and performance evaluation of solar collectors, it is sufficient to assume a constant environment for the first step. Therefore, only the instantaneous efficiency is considerd in the following discussion, and the daily efficiency is not treated. As one example, the insolation, I, and ambient temperature, T_a, are fixed as $I=650$ W/m^2, $T_a=300$ K The stagnation temperature, T_{st}, is obtained by setting $\eta_t=0$ in equation (3), as

$$T_{st}\equiv T_a+\tau\alpha I/U_L=449\text{ K} \tag{5}$$

With respect to this case, η_t is depicted in Fig. 1 as a function of T_i and $\dot{m}$.

The exergy gain, $\Delta\epsilon$, in the collector, which means the maximum available work obtained from the thermal energy gain is given [4,5] by

$$\Delta\epsilon=\Delta h-T_a\Delta s \tag{6}$$

Assuming that the specific heat, C_P, at constant pressure is constant, Δh and Δs in equation (6) can be integrated, and the exergy efficiency, η_e, is expressed as follows

$$\eta_e\equiv\frac{\Delta\epsilon}{IA_c}=(\dot{m}C_P/IA_c)\{T_o-T_i-T_a\ln(T_o/T_i)\} \tag{7}$$

Eliminating T_o by equation (1), η_e is also expressed as a function of T_i and $\dot{m}$, which is shown in Fig. 2. In the figure, the line $\eta_t=0.4$ is also depicted.

From Fig. 2, it can be seen that in the range of $T_i\leqq 367$ K $(=\sqrt{T_{st}T_a})$, the optimum flow rate, $\hat{\dot{m}}$, which gives the maximum exergy efficiency under a constant, T_i, is characterized by $\eta_t=0.4$. In the range of $T_i\geqq 367$ K, however, $\hat{\dot{m}}$ becomes infinite. This is unsuitable for the optimum condition because infinite pumping power is required to realize this condition. This difficulty can be resolved by introducing the pressure drop inside the collector, and is discussed in the next section.

Next, the maximum exergy efficiency $\eta_e^{\max}$ over all of T_i and $\dot{m}$ is considered. In Fig. 2, $\eta_e^{\max}$ is equal to 7.22 percent, and is given by $T_i=T_i^{\max}=367$K, $\dot{m}=\dot{m}^{\max}\rightarrow\infty$. In the range of $\dot{m}C_P>>F'U_LA_c$, the collector property equation (1) can be approximated with Taylor expansion, and the above conclusion can be derived analytically as follows:

Expanding equations (1) and (7) with $F'U_LA_c/\dot{m}C_P$ up to the second order, we obtain

$$T_o=T_{st}-(T_{st}-T_i)(1-A/\dot{m}C_P+A^2/2\dot{m}^2C_P^2) \tag{8}$$

$$\eta_t=B(T_{st}-T_i)(1-A/2\dot{m}C_P) \tag{9}$$

$$\eta_e=B(T_{st}-T_i)\{1-T_a/T_i-(A/2\dot{m}C_P)(1-T_{st}T_a/T_i^2)\} \tag{10}$$

where, $A\equiv F'U_LA_c$, $B\equiv A/IA_c$

So that

$$\frac{\partial\eta_e}{\partial T_i}=B\{-1+T_{st}T_a/T_i^2+(A/2\dot{m}C_P)(1+T_{st}T_a/t_i^2-2T_{st}^2T_a/T_i^3)\} \tag{11}$$

$$\frac{\partial\eta_e}{\partial\dot{m}}=(AB/2\dot{m}^2C_P)(T_{st}-T_i)(1-T_{st}T_a/T_i^2) \tag{12}$$

