Full polarization measurement of synchrotron radiation with use of soft x-ray multilayers

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Using two Ru/Si multilayers as a phase shifter and an analyzer, we have measured the state of polarization for 12.8-nm synchrotron radiation (SR) of the beam line 11A at the Photon Factory. It has been found that the state of polarization depends largely on the vertical inclination angle of the first mirror of the beam line. From the phase information, we have determined parameters of the polarization ellipse including handedness.

I. INTRODUCTION

Spectroscopic investigations with use of circularly polarized synchrotron radiation (SR), as well as linearly polarized SR, have attracted much interest in recent years to research into the magnetic properties of materials. As circularly polarized SR, off-axis SR[1] from a bending magnet or SR from a special insertion device[2] is provided. It is quite important for the experiments to know the state of polarization of the emergent light from a monochromator.

Suitable phase shifters for polarization measurements in the vacuum ultraviolet (VUV) region have not been developed so far. Heinzmann[1] measured the degree of linear polarization (P_L) of SR of wavelength 40–100 nm at BESSY using a conventional VUV polarizer (analyzer) which was composed of three or four Au-coated mirrors. He calculated the degree of circular polarization (P_C) on the assumption $P_L^2+P_C^2=1$. Schledermann *et al.*[3] proposed to use a conventional VUV polarizer as a phase shifter by utilizing its retardation upon reflections. Gaupp *et al.*[4] and Koide *et al.*[5] carried out polarization measurements using such VUV polarizers at a wavelength of 80 nm and in a range of 15.5–49.6 nm, respectively. By measuring the intensity of transmitted light for several independent combinations of azimuthal angles of a phase shifter and an analyzer, they determined the state of polarization of light including the degree of polarization.

However, it is very difficult to do polarization measurement by the use of the conventional VUV polarizers in the soft x-ray (SXR) region, because their throughput and polarizance (degree of polarizing power) become poor for SXR. For example, throughput and polarizance of the reflectance polarizer are 5