Effectiveness of a Geometric Programming Algorithm for Optimization of Machining Economics Models

Jae Chul Choi† and Dennis L. Bricker‡

Department. of Industrial Engineering, The University of Iowa, Iowa City, Iowa 52242

Scope and Purpose - Many engineering problems require the optimal solution of nonlinear programming problems of a complex nature, and the practitioner must choose an optimization algorithm which is appropriate for his or her specific problem. There is a large body of literature in which numerous methods have been suggested and tested for the solution of machining economics problems, in which the optimal cutting parameters (feed rate, speed, depth of cut, etc.) are to be determined. Duffuaa, Shuaib, and Alam, in a paper published in this journal [1], compared the performance of several algorithms in solving problems of this type, and concluded that the Generalized Reduced Gradient (GRG) algorithm performed better than others which were included in the study. Among the algorithms which they evaluated was a Geometric Programming (GP) algorithm, which fared badly in the comparison. However, the GP algorithm which was evaluated was not representative of the "state-of-the-art". In this paper, we demonstrate that, applied to the five problems used in Duffuaa et al. [1], a GP algorithm is competitive with the GRG algorithm.

Abstract - Machining economics problems usually contain highly nonlinear equations which may present difficulties for some nonlinear programming algorithms. An earlier article by Duffuaa et al. [1] compared the performance of several nonlinear programming algorithms, including a geometric programming algorithm, applied to five machining economics problems. Those authors concluded that the Generalized Reduced Gradient (GRG) algorithm is the most suitable method for solving such problems. In this paper, we point out shortcomings in that conclusion and demonstrate the effectiveness of the Geometric Programming technique in such problems compared with the results of GRG which were presented.

---

†Jae Chul Choi is Research Associate at The University of Iowa, where he earlier received his M.S. and Ph.D. degrees in Industrial and Management Engineering.
‡Dennis L. Bricker is Associate Professor of Industrial Engineering at The University of Iowa. He received B.S. and M.S. degrees in Mathematics from the University of Illinois, and M.S. and Ph.D. degrees in Industrial Engineering and Management Sciences from Northwestern University. Please address correspondence to the address above or (e-mail) dennis-bricker@uiowa.edu.

1. INTRODUCTION

In machining economics problems, the primary objective in general is to determine the optimal cutting parameters which minimize production cost while satisfying certain design conditions. This is usually achieved by developing a mathematical programming model which puts restrictions on the choice of cutting parameters. Because of the complexity of many of these constraints, these models need efficient and robust methods for their solution.

A large number of methods or techniques have been suggested and evaluated for solving these problems. Duffuaa et al. [1] tested six methods to identify the most suitable method(s) for optimizing a set of machining economics models from the literature. The Generalized Reduced Gradient (GRG) algorithm, as implemented in Generalized Interactive Nonlinear Optimizer (GINO), emerged as the best method and, therefore, was recommended for use in solving this class of problems. However, the authors of that study dismissed too easily the usefulness of Geometric Programming (GP) techniques, which have often been utilized in solving such problems [2,3,4,5,6,7].

GP was originally developed by Duffin et al. [8] as a modeling method which would allow a more intimate analysis of all design variables and their relative importance to the overall design. A number of algorithms for optimizing GP models have been developed and improved (cf.[9,10]). The GP algorithm evaluated by Duffuaa et al. [1], namely the Gomtry algorithm of Blau, was judged to be not sufficiently robust to solve machining economics problems. The generalization that all GP algorithms lack robustness is, however, without justification. Our purpose in this present paper is to correct this misperception.

