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Modeling of a Power Transformer Winding for Deformation Detection Based on Frequency Response Analysis*

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Abstract: The paper presents a mathematical model of disc-type power transformer winding for frequency response analysis (FRA) based on traveling wave and multiconductor transmission line theories. Each disc of the model is described by traveling wave equations, which are connected to each other in a form of multiconductor transmission line model. The model is applied to FRA simulation in order to study winding axial and radial deformation and its delectability. Comparison of the simulated winding deformation cases with the reference FRA traces is given and discussed to explore the potentials of the proposed model for winding fault detection.

Key Words: Transformer winding, frequency response analysis, transfer function, deformation detection

1 INTRODUCTION

Among various techniques applied to power transformer condition monitoring, only frequency response analysis (FRA) is suitable for reliable winding distortion and deformation assessment and monitoring. It is based on a statement that the response shape in higher frequencies is uniquely determined by the changes of internal distances and profiles of a transformer winding which are concerned with its deviation or geometrical deformation [1]. Thus, the simultaneous measurements of amplitude ratio and phase difference between the input and output signals of a transformer winding give its frequency response in a wide range up to several mega hertz using the Fourier transform.

Despite of extensive FRA practice, transformer winding condition assessment is usually conducted by trained experts. The obtained FRA traces are compared with the various reference ones taken from the same winding during previous tests or from the corresponding winding of a "sister" transformer, or from other phases of the same transformer. The shifts in resonant frequencies and magnitude of FRA trace indicate a potential winding deformation. However, the questions of potential deformation location in a winding are still required to be investigated.

In order to analyse signal propagation along a transformer winding multiconductor transmission line theory [2] was employed[3-6]. Each turn of a winding is represented by a single transmission line which makes these models complex to operate in case of describing a winding with a large number of turns. Based on the traveling wave theory [7] the model of a single layer helical winding for FRA testing [8] was developed. However, in practice, most power transformers have disc-type high voltage windings which are of the major interest for FRA diagnosis.

In this paper a modified power transformer winding model, which is the consensus between physical veracity from one side and comparative simplicity from another side, is presented. This model is applied to FRA simulation in order to study frequency responses of a disc-type winding at various fault conditions. Simulations and discussions are presented to explore the potentials of the proposed model to transformer fault detection.

2 MATHEMATICAL MODEL OF POWER TRANSFORMER WINDING

2.1 Equivalent Circuit of Power Transformer Winding

The essential part of high voltage transformer windings is designed as a continuous disc-type winding, where each disc is consisted of a number of turns wounded in a radial direction. A continuous disc-type winding has interdisc connections on internal and external sides of a winding and, therefore, wave propagation along each following disc is in opposite direction with respect to the previous one. For the sake of simplicity it is assumed the one-directional flow of an injected signal. Moreover, the conductor length of each disc of the winding is also assumed to be equal. The disc transformer winding at a FRA test can be represented by an equivalent circuit illustrated in Fig.1, where the following notations for the winding parameters in per unit of length of a conductor are in use: K and G, C and g, $C_{\rm d}$ and $G_{\rm d}$ are average interturn, ground and interdisc capacitances and conductivities respectively; r and L are an average disc resistance and inductance correspondingly; M is an average mutual inductance between discs; $Z_{\rm inp}$, $Z_{\rm out}$ are impedances of input and output measurement cables; l is a conductor length of a disc; a is a turn length; n is a total number of discs in the winding.

As clear from the equivalent circuit the disc numeration is marked from the top till down and the direction of the spatial coordinate x is denoted from the left terminal of each disc towards the right end. Each disc has its own parameters signified by a corresponding disc number and the discs are connected to each other by a curve line. Thus, notations $u_1(t,0)$, $i_1(t,0)$ and $u_n(t,l)$, $i_n(t,l)$ denote the voltages and currents at an input and an output ends of the winding, *i.e.* at $x=0$ for the first disc and $x=l$ for the $n^{\rm th}$ disc.

