Diversity of Theories in Mathematics Education – How can we deal with it?

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Abstract: This article discusses the central question of how to deal with the diversity and the richness of existing theories in mathematics education research. To do this, we propose ways to structure building and discussing theories and we contrast the demand for integrating theories with the idea of networking theories.

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In their introductory article for the ZDM volumes on theories Sriraman & English (2005 gave an impression of the diversified field of theories in mathematics education. Starting from the often repeated criticism of the discipline's lack of focus, its diverging theoretical perspectives, and a continued identity crisis (Steen, 1999), the editors call for the ambitious project to "take stock of the multiple and widely diverging mathematical theories, and chart possible courses for the future (Sriraman & English 2005, p. 450). These two ZDM volumes are supposed to "re-initiate the discussion on the critical role of theories for the future of our field" (ibid, p. 451).

Our contribution to this discussion is influenced by the working group on theoretical perspectives in mathematics education at the Fourth Congress of the European Society for Research in Mathematics Education in Spain 2005 (see Dreyfus et al., 2006). In certain aspects, this article conveys a European perspective to the discussion. So, what *is* the critical role of theories in our field? The *general* role of theories is perhaps not that critical: There seems to be a certain consensus in the scientific discipline that theories are very important on many different levels. First of all, all observation is theory laden. That means we need theories because they provide the language to interpret and talk about empirical phenomena and to draw consequences for practice. As a scientific discipline we use theories for choosing research questions, for choosing adequate methods and especially for the way in which results are interpreted and later integrated into greater argumentative contexts.

Bishop characterises theory as an outcome of research and theorizing as its essential goal:

> "It [theory] is the way in which we represent the knowledge and understanding that comes from any particular research study. Theory is the essential product of the research activities, and theorizing, therefore, its essential goal." (Bishop 1992, p. 711)

For us, the most central question is not to ask *whether* we need theory or *which* theory we need. Instead, we ask *how to deal* with the diversity of manifold, partly overlapping and partly contradictory theories and the connected diversity of conceptual descriptions for similar phenomena. We have approached this question through several steps:

- How can theories be distinguished?
- Why do there exist different theories?
- What does it mean to elaborate theories further?
- How should we deal with the diversity of theories?

How can theories be distinguished?

The disciplinary discourse about theories suffers from the fact that the term "theory" is used in very different ways. Mason & Waywood (1996) distinguished three different senses of the term theory: theory is

> - "an organised system of accepted knowledge that applies in a variety of circumstances to explain a specific set of phenomena as in 'true in fact and theory';
> - an hypothesis, or possibility such as a concept that is not yet verified but that if true would explain certain facts or phenomena, as in 'he proposed a fresh theory of alkalis that later was accepted in chemical practices';
> - a belief that can guide behaviour, as in the 'architect has a theory that more is less' […]
>
> What is common in the use of the word 'theory' is the human enterprise of making sense, in providing answers to people's questions about why, how, what. How that sense-making arises is itself the subject of theorizing.
> To understand the role of theory in a research program is to understand what are taken to be the things that can be questioned and what counts as an answer to that questioning."
> (Mason & Waywood 1996, pp. 1055)

On the basis of this first account for theory, Mason and Waywood distinguish between different characters of theories: foreground theories, background theories and implicit theories. Since the distinction has turned out to be more useful for a discussion about theories than the search for an exact definition, it shall be recalled here:

Foreground theories are mostly local theories *in* mathematics education:

> "The process of asking and answering questions gives rise to explicit theorizing in the second sense of hypothesizing. We will call this sense foreground theory because the foreground aim of most mathematics education about what does and can happen within and without educational institutions." (ibid., p. 1056)

In contrast, *background theory* is a (mostly) consistent philosophical stance *of* or *about* mathematics education which

> "plays an important role in discerning and defining what kind of objects are to be studied, indeed, theoretical constructs act to bring these objects into being. Background theories encompass an object, aims, and goals of research including what constitutes a researchable question, methods, and situation as perceived by he researcher." (Mason & Waywood 1996, p. 1058).

Most theories about and within mathematics education share their *research object* up to a certain point: it is about aspects of mathematics teaching and learning. But they differ in the *situations* that are considered, what exactly in these situations is theoretically conceptualized, in the *methods* that are used for generating results for theory building and in their *aims* (cf. Figure 1).