Setting equations (11) and (12) equal to zero, and assuming $T_i=T_{st}$, we obtain $\dot{m}\rightarrow\infty$ and $T_i=\sqrt{T_{st}T_a}$. Substituting the solutions into equations (8–10), the following relations are obtained

$$T_o^{\max}=T_i^{\max}=\sqrt{T_{st}T_a}=367\text{ K} \tag{13}$$

$$\eta_t^{\max}=\frac{F'\tau\alpha}{1+\sqrt{T_a/T_{st}}}=0.396 \tag{14}$$

$$\eta_e^{\max}=\mathrm{F}'\tau\alpha\left(\frac{\sqrt{T_{st}}-\sqrt{T_a}}{\sqrt{T_{st}}+\sqrt{T_a}}\right)=7.22\times10^{-2} \tag{15}$$

which coincide with the results of Fig. 2. It is notable that equation (13) is independent of the collector efficiency factor F', which contains the fin efficiency and the heat transfer coefficient between the fluid and the tube wall. In this con-

Nomenclature

A_c = collector area
C_P = specific heat at constant pressure
D_i = inner diameter of absorber tube
f = friction factor
F' = collector efficiency factor
F_R = heat removal factor
G = volumetric flow rate
I = solar radiation incident on the collector
L = absorber tube length
$\dot{m}$ = mass flow rate
T_a = ambient temperature
T_i = collector inlet temperature
T_m = mean fluid temperature
T_o = collector outlet temperature
T_{st} = stagnation temperature
U_L = collector loss coefficient
V = mean fluid velocity
$\Delta\epsilon$ = exergy gain in the collector
Δh = entalpy difference between inlet and outlet fluid
ΔP = pressure drop of the collector
Δs = entropy difference between inlet and outlet fluid
η_e = exergy efficiency
η_t = thermal efficiency
H_e = net exergy efficiency
H_t = net thermal efficiency
H_t' = apparent thermal efficiency
ρ = mean fluid density
$\tau\alpha$ = collector transmissivity absorptivity product

Superscripts

$\hat{}$ = the condition giving the (net) maximum exergy efficiency under a constant T_i
max = the condition giving the (net) maximum exergy efficiency over all of T_i and $\dot{m}$

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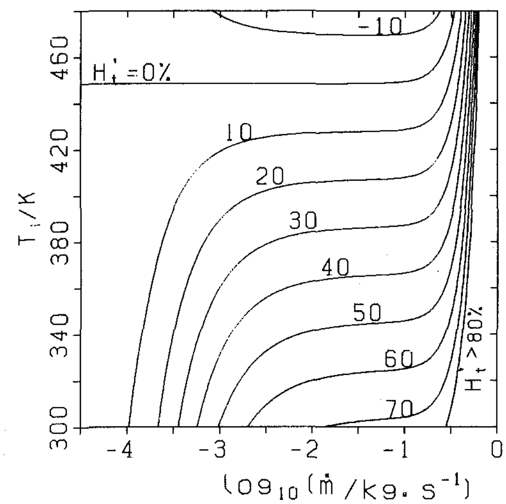

Fig. 3 Contour map of apparent thermal efficiency H_t'

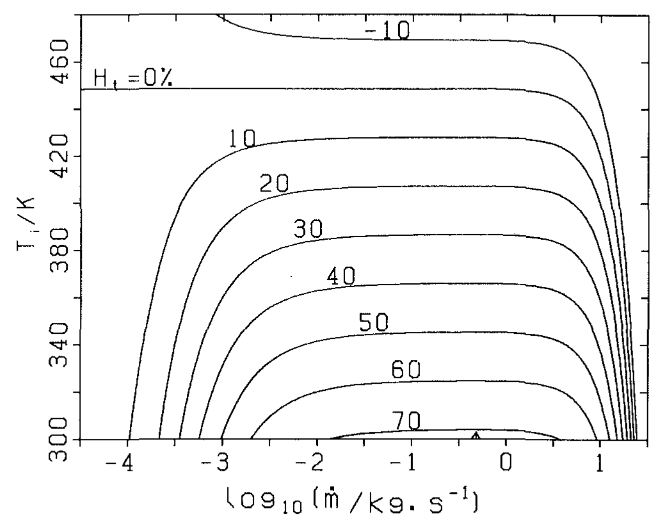

Fig. 4 Contour map of net thermal efficiency H_t

dition, as the collector has an almost isothermal temperature distribution owing to $\dot{m} \rightarrow \infty$, $\eta_e^{\max}$ is also obtained by multiplying $\eta_t^{\max}$ by the Carnot efficiency $(1 - T_a/\sqrt{T_{st}T_a})$. Consequently, these results agree with those obtained for the isothermal collector model in [5]. This means that to obtain the maximum capability of collectors, it is necessary to have a uniform temperature distribution at $\sqrt{T_{st}T_a}$ because insolation is uniform. Accordingly, $\eta_e^{\max}$ indicates the idealized maximum capability of the collector under a constant insolation.