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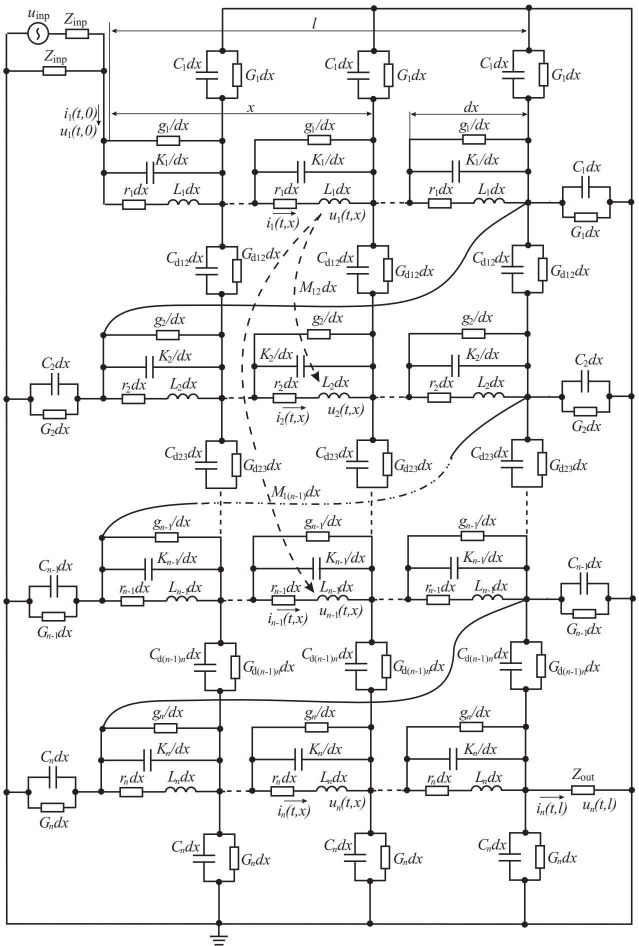

Fig. 1 The equivalent circuit of a disc-type transformer winding

2.2 Mathematical Description of Power Transformer Winding

In order to describe the wave propagation along a winding disc, a mathematical model of a uniform single layer transformer winding is applied [7, 8]. Using the notations in Fig.1, the following mathematical equations for the first disc are derived in the frequency domain:

$$\begin{aligned}\frac{\partial U_1(s,x)}{\partial x} = &- Z_1 I_1(s,x) - sM_{12} I_2(s,x) - \dots \\ &- sM_{1(n-1)} I_{n-1}(s,x)\end{aligned} \tag{1}$$

$$\begin{aligned}\frac{\partial I_1(s,x)}{\partial x} = &- Y_1 U_1(s,x) + a^2 Y_{\mathrm{s}1} \frac{\partial^2 U_1(s,x)}{\partial x^2} \\ &+ Q_{12} U_2(s,x)\end{aligned} \tag{2}$$

where s denotes to the Laplace transform operator, $U_1(s,x)$ and $I_1(s,x)$ are the Laplace transforms of the voltage $u_1(t,x)$ and current $i_1(t,x)$ correspondingly. Z_1, Y_1, $Y_{\mathrm{s}1}$ and Q_1 are the impedance, admittances of the first disc and interdisc admittance per unit conductor length described as follows:

$$\begin{aligned}Z_1 &= L_1 s + r_1 \\ Y_1 &= (C_1 + C_{\mathrm{d}12})\, s + (G_1 + G_{\mathrm{d}12}) \\ Y_{\mathrm{s}1} &= K_1 s + g_1 \\ Q_{12} &= C_{\mathrm{d}12}\, s + G_{\mathrm{d}12}\end{aligned} \tag{3}$$