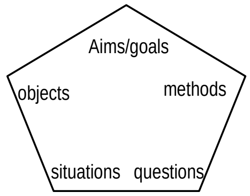

Figure 1: Aspects of research and theory

Since we do not, for example, necessarily count the methods as a part of the background theory, we prefer here the term aspects of research and theory. To the four aspects of research (aims/goals, objects, methods, situations), given by Mason & Waywood (1996), we added also the *sort of questions considered to be relevant*, since we know from higher education research and comparative research about scientific cultures that the questions which are considered to be relevant form an important part of the scientific culture of each research group and community (cf. Arnold & Fischer, 2004).

Beyond these research and theory aspects, every theory is based on epistemological and methodological ideas as well as on philosophical ideas about the nature and aim of education, the nature of mathematics and the nature of mathematics education. Since these "philosophies of mathematics education" are often implicit, Mason & Waywood classify them as *implicit theories* and emphasis that they need not necessarily form a consistent corpus of beliefs. It is exactly this incompleteness and possible inconsistency that leads us to chose the term 'ideas' instead of 'theories' for the fragments of the philosophical base. For the ensemble of ideas, we prefer the term *philosophical base* and consider it as a possible part of the background theory which ought to become explicit.

The background theory and its philosophical base are deeply interwoven. For example, the choice of the theoretical perspective on an object and the observed situation influences the aims, the posed questions and the activated methods. Vice versa, the objects, situations, aims, questions, and methods, often suggest a certain theoretical perspective. Furthermore, the different perspectives are deeply connected with epistemological and methodological points of view which shape an integral part of the philosophical base. Adopting the perspective of social constructivism on mathematical knowledge, for example, is usually based on the idea that knowledge is socially constituted (e.g. Krummheuer, 1995). In contrast, adopting a constructivist perspective starts from the philosophical idea that knowledge is mainly individually constructed (e.g. Harel & Lim, 2004). Using a semiotic perspective on mathematical knowledge normally implies that mathematical knowledge is mediated by signs regarded from the view of a special semiotic concept of signs (e.g. Hoffmann & Roth, 2004). Obviously, this distinction between three different perspectives (the social constructivist, the constructivist and the semiotic perspective) is not complete; there are also cultural, socio-cultural, institutional, systemic, mathematical, epistemological, psychological, social, individual and other perspectives. Perspectives can be used on different levels of specificity and they may overlap, as visualized in Figure 2.[1]

Now, what exactly is meant by the term perspective? The Latin word 'perspicere' means to look through something or to have a clear view of something. Therefore, adopting a special perspective means viewing something in a special way or from a special standpoint. Thus, if we take a social perspective, then we look at a situation as a social event. Adopting a perspective means directing the view towards a thing in a special way. However, if we use a special theoretical perspective then we watch something from a special theoretical angle: An interactionist perspective informs us that some kind of interactionism is taken as background theory.

An interesting example of the importance of perspectives is given by Even & Schwarz (2003): The researchers have analyzed the same data from two different theoretical perspectives. They show that adopting a special theoretical perspective means posing special questions which only make sense within their background theories. The kind of question is interwoven with a special view of the situation. Changing the theoretical perspective can mean changing the research question and the research object at the same time.

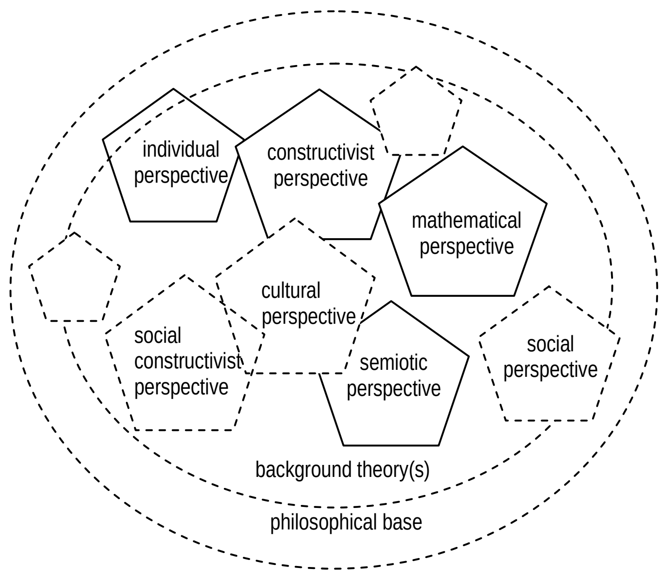

Figure 2: Variety of perspectives

The philosophical base is *not necessarily implicit*, on the contrary, researchers should always aim at making their implicit philosophical bases as explicit as possible. This claim is not new; it is for example basic for the discussion in the field of philosophy of mathematics educa-