Introduction of Pressure Drop

The mechanical energy of the fluid generated by pumping is converted to thermal energy by friction. No energy loss occurs in this process if the thermal insulation is perfect. But the exergy of the fluid inevitably decreases by friction, so that it is necessary to take the pressure drop into consideration in order to treat the friction process as an exergy loss. In the present study, only the pressure drop of the collector absorber tube is treated, which is given by

$$\Delta P = \rho V^2 fL/2D_i \tag{16}$$

where f is the friction factor assumed to be constant at 0.03. It is a typical value for a copper tube of 10-mm i.d. in a turbulent flow region [8].

The friction heat generated by ΔP is given by $G\Delta P$ for a liquid phase medium, where G is the volumetric flow rate. Furthermore, $G\Delta P$ corresponds to the decrease of the mechanical energy of the fluid, and also means the minimum value of the required pumping power.

Setting $\rho = 1000$ kg/m^3, $D_i = 0.01$ m and $L = 10$ m, we obtain

$$G\Delta P = 2430\dot{m}^3 \quad \text{(W)} \tag{17}$$

In order to introduce the effect of friction heat, the following displacement is required in equation (1)

$$\tau\alpha I \rightarrow \tau\alpha I + G\Delta P/A_c F'$$

In this section, the total efficiency, H, which includes the effect of the friction heat, is considered. At first, the apparent thermal efficiency, H_t', defined similarly to equation (1) becomes

$$H_t' = F_R\{\tau\alpha + G\Delta P/IA_cF' - (T_i - T_a)U_L/I\} \tag{18}$$

which is shown in Fig. 3. Comparing Fig. 3 with Fig. 1, it can be seen that the friction heat dominates in the range of $\dot{m} > 10^{-1}$kg/s, and the definition of H_t' becomes unsuitable. This is because the friction heat is contained in the thermal energy gain in equation (18). Therefore, the friction heat must

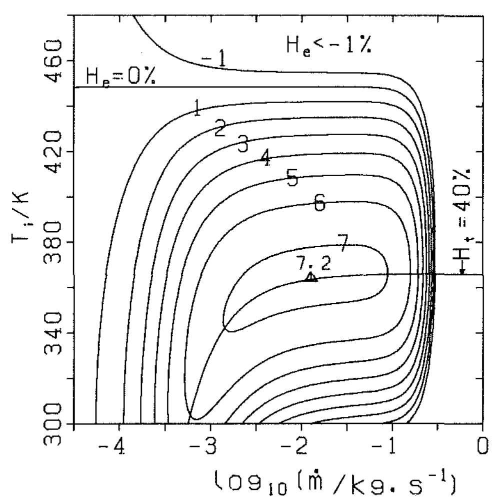

Fig. 5 Contour map of net exergy efficiency H_e

be subtracted from the apparent thermal energy gain, and the net thermal efficiency H_t can be introduced as

$$H_t \equiv H_t' - \frac{G\Delta P}{IA_c} \tag{19}$$

which is shown in Fig. 4. It can be seen that H_t is almost analogous to η_t in the range of $\dot{m} < 1$ kg/s. However, H_t becomes negative with $\dot{m} \rightarrow \infty$. Expanded with $1/\dot{m}$, H_t becomes

$$H_t = F'\{\tau\alpha - (T_i - T_a)U_L/I\} - \frac{1}{IA_c}\left\{\frac{c_1}{2}\left(\frac{F'U_LA_c}{C_p}\right)\dot{m}^2 - \frac{c_1}{6}\left(\frac{F'U_LA_c}{C_P}\right)^2\dot{m} + \frac{c_1}{24}\left(\frac{F'U_LA_c}{C_P}\right)^3\right\} + O\left(\frac{1}{\dot{m}}\right) \tag{20}$$

where $c_1 = 2430$, and $O(1/\dot{m})$ is the term of the order $1/\dot{m}$ in $\dot{m} \rightarrow \infty$. Equation (20) shows negative H_t with $\dot{m} \rightarrow \infty$. This is because the temperature rise due to the friction heat causes additional heat loss, and the apparent heat gain becomes lower than the friction heat. However, the range of $\dot{m} > 1$ kg/s is not a realistic condition because the friction heat exceeds 2430 W, which is much greater than the insolation.