The expressions for the i^{th} disc, where $i = 2, \cdots, n$ are similar to equations (1) – (3) with minor modifications. Combination of the equations corresponded to all n discs in the matrix form gives the following matrix equation:

$$\begin{aligned}\frac{\partial}{\partial x} \boldsymbol{U}(s,x) &= -\boldsymbol{Z}\, \boldsymbol{I}(s,x) \\ \frac{\partial}{\partial x} \boldsymbol{I}(s,x) &= -\boldsymbol{Q}\, \boldsymbol{U}(s,x) + \boldsymbol{Y_S} \frac{\partial^2}{\partial x^2} U(s,x)\end{aligned} \tag{4}$$

where

$$\boldsymbol{Z} = \begin{bmatrix} Z_1 & sM_{12} & \cdots & sM_{1n} \\ sM_{21} & Z_2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & sM_{(n-1)n} \\ sM_{n1} & \cdots & sM_{n(n-1)} & Z_n \end{bmatrix} \tag{5}$$

$$\boldsymbol{Q} = \begin{bmatrix} Y_1 & -Q_{12} & \cdots & 0 \\ -Q_{21} & Y_2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & -Q_{(n-1)n} \\ 0 & \cdots & -Q_{n(n-1)} & Y_n \end{bmatrix} \tag{6}$$

$$\boldsymbol{Y_S} = \operatorname{diag}\left(a^2 Y_{\mathrm{s}1},\, a^2 Y_{\mathrm{s}2}, \dots,\, a^2 Y_{\mathrm{s}n}\right) \tag{7}$$

The impedances and admittances Z, Y, Y_{s} of each disc and interdisc admittances Q per unit conductor length are defined as follows for $i = 1, \cdots, n$:

$$Z_i = L_i s + r_i \;\text{ and }\; Y_{\mathrm{s}i} = K_i s + g_i \tag{8}$$

for $i = 2, \cdots, n-1$:

$$\begin{aligned}Y_i &= (C_i + C_{\mathrm{d}(i-1)i} + C_{\mathrm{d}i(i+1)})\, s \\ &\quad + (G_i + G_{\mathrm{d}(i-1)i} + G_{\mathrm{d}i(i+1)}) \\ Q_{(i-1)i} &= C_{\mathrm{d}(i-1)i}\, s + G_{\mathrm{d}(i-1)i} \\ Q_{i(i+1)} &= C_{\mathrm{d}i(i+1)}\, s + G_{\mathrm{d}i(i+1)}\end{aligned} \tag{9}$$

and

$$Y_n = (C_n + C_{\mathrm{d}(n-1)n})\, s + (G_n + G_{\mathrm{d}(n-1)n}) \tag{10}$$

The equations (4) can then be rearranged in the matrix form as below:

$$\frac{\partial}{\partial x} \begin{bmatrix} \boldsymbol{U}(s,x) \\ \boldsymbol{I}(s,x) \end{bmatrix} = \boldsymbol{A} \begin{bmatrix} \boldsymbol{U}(s,x) \\ \boldsymbol{I}(s,x) \end{bmatrix} \tag{11}$$

where

$$\boldsymbol{A} = \begin{bmatrix} 0 & -\boldsymbol{Z} \\ -\boldsymbol{Y} & 0 \end{bmatrix}, \quad \text{and} \quad \boldsymbol{Y} = \frac{\boldsymbol{Q}}{\boldsymbol{II} - \boldsymbol{Y_S} \boldsymbol{Z}} \tag{12}$$

and II is the identity matrix 0 is a zero matrix.
Consequently, the following equations are derived:

$$\begin{aligned}\frac{\partial^2 \boldsymbol{U}(s,x)}{\partial x^2} &= \boldsymbol{Z}\boldsymbol{Y}\boldsymbol{U}(s,x) \\ \frac{\partial^2 \boldsymbol{I}(s,x)}{\partial x^2} &= \boldsymbol{Y}\boldsymbol{Z}\boldsymbol{I}(s,x)\end{aligned} \tag{13}$$