2. THE GEOMETRIC PROGRAMMING PROBLEM

The general primal problem of GP is to

$$\text{Minimize } y_0(x) = \sum_{t=1}^{T_0} \sigma_{0t}\, c_{0t} \prod_{n=1}^{N} x_n^{a_{0tn}} \tag{1}$$

$$\text{subject to } y_m(x) = \sum_{t=1}^{T_m} \sigma_{mt}\, c_{mt} \prod_{n=1}^{N} x_n^{a_{mtn}} \le \sigma_m,\ m=1,\cdots,M \tag{2}$$

$$x_n > 0,\ n=1,\ldots,N$$

where $\sigma_{mt} = \pm 1, \quad t=1,\cdots\cdots,T_m,\ \ m=0,\cdots,M$

$\sigma_m = \pm 1, \quad m=0,\cdots,M$

The exponents a_{mtn} are arbitrary real numbers, but the coefficients c_{mt} are assumed to be positive constants and the design variables x_n are required to be strictly positive. If the objective function and constraints of GP have only positive coefficients, i.e., σ_{mt} and σ_m are all +1, then the GP problem is referred to as a posynomial GP problem. The corresponding posynomial GP dual problem is to

$$\text{Maximize } v(\delta,\lambda) = \prod_{m=0}^{M} \prod_{t=1}^{T_m} \left(\frac{c_{mt}\lambda_m}{\delta_{mt}}\right)^{\delta_{mt}} \tag{3}$$

$$\text{subject to} \quad \sum_{m=0}^{M} \sum_{t=1}^{T_m} a_{mtn}\delta_{mt} = 0,\ n=1,\cdots,N \tag{4}$$

$$\lambda_m = \sum_{t=1}^{T_m} \delta_{mt},\ m=0,1,\cdots,M \tag{5}$$

$$\lambda_0 = 1 \tag{6}$$

$$\delta_{mt} \ge 0,\ t=1,\cdots,T_m,\ \ m=0,1,\cdots,M \tag{7}$$

This dual problem offers several computational advantages, primarily the fact that the constraints are linear. If an optimal dual solution (δ^*,λ^*) is known, then the following relationships may be used to compute a primal solution x* in nonpathological cases:

$$\delta^*_{mt}\, y_m(x^*) = \lambda^*_m\, c_{mt} \prod_{n=1}^{N} x_n^{*a_{mtn}},\ t=1,\cdots T_m,\ m=0,1,\cdots M \tag{8}$$

where $y_0(x^*)=v(\delta^*,\lambda^*)$ and, for m>0, $y_m(x^*)=1$ if $\lambda_m\neq 0$.

Note that, from these relationships, one may obtain a system of equations linear in the logarithms of the optimal values of the primal variables:

$$\sum_{n=1}^{N} a_{mtn} \ln x_n = \ln\left(\frac{\delta^*_{mt} y_m(x^*)}{\lambda^*_m c_{mt}}\right), \qquad m=0,\cdots,M \quad \text{such that } \lambda^*_m\neq 0 \text{ and } t=1,\cdots T_m \tag{9}$$

If one or more terms of the GP model have negative coefficients, then it is referred to as a signomial, or generalized, GP model. Signomial GP problems are best solved by a method analogous to sequential quadratic programming (SQP), but using a posynomial approximation to each of the signomial functions. This is accomplished by the condensation technique, first suggested by Duffin and Peterson [11], which approximates a posynomial function by a monomial, i.e., a single term. Each signomial function $y_m(x)$ may be expressed as the difference of two posynomial functions $y^+_m(x) - y^-_m(x)$. Thus, the signomial constraint $y_m(x) \le \sigma_m$ may be written, depending upon whether σ_m is positive or negative, as

$$\frac{y^+_m(x)}{1+y^-_m(x)} \le 1$$

or

$$\frac{1+y^+_m(x)}{y^-_m(x)} \le 1$$

respectively. By approximating the denominator by a monomial, the signomial constraint may then be approximated by a posynomial constraint. If the objective function $y_0(x)$ is not posynomial, then minimizing a new variable x_0 subject to a (signomial) constraint $x_0^{-1}\, y_0(x) \le 1$, yields a GP problem with posynomial objective function. Hence, a signomial GP problem may be approximated by a posynomial GP problem.