In [1-3], optimum control is determined by the criterion of maximizing the difference between collected thermal energy and required pumping power. As $G\Delta P$ also corresponds to the minimum value of the required pumping power, equation (19)

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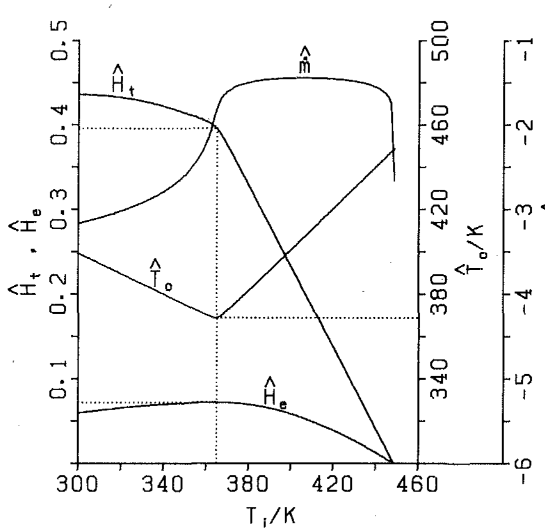

Fig. 6 Optimum operating conditions under constant inlet temperature

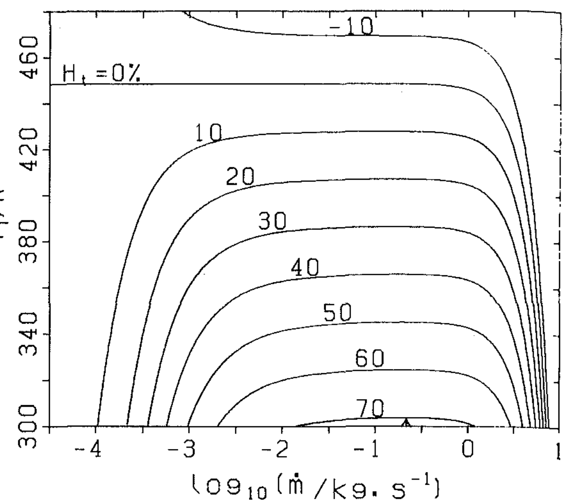

Fig. 8 Contour map of net thermal efficiency H_t = f = 0.3

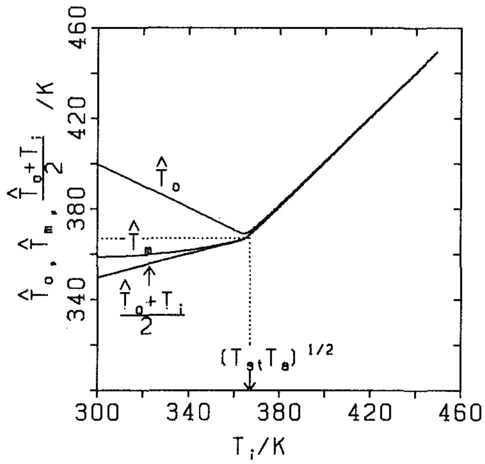

Fig. 7 Fluid temperature under optimum conditions

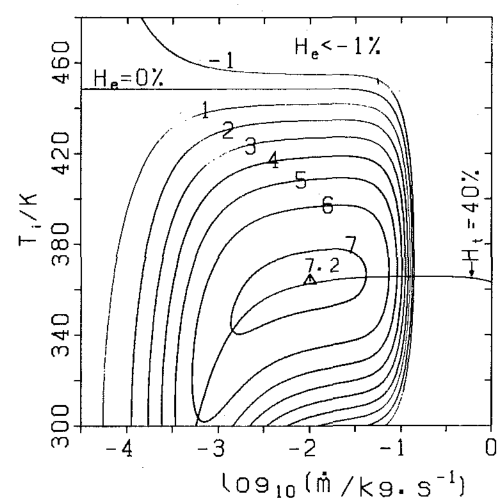

Fig. 9 Contour map of net exergy efficiency H_e in f = 0.3

is equivalent to the criterion of these references. However, it is shown in Fig. 4 that the curve of H_t has very broad peaks and the optimum condition under a constant T_i cannot be determined definitely. This is because the mechanical energy of the fluid is converted to thermal energy by friction, and no energy loss occurs directly in this process. This criterion function equalizes the value of thermal energy and pumping power. As thermal energy and mechanical energy are substantially different in nature, this study adopts exergy instead of thermal energy for the criterion of optimum control. For this purpose, the net exergy efficiency, H_e, must be introduced, which is defined by

$$H_e \equiv [\dot{m}C_P\{T_o - T_i - T_a \ln(T_o/T_i)\} - G\Delta P]/IA_c \qquad (21)$$

where the first term means the exergy gain owing to the fluid temperature increase caused by the insolation and friction. The second term means the decrease of mechanical energy owing to friction. In equation (21), the first term can be transformed into a nondimensional form [5]. But the second term, $G\Delta P$, is a cubic expression of $\dot{m}$, and contains no common parameter with the first term except $\dot{m}$. Consequently, there is no advantage in transforming equation (21) into a nondimensional form. Hence, the dimensional form is used in the present study, and as fundamental