[1] The diagram in Figure 2 visualizes these manifold perspectives. A current research project might focus on some of them, not on all. Some perspectives overlap or penetrate each other, some are explicitly situated within a background theory and others still have implicit parts. There are probably important perspectives that remain unnamed. This variety of perspectives is regarded with respect to their background theory(ies). Limitations of background theories and philosophical bases are considered to be permeable.

tion (see e.g. Ernest, Prediger & Viggiani-Bicudo, 2006). The process of making explicit the implicit base of the background theory is one important way of developing theories to maturity (see below). Then, many aspects of the philosophical base can become a part of the background theory itself by making and justifying them explicitly.

On the other hand, we agree with Mason & Waywood about the important influence of researchers' subjective, often *implicit theories* (or theory fragments, respectively). However, the attention to implicit theory fragments should be given to *all aspects* of subjective theories, and this comprises not only philosophical concerns but also for example beliefs about the worth of theories, aims, or values. These subjective theories of researchers and the impact on their research would constitute an interesting object of empirical research (e.g. within the research program 'Subjective Theories', proposed by Groeben et al., 1988) since up to now, the discourse on implicit theories of mathematics education researchers is more philosophically than empirically based.

The distinction between foreground theory and background theory given by Mason & Waywood (1996) is, in our view, not an objective or time independent criterion for characterising theories. In contrast, the status of some parts of the theory can change from foreground to background theory or vice versa within the research process.

Over and above that, the distinction between foreground and background theory is a helpful conceptual tool for characterizing and comparing different theories and their functions in research processes, since we can characterize foreground theories with respect to important aspects of their respective background theories. At first sight, foreground theories can be distinguished with respect to their objects according to size (such as individuals or groups), functions (such as supporting learning or hindering learning), kind (such as cognitive processes, social processes, signs, mistakes), etc. But in order to understand the roots of these differences, a second look at the background theory is necessary. This might reveal that different kinds of questions are considered to be important, like questions related to the difference between change and stability orientation for theories on students' learning (see Ulich 1976), or that fundamentally different aims guide the research processes, like supporting students' concept building on the one hand and describing students' concept building in every day classes on the other.

With respect to the aims of a research process and a theory, we can distinguish between *descriptive*, *explanatory*, and *prescriptive theories*. This goes back to a classical distinction introduced by Dilthey (1883) in the humanities between explaining (as an analysis of causal relations) and understanding (as a dense description and the specification of sense, e.g. from the actor's inner perspective). We distinguish between theories that aim at elucidating phenomena in their exact conditions and functions (*explain how*) or causes (*explain why*) and theories which intend more cautiously to analyze complex phenomena in their connections and to describe by means of a well-defined corpus of concepts (see Schoenfeld's 2002 standards for judging theories). Whereas explanatory as well as descriptive theories aim to analyse empirical phenomena, prescriptive theories formulate propositions on how reality *should* be. For example, Heymann's (2003) theory on general education integrates empirical results, but mainly it is a normative theory about what students should learn in mathematics classrooms. Many didactical approaches which consider how learning environments should be designed have a prescriptive character. However, these different aims cannot offer a disjoint classification but a typing that respects the fact that theories can have explanatory, descriptive and predictive parts.

Why do different theories exist?

Wherever discourse about the diversity of theories takes place, the question is raised: Why is the marketplace of theories so diversified?

> "One plausible explanation for the presence of multiple theories of mathematical learning is the diverging, epistemological perspectives about what constitutes mathematical knowledge. Another possible explanation is that mathematics education, unlike 'pure' disciplines in the sciences, is heavily influenced by cultural, social, and political forces." (English & Sriraman 2005, p. 452)

The European example shows that both aspects are heavily interwoven, since it is exactly in the background theories and the diverging philosophical bases that the traditions of different national or regional communities differ (see Dreyfus et al., 2006 and Ernest, Prediger & Viggiani-Bicudo, 2006). In addition to these regional traditional differences, there have existed different priorities in making explicit the foreground theories, the background theories and their philosophical bases, and very different degrees of being explicit about each part.

This complex diversity is visible in the (often criticized) existence of many different and partly incompatible theories. In contrast to this criticism, we do not consider this diversity to be a defect but a special characteristic of our field of research: *Diversity is richness!* When we understand it as an indicator for the complexity and transdisciplinary character of our research objects, we should not aim to reduce this complexity. Instead, the richness gained should be better exploited (see below).

What does it mean to elaborate theories further?