The solution of matrix equation (11) for the voltages and currents at the end of transformer winding when $x = 0$ and $x = l$ is obtained in the form below [2]:

$$\begin{aligned}\begin{bmatrix} \boldsymbol{U}(s,l) \\ \boldsymbol{I}(s,l) \end{bmatrix} &= \Phi(l) \begin{bmatrix} \boldsymbol{U}(s,0) \\ \boldsymbol{I}(s,0) \end{bmatrix} \\ &= \begin{bmatrix} \boldsymbol{\Phi_{11}}(l) & \Phi_{12}(l) \\ \boldsymbol{\Phi_{21}}(l) & \Phi_{22}(l) \end{bmatrix} \begin{bmatrix} \boldsymbol{U}(s,0) \\ \boldsymbol{I}(s,0) \end{bmatrix}\end{aligned} \tag{14}$$

where Φ is a $2n \times 2n$ chain parameter matrix.
Since the admittance matrix $\boldsymbol{Y}$ is nonsymmetrical, i.e., $\boldsymbol{Y}^{\mathrm{T}} \neq \boldsymbol{Y}$, where $\boldsymbol{Y}^{\mathrm{T}}$ denotes the transpose of $\boldsymbol{Y}$, the equations (13) are difficult to decouple like in the case of multiconductor transmission line equations using the similarity transformations in order to obtain an analytical solution.
On the other hand, since the turn length a varies depending on turn number in a winding disc, which affects calculation of Y, the matrix $\Phi(l)$ can be approximated as follows:

$$\Phi(l) = \Phi(a_m) \times \cdots \times \Phi(a_2) \times \Phi(a_1) \tag{15}$$

which can be calculated in an iterative numerical form using a matrix exponential function [2]:

$$\Phi(l) = \mathbf{e}^{\mathbf{A}a_m} \times \cdots \times \mathbf{e}^{\mathbf{A}a_2} \times \mathbf{e}^{\mathbf{A}a_1} \tag{16}$$

where a_i is the turn length of the i^{th} turn and m is the number of turns in a disc.
By calculating the chain parameter matrix $\Phi(l)$ it is possible to derive expressions for the transformer winding transfer function, which is presented in next subsection.

2.3 Transfer Function of Power Transformer Winding for Frequency Response Analysis

With the purpose to derive the transformer winding transfer function expressions, equation (14) is rearranged in the form as follows:

$$\begin{bmatrix} \boldsymbol{I}(s,0) \\ \boldsymbol{I}(s,l) \end{bmatrix} = \begin{bmatrix} \Upsilon_{11}(l) & \Upsilon_{12}(l) \\ \Upsilon_{21}(l) & \Upsilon_{22}(l) \end{bmatrix} \begin{bmatrix} \boldsymbol{U}(s,0) \\ \boldsymbol{U}(s,l) \end{bmatrix} \tag{17}$$

where [2]

$$\begin{aligned} \Upsilon_{\mathbf{11}}(l) &= -\Phi_{\mathbf{12}}(l)^{-1}\,\Phi_{\mathbf{11}}(l) \\ \Upsilon_{\mathbf{12}}(l) &= \Phi_{\mathbf{12}}(l)^{-1} \\ \Upsilon_{\mathbf{21}}(l) &= \Phi_{\mathbf{21}}(l) - \Phi_{\mathbf{22}}(l)\,\Phi_{\mathbf{12}}(l)^{-1}\,\Phi_{\mathbf{11}}(l) \\ \Upsilon_{\mathbf{22}}(l) &= \Phi_{\mathbf{22}}(l)\,\Phi_{\mathbf{12}}(l)^{-1} \end{aligned} \tag{18}$$