parameters, T_i and $\dot{m}$ are adopted. The contour map of H_e is given in Fig. 5 as a function of T_i and $\dot{m}$. In Fig. 5, the line of H_t = 0.4 is also shown. In this figure, the optimum flow rate under a constant T_i described by $\hat{\dot{m}}$ roughly gives $H_t = 0.4$ in the range of $T_i = \sqrt{T_{st}T_a}$, and becomes constant in $T_i \geqq \sqrt{T_{st}T_a}$. In order to show this clearly, the optimum value of H_t, H_e, and T_o under a constant T_i designated by the superscript ˆ, and $\dot{m}$ are illustrated as a function of T_i in Fig. 6. In this figure, $\hat{H}_t$ varies from 43.5 to 39.6 percent in the range of $T_i \leqq \sqrt{T_{st}T_a}$, which is approximately constant, and $\hat{H}_e$ varies from 5.94 to 7.21 percent in this range. On the other hand, in the range of $T_i \geqq \sqrt{T_{st}T_a}$, $\hat{\dot{m}}$ becomes almost constant. The characteristic of a constant $\hat{H}_t$ in $T_i \geqq \sqrt{T_{st}T_a}$ also means that the mean fluid temperature should be constant. According to the Hottel-Whillier model, the fluid temperature has an exponential distribution (see equation (1)), having the following average fluid temperature, T_m

$$T_m = T_{st} - \dot{m}C_P(T_o - T_i)/F'U_LA_c \qquad (22)$$

In Fig. 7, $\hat{T}_m$, which is the optimum value of T_m, $\hat{T}_o$, and $\hat{T}_o + T_i/2$ are shown in the same way as Fig. 6. It indicates that $\hat{T}_m$ is approximately constant at $\sqrt{T_{st}T_a}$ in the range of $T_i \leqq \sqrt{T_{st}T_a}$, which varies from 359 to 367 K. In the range of $T_i \geqq \sqrt{T_{st}T_a}$, $\hat{T}_m$ increases with T_i. But the operation in this range should be avoided because $\hat{H}_t$ *and* $\hat{H}_e$ decrease rapidly. Consequently, the optimum collector operating conditions can be characterized by an approximately constant average fluid temperature at $\sqrt{T_{st}T_a}$. The same figure as Fig.

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7 is obtained in the case of $F'=1.0$ and $F'=0.5$, so that this result is not affected by the collector efficiency factor F'.