Sriraman & English (2005, e.g. p. 453) discuss the claim that theories in mathematics education should be further developed. The question is, exactly does it mean to develop theories further? This depends on the theory's character, since explanative and descriptive theories develop differently from prescriptive theories.

Empirically grounded theories develop in a spiral process of empirical analysis and theory construction. For example, Bikner-Ahsbahs (2005) begins the development of the theory about interest-dense situations with a first conceptual component in the context of background theories. Then, the analysis of data leads to a first hypothesis which can again be tested through analysing data. New hypotheses are generated, etc. In this spiral process between theory development and empirical analysis and testing, non-consistent components are systematically sorted out. In this sequential chain of construction steps, we have loss and gains: Whereas information about the research objects is lost, theoretical propositions are successively won.

In contrast to such processes of empirically based theory building, the development of prescriptive theories e.g., Prediger, 2004) is characterized more by argumentative connections to other theory elements and by the successive process of making explicit the philosophical base. Nevertheless, empirical correspondences and relevance for classrooms practices play an important role as criteria of relevance and acceptance.

Beyond these *ways* of theory development, there arises the question of *directions* in which theories can or should develop. In our view, this question should be discussed in the communities' discourse on theory. We consider at least three directions to be important:

- *empirical scope*: Formal foreground theories have a large empirical scope. They characterize empirical phenomena in a global way and often cannot exactly be concretised through empirical examples (Lamnek 1995, p. 123). On the other hand, local and contextualized theories have a limited scope but their statements can be made concrete by the empirical content more easily (see Krummheuer 2001, p. 199). This proximity to empirical phenomena leads Krummheuer to consider contextualized theories as a suitable background to guide practice in schools. However, developing local theories in order to enlarge their empirical scope can be an important direction of theory development.
- *degree of maturity*: Starting from the (above developed) claim that a good theory should make its background theories and its underlying philosophical base (especially its epistemological and methodological foundations) as explicit as possible, the maturity of a theory can be measured by the degree of its explicitness: The more implicit suppositions are explicitly stated and the more parts of the philosophical base form explicit parts of the background theory, the more we would consider the theory to be mature.
- *connectivity:* Science is characterized by argumentation and interconnectedness, as Fischer (e.g. 1993) emphasizes. This can for example be realized by establishing relationships to linked theories, by declaring communalities and differences. Hence, establishing argumentative connectivity is another important direction for the development of theories.

The growing discourse on theory within the scientific community deals already with the question whether there are more directions of development and whether we can formulate standards for degrees of theory development. Schoenfeld (2002) proposes eight standards for evaluating theories: descriptive power; explanative power; scope; predictive power; rigor and specificity; replicability, generality, and trustworthiness; and multiple sources of evidence (triangulation). Not all these standards fit for every theory, but they can give some orientation in what other directions theories could be further developed.

How to develop theories further is not only an isolated question guiding separate research pathways. It is fundamentally interwoven with the question of how the research community as a whole, with its manifold different theories, can develop further. This second question is far from being clear. We consider it to be a crucial point for our discipline.

How should we deal with the diversity of theories?

On the basis of accepting the large variety of different theoretical frameworks as an important *resource*, not as a defect, the CERME working group discussed the question of how to deal with the divergence of theories. Different strategies were proposed and discussed (see Dreyfus et al., 2006). We will further elaborate the following ideas here:

- unifying
- integrating
- competing and comparing
- networking

A classical answer to the question of how to cope with the diversity of theories is the demand for *unifying* theories. This can be very fruitful for local theories which deal with the same phenomena and equal background theories but use diverging conceptual systems for describing the same phenomena. In such a case, it is helpful to unify the theories in order to overcome the unnecessary diversity of similar approaches. On the other hand, the fact that many other theories operate with contradicting background theories and incompatible philosophical bases shows that it is neither possible nor desirable to unify *all* theories. Unifying all theories would only be possible if there existed one common base and one common perspective. But one single base for the whole discipline would not be adequate due to the complexity of the research objects.

Integrating is our name for a promising strategy that does not aim at one unified theory but tries instead to combine a small number of local theories with compatible (but not necessarily equal) background theories and perspectives. One convincing example is Steinbring's approach of integrating an epistemological perspective and a social perspective on the development of building mathematical concepts (Steinbring, 2004). This strategy can help to exploit the diversified field in a productive way. In contrast, it would be dangerous to combine different parts of incompatible theories into arbitrary patchwork-theories. Especially when background theories contradict, there is a danger of building inconsistent theoretical parts without a coherent philosophical base. While integrating theories, it is necessary to check the compatibility of the background theories and to make explicit how compatibility can be characterized.