Using the derived from Fig.1 boundary equalities for equation (17) [9]:

$$\begin{aligned} U_{i+1}(s,0) &= U_i(s,l), \quad \text{for} \quad i = 1, \ldots, n-1 \\ I_{i+1}(s,0) &= I_i(s,l), \quad \text{for} \quad i = 1, \ldots, n-1 \end{aligned} \tag{19}$$

equation (17) can be rewritten as follows [4, 9]:

$$\begin{bmatrix} U_1(s,0) \\ U_2(s,0) \\ \vdots \\ U_n(s,0) \\ U_n(s,l) \end{bmatrix} = \begin{bmatrix} & & \\ & \Omega(l) & \\ & & \end{bmatrix} \begin{bmatrix} I_1(s,0) \\ 0 \\ \vdots \\ 0 \\ I_n(s,l) \end{bmatrix} \tag{20}$$

where $\Omega(l)$ is of $(n+1) \times (n+1)$ order [9].
One of the practical approaches to carry out FRA tests is to measure the neural current flowing to ground [10]. Therefore, a FRA simulation can be carried out assuming a grounded winding. Thus, the following terminal condition is applied to the model:

$$U_n(s,l) = 0 \tag{21}$$

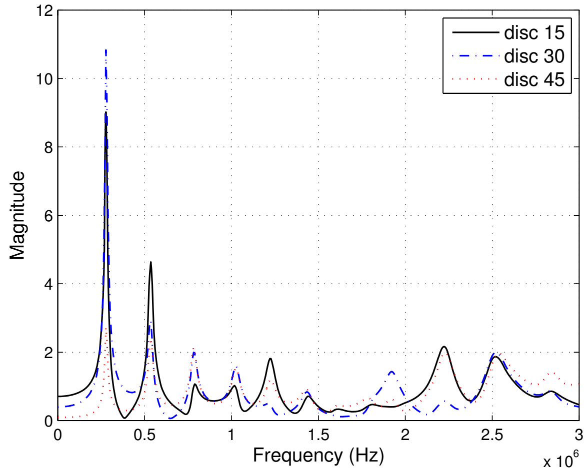

Fig. 2 Magnitude of transfer function of the discs 15, 30 and 45 to the input

Consequently, the ratio of the ground current $I_n(s,l)$ to the input signal $U_1(s,0)$ gives the admittance of the winding:

$$\begin{aligned} H(s) &= \frac{I_n(s,l)}{U_1(s,0)} \\ &= \frac{\Omega_{(n+1,1)}}{\Omega_{(1,n+1)}\Omega_{(n+1,1)} - \Omega_{(1,1)}\Omega_{(n+1,n+1)}} \end{aligned} \tag{22}$$

For the simulation study of FRA traces at different winding positions the transfer function is calculated as the ratio of a voltage $U_k(s,l)$ from the terminal of an arbitrary disc k to the input of the winding $U_1(s,0)$ [6]:

$$\begin{aligned} H(s) &= \frac{U_k(s,l)}{U_1(s,0)} \\ &= \frac{\Omega_{(k+1,1)}\Omega_{(n+1,n+1)} - \Omega_{(k+1,n+1)}\Omega_{(n+1,1)}}{\Omega_{(1,1)}\Omega_{(n+1,n+1)} - \Omega_{(n+1,1)}\Omega_{(1,n+1)}} \end{aligned} \tag{23}$$

Using the above obtained equations (22) and (23) for the transfer functions of transformer winding with substitution $s = j\omega$, where ω is an angular frequency, it is possible to simulate the frequency response of the winding.