3. COMPUTATIONAL RESULTS

Five machining economics problems were used by Duffuaa et al.[1] and may be found there, together with references to their sources. (Note, however, the typographical error in the first problem which was corrected in [14,15].) These five problems have been again solved with a GP algorithm in order to compare the results with those of the GRG algorithm. The first problem, namely, that of Iwata et al., has an exponential function in its objective; this we have approximated by a posynomial function. All the problems are solved as posynomial GP problems, except that of Hati and Rao, which is a signomial GP and solved by the sequential posynomial GP method described above. The algorithm used to solve the posynomial GP problems is that of Bricker and Rajgopal [12], which is comparable to the well-known GGP algorithm of Dembo [13]. Both yield dual feasible solutions and employ as a stopping criterion the maximum permissible relative infeasibilities in the primal constraints and the duality gap in the objective.

Each problem was solved using each of the four initial points in [1]. For the purposes of comparison, Table 1 includes both the results from [1] for the GRG algorithm and the results of the current test for the GP algorithm. With the exception of the second problem, the results yielded by the GP algorithm are identical to those reported for GRG. (It is suspected that the objective function value 152.96 given by Duffuaa and Shuaib in [15] is in error, since the cutting parameters found by both methods appear identical.)

Finally, a comparison of problem 2 as given in [1] (and its correction in [15]) with the original problem as given by Hati and Rao [16] indicates a discrepancy in the exponent of the depth of cut d (treated as a constant, 2.5). Hati and Rao stated the objective function as

$$n\left(3.14159 \times 10^{3}\, V^{-1} f^{-1} + 2.87979 \times 10^{-8}\, V^{4} f^{0.75} d^{0.75} + 10\right)$$

Table 2 shows the results yielded by the GP algorithm for this original version of the problem. The optimal cutting parameters remain unchanged compared to the version of the problem used by Duffuaa et al., although the optimal cost has increased.

In summary, this computational test demonstrates that a geometric programming algorithm can yield results equally as good as the GRG algorithm, with the same reliability and insensitivity to starting point which was experienced by Duffuaa et al.

Table 1. Computational Results

Problem	Method	Input vector		Optimal solution				
		Speed	Feed	Speed	Feed	Cost	Depth	Passes
Iwata	GRG	190.0	0.230	216.08	0.388	108.03	2.0	1
		200.0	0.230	216.08	0.388	108.03	2.0	1
		190.0	0.320	216.08	0.388	108.03	2.0	1
		200.0	0.320	216.08	0.388	108.03	2.0	1
	GP	190.0	0.230	216.0	0.389	108.03	2.0	1
		200.0	0.230	216.0	0.389	108.03	2.0	1
		190.0	0.320	216.0	0.389	108.03	2.0	1
		200.0	0.320	216.0	0.389	108.03	2.0	1
Hati & Rao	GRG	146.0	0.375	148.22	0.3618	152.96	2.5	2
		196.0	0.375	148.22	0.3618	152.96	2.5	2
		146.0	0.575	148.22	0.3618	152.96	2.5	2
		196.0	0.575	148.22	0.3618	152.96	2.5	2
	GP	146.0	0.375	148.22	0.3618	149.85	2.5	2
		196.0	0.375	148.22	0.3618	149.85	2.5	2
		146.0	0.575	148.22	0.3618	149.85	2.5	2
		196.0	0.575	148.22	0.3618	149.85	2.5	2
Petropo -ulos	GRG	185.0	0.150	174.38	0.232	12.10	3.0	1
		215.0	0.150	174.38	0.232	12.10	3.0	1
		185.0	0.200	174.38	0.232	12.10	3.0	1
		215.0	0.200	174.38	0.232	12.10	3.0	1
	GP	185.0	0.150	174.39	0.232	12.10	3.0	1
		215.0	0.150	174.39	0.232	12.10	3.0	1
		185.0	0.200	174.39	0.232	12.10	3.0	1
		215.0	0.200	174.39	0.232	12.10	3.0	1
Ermer	GRG	135.0	0.0011	143.90	0.0014	6.26	0.2	1
		170.0	0.0011	143.90	0.0014	6.26	0.2	1
		135.0	0.0035	143.90	0.0014	6.26	0.2	1
		170.0	0.0035	143.90	0.0014	6.26	0.2	1
	GP	135.0	0.0011	143.90	0.0014	6.26	0.2	1
		170.0	0.0011	143.90	0.0014	6.26	0.2	1
		135.0	0.0035	143.90	0.0014	6.26	0.2	1
		170.0	0.0035	143.90	0.0014	6.26	0.2	1
Ermer & Kromod -ihardjo	GRG	320.0	0.0018	433.60	0.0038	1.553	0.2	1
		440.0	0.0018	433.60	0.0038	1.553	0.2	1
		320.0	0.0039	433.60	0.0038	1.553	0.2	1
		440.0	0.0039	433.60	0.0038	1.553	0.2	1
	GP	320.0	0.0018	433.60	0.0038	1.553	0.2	1
		440.0	0.0018	433.60	0.0038	1.553	0.2	1
		320.0	0.0039	433.60	0.0038	1.553	0.2	1
		440.0	0.0039	433.60	0.0038	1.553	0.2	1