In order to take the complexity and transdiciplinarity of our field into account seriously, the strategies *competing* and *comparing* can be interesting. For example, by treating the same set of data from the basis of different theories, similarities and differences of theories can be specified although the diversity is respected. One example for this approach is given by Even & Schwarz (2003). They investigated the same data of a classroom situation from a cognitive perspective on the one hand and from the perspective of activity theory on the other. They wanted to make clear what happened in an unsuccessful learning process and why. Analyses from a cognitive perspective showed that for the students "to use multiple representations to solve [this] problem is a cognitive obstacle…" (Even & Schwarz 2003, p. 295). Analyses of the activities and motives and goals of the activities shed light on the situation from another view: the students and the teacher participated in different activities or in other words, the "students' ways of participating in the lesson were different from what the teacher had wished for" (Even & Schwarz 2003, p. 308). Although working on the same data with the same aim the two researchers' analyses were guided by two different research questions and therefore had two different complementary outcomes. Even and Schwarz came to the conclusion that the classroom situation "is too complex to be understood only from one perspective" (Even & Schwarz 2003, p. 309). This example shows how the strategy can deepen our understanding of theories with respect to their advantages and disadvantages.

The CERME working group on theories (Dreyfus et al. 2006) considered the *networking* strategy to be the perhaps most promising strategy. The main idea of networking is to exploit the diversity of approaches constructively by first analyzing the same phenomena from complementary perspectives. This allows *comparing and competing* as well as *integrating* local theories. The complementary or integrative understanding of a phenomenon generates a deeper or more comprising understanding of the phenomenon, especially when more than two theories are involved.

To understand what networking theories might mean, let us have a look at the analyses of Even and Schwarz (2003) again. We take this research project as a starting point to explain what a networking strategy could mean beyond comparing and competing. Following a networking strategy, we would try to complete our understanding more and more by using additional perspectives. An interactionist perspective probably would show us what kind of interactional patterns made this missing fit of the activities possible. A semiotic analysis might show us something about the process of sign building and its epistemological problems and barriers. A conflict theory might tell us something about the underlying conflict in the class. An exchange theory probably would give information about benefit and cost: What are the benefits and the costs the teacher and the student have by keeping this situation going. Step by step we would complete and deepen our understanding of the empirical situation.

Initially, such a multi-perspective analysis would lead to complementary descriptions of the same situation and, in this way, give rise to new questions: Is the cognitive problem of the students in the class a reason for the genesis of the different kinds of activities the students and the teacher participate in or vice versa? Are the results Even and Schwarz observed an outcome of mutual interdependence? Is there a perspective which might help us to answer these questions? Would an interactionist analysis together with an analysis about benefit and cost be suitable? These questions show that networking is more than just integrating or comparing.

Even if we do not get satisfying simple answers about special situations, we could gain a deeper understanding of the mutual interdependences within the lesson and we probably might learn something about the impact of small changes on processes of teaching and learning.

But we also might learn more. Using a networking strategy can not really be done by researchers who come from a single research culture. Working on networking theories means building networks of research groups as well. In order to be understood, researchers from different cultures will be forced to make their philosophical bases more and more explicit. Hence, working on networking theories will help us to develop our own theories and, hopefully, will lead to the development of kinds of theories which fit the complexity of our research object better then before.

Conclusion

The European tradition in mathematics education research is affected by a large diversity of theories and theoretical frameworks. *Our strategy is to exploit this diversity as an eminently fruitful source for the development of a disciplinary identity*. We consider it as an indicator for the necessity of a particular way of theory-building-development that takes the complexity of our research objects into account seriously. Concretely, strategies of comparing, competing, networking, or integrating are helpful tools to exploit this diversity.

In consequence, we need a meta-theory, a method, a methodological principle or a local strategy for particular theory constructions that is appropriate to the complexity of our research objects. On the practical side, we need the concrete cooperation of different research groups with diverging traditions and the common aim of working on such a meta-theory. We have the vision that many researchers in our field might work on networking theories in mathematics education (see e.g. the Enactivist Research Group, Reid, 1996), that can be applied to local phenomena and allow us to get a deeper understanding of empirical learning processes. However, we do not know how these networks might look, how they can be (re-)constructed and how they can be developed further.

Nevertheless, we are sure that the discourse on adequate ways of theory-building can push the mathematics education research community forward and can help to distinguish it from other "neighbour disciplines".

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