3 FREQUENCY RESPONSE SIMULATION STUDIES

The proposed model is applied to FRA simulation studies of a continuous disc power transformer winding. Typical technical parameter values of a high voltage transformer winding have been selected for the simulation purposes.
It is assumed that the capacitances, conductor resistance and insulation conductivities of a winding are estimated in a similar manner to those obtained for multi conductor line models based on the geometry of a winding and the permittivity of winding insulation [3, 5, 6]. Self and mutual inductances of the discs are possible to calculate using the expressions derived in [11] and adjusted to per unit of conductor length base.
Equation (23) is utilized to obtain the frequency response characteristics of the transfer function taken from the terminals of the discs 15, 30 and 45, which correspond to the top,

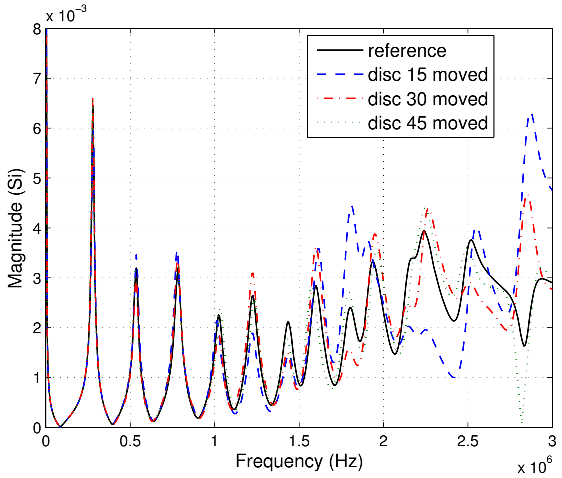

Fig. 3 Simulated axial movements of winding discs

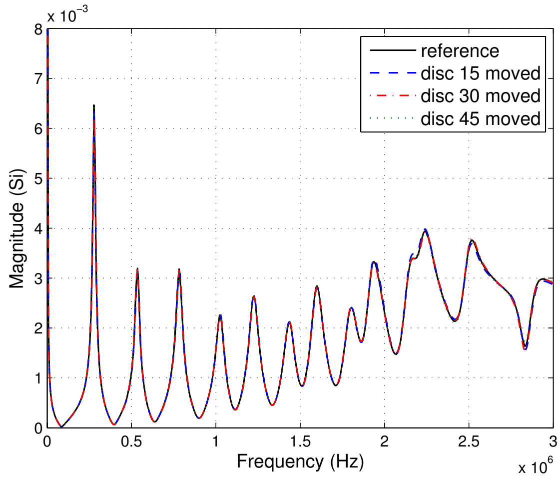

Fig. 5 Simulated radial movements of winding discs

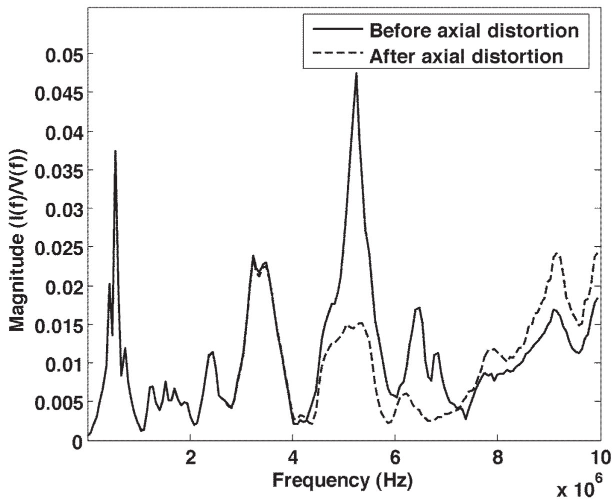

Fig. 4 Measured axial movement (reprinted from [12])

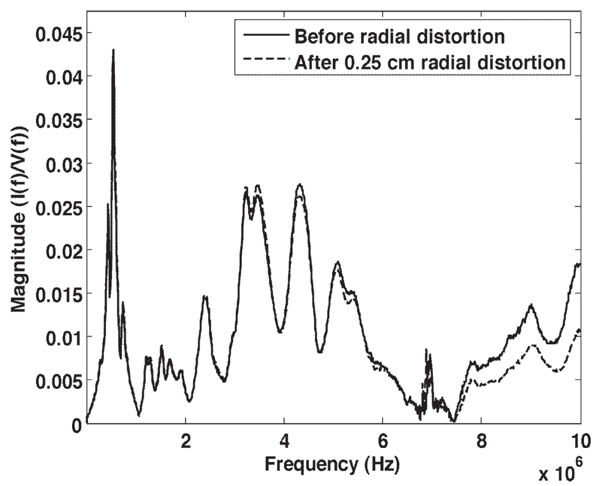

Fig. 6 Measured radial movement (reprinted from [12])