Table 2. Hati & Rao's original problem

Input vector		Optimal solution				
Speed	Feed	Speed	Feed	Cost	Depth	Passes
146.0	0.375	148.22	0.3618	162.95	2.5	2
196.0	0.375	148.22	0.3618	162.95	2.5	2
146.0	0.575	148.22	0.3618	162.95	2.5	2
196.0	0.575	148.22	0.3618	162.95	2.5	2

REFERENCES

1. S. O. Duffuaa, A. N. Shuaib and M. Alam. Evaluation of optimization methods for machining economics models. *Computers Ops Res.* **20** 227-237 (1993)
2. P. Petropoulos. Optimal selection of machining rate variables by geometric programming. *Int. J. Prod. Res*. **11**, 305-314 (1973)
3. D. S. Ermer. Optimization of the constrained machining economics problem by geometric programming. *Trans. ASME*. **93**, 1067-1072 (1971)]
4. D. S. Ermer and S. Kromodihardjo. Optimization of multipass turning with conatraints. *J. Engng Indust. Trans. ASME*. **103**, 462-468 (1981)
5. A. G. Walvekar and B. K. Lambert. An application of geometric programming to machining variable selection. *Int. J. Prod. Res*. **8**, 241-245 (1969)
6. B. K. Lambert and A. G. Walveker. Optimization of multi-pass machining operations. *Int. J. Prod. Res.* **16**, 259-265 (1978)
7. K. Unklesbay and D. L. Creighton. The optimization of multi-pass machining processes. *Engng Optimiz.* **3**, 229-238 (1978)
8. R. J. Duffin, E. L. Peterson and C. M. Zener. *Geometric Programming*, John Wiley & Sons, New York. (1967)
9. J.G. Ecker. Geometric programming, in R.R. Levary (ed.), *Engineering Design: Better Results through Operations Research Methods.* North-Holland, Amsterdam (1988)
10. M.J. Rijckaert and E.J.C. Walraven. Reflections on geometric programming, in K. Shittkowski (ed.), *Computational Mathematical Programming.* Springer-Verlag, Berlin-Heidelberg (1985)
11. R. J. Duffin and E. L. Peterson. Duality theory for geometric programming. *SIAM. J. Appl. Math.* **14**, (1966)

12. D. L. Bricker and J. Rajgopal. Yet another geometric programming dual algorithm, *Operations Research Letters,* **2**, 177-180 (1982)
13. M. Avriel., R. S. Dembo and U. Passy. Solution of generalized geometric progrrams, *International Journal of Numerical Methods in Engineering,* **9**, 149-168 (1975)
14. H. Sarper. Comments and corrections for the machine economics optimization article by Duffuaa, Shuaib and Alam, *Computers and Operations Research,* **22**, 247-248 (1995)
15. S.O. Duffuaa and A.N. Shuaib. Rejoinder to Sarper's note on the machine economics optimization article by Duffuaa, Shuaib and Alam, *Computers and Operations Research,* **22**, 249-250 (1995)
16. S.K. Hati and S.S. Rao. Determination of machining conditions --probabilistic and deterministic approaches. *Journal of Engineering for Industry, Transactions of ASME,* 354-359 (1976)