Following the conclusion of the prior section, the optimum fluid temperature is $\sqrt{T_{st}T_a}$ if it has a uniform temperature distribution. As a conclusion of this section, under the optimum control, the mean fluid temperature ought to be constant at $\sqrt{T_{st}T_a}$. This is a simple principle for the optimum control of solar collectors. However, in order to apply it to solar systems including a storage unit [1-3], the exergy loss inside the storage unit must be taken into account. For this purpose, the exergy loss caused by mixing the fluids of different temperature must be included in addition to the heat loss.

As for the maximum value of H_e designated by $H_e^{\max}$, it has almost the same value 7.21 percent as $\eta_e^{\max}$, and $H_t^{\max}$ is also equal to $\eta_t^{\max}$ which is 39.6 percent. $T_i^{\max}$ and $T_o^{\max}$ are 365 K and 369 K, respectively. The discrepancy from $\sqrt{T_{st}T_a} = 367$ K is due to friction, but it hardly effects $H_e^{\max}$ and $H_t^{\max}$. In the maximum condition, $\dot{m}$ is $10^{-1.81}$ kg/s, and $G\Delta P$ is only 9.03 mW, which is insignificant compared to the insolation. In Fig. 8 and 9, H_t and H_e are shown for the case of $f=0.3$, which is ten times as large as the former case and is rather larger than the usual case [8]. But the main points of these figures are similar to those of Fig. 4 and 5 except for the shift of the efficiency drop in the large $\dot{m}$ region in the direction of the smaller $\dot{m}$ region. The effect of the friction factor is not significant because the friction heat term is a cubic function of $\dot{m}$ but is only a linear function of the friction factor. Furthermore, exergy and energy gains are almost consant in the range where friction heat becomes significant. Consequently, friction factor has no influence on the principle of the optimum control other than the constant value of flow rate in $T_i > \sqrt{T_{st}T_a}$, that is $10^{-2.01}$ kg/s in the case of $f=0.3$. The friction factor has little effect on $H_t^{\max}$ and $H_e^{\max}$, which are 7.21 percent and 39.6 percent, respectively, and $G\Delta P$ in that condition is 22.7 mW when $f=0.3$. Thus, the maximum capability is almost unchanged by introducing the pressure drop, and is given by equations (13-15), which are expressed in a simple form using T_{st}, $\tau\alpha$ and F'. This means that in order to utilize the maximum capability of the collector, it should be used in a uniform temperature distribution at $\sqrt{T_{st}T_a}$ within the limits of unremarkable friction loss. Equations (13), (14), and (15) can give a useful basis for the performance evaluation, design, and improvement of solar collectors without considering pressure drop.

In the present study, only one model case is analyzed. But the conclusions obtained are applicable to most solar collectors fitting the Hottel-Whillier model, because the characteristics of Fig. 5 are unchanged by variation in the parameters of the collector and the pressure drop.

Since the concept of exergy is based on the entropy generation owing to the irreversibility of process, it has some academic interest. Furthermore, as exergy has the meaning of the maximum available work obtained from the thermal energy gain, it is valuable for practical application.

Conclusions

The performance of solar collectors is analyzed from the standpoint of exergy. As a result, it is shown that the criterion based on thermal efficiency and pumping power does not give the optimum operating conditions if the friction heat is included in the analysis. The friction process, which is an energy conversion from mechanical energy to thermal energy, can be treated as exergy loss. Comparing it with the exergy gain extracted from insolation, an optimum control can be established, and a simple principle is obtained. That is, in the range of $T_i \leqq \sqrt{T_{st}T_a}$ (K), the average fluid temperature should be constant at $\sqrt{T_{st}T_a}$ (K), and in $T_i \geqq \sqrt{T_{st}T_a}$ (K), the mass flow rate should be constant, while the collector performance declines rapidly in this range.

The maximum capability of collectors is also obtained. This is expressed by a simple relationship among the stagnation temperature, optical efficiency, and collector efficiency factor. The maximum value of exergy efficiency is hardly affected by considering the pressure drop. It can give a useful basis for the performance evaluation and design of solar thermal systems.

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