middle and bottom parts of the winding accordingly. The simulation results are presented in Fig.2. It can be deduced that the model is able to reflect interactions between internal capacitances and inductive elements and, thus, can be utilized for the resonance frequency analysis of power transformer windings, similar to the research reported in [6], or for development of a winding deformation detection technique.

In order to investigate the sensitivity of the model and its ability to simulate various faulty conditions, the following cases are studied [12]:

- minor axial winding disc movement in different parts of the winding, simulated by 50% changing of inter-disc capacitances C_d of the correspondent discs from its original value;
- minor radial winding disc movement in different parts of the winding, simulated by 50% changing of ground capacitances C of the correspondent discs from its original value.

Using equation (22) the axial movement of three winding discs, representing top, middle and bottom part of the winding, were simulated. The simulation results are presented in Fig.3. The analysis of the FRA traces shows that there are more significant changes in the higher frequency region mainly due to magnitude deviation, and a slight resonant frequencies shifting. There is also a strong effect of the displacement position on the FRA results. In general, the simulated results are consistent with those reported in [10] and [12] both from computer simulations and experimental tests. As an example, Fig.4 presents measured FRA results, taken from a distribution transformer winding, with an artificially created axial movement introduced [12]. As clear from Fig.4, there is an apparent magnitude deviation of the distorted winding trace occurred at higher frequencies with respect to the reference.

To simulate minor radial movement of winding discs it is assumed that the ground capacitances C is changed. Fig.5 illustrates the simulation results, whereas in Fig.6 the winding admittance, taken experimentally with an artificially created radial movement [12], is presented. In general, there are no apparent deviations between the reference and other traces. However, more detailed observation shows tiny deviations both in magnitude and resonant frequencies primarily in high frequency region, which may possibly be evaluated by using additional automated pattern recognition methods [12].

To summarize above presented simulation results of various power transformer winding conditions it can be concluded that the proposed model correctly reflects the interaction between capacitances and inductive elements in the power transformer winding. Thus, it can be applied to study of signal propagation along a disc-type transformer winding in the frequency domain as well as for transformer winding deformation detection.

4 CONCLUSION

A modified mathematical model of a disc-type transformer winding is proposed. During the derivation of the model, each disc is represented by the equations describing traveling wave propagation in a uniform single layer winding. It allows to reduce significantly the order of the model determined only by the number of discs in a modeled winding. Whereas known multiconductor line models of transformer winding are of the order of total turn number which is, for example, 7 times more with respect to the above simulated winding.

Using the derived transfer function expressions, numerical simulations of disc axial and radial movements in the disc-type transformer winding are undertaken. Apparent deviation of the modeled frequency responses at different locations of the winding disc deformation shows that the model is suitable for fault simulation and can be utilized to winding deformation detection.

In the current study, only simulations are involved to analyze the FRA traces of the proposed disc-type transformer winding model. In general, model verification needs to be performed using a real FRA data. It concerns primarily the winding parameter estimation methods, especially correct accounting of its frequency dependant behavior, which may be revised.

One way to overcome this difficulty is to apply evolutionary optimization techniques for winding parameter's identification, such as Genetic Algorithms and Particle Swarm Optimizer, etc. However, the main concern is the extension of a number of parameters to be optimized in case of a faulty condition winding, causing nonuniform distribution of parameter values and, thus, resulting to significant computational time demanding.

Consequently, further studies will be concentrated on the utilizing of the proposed model for development of a transformer winding deformation detection procedure upon the measurement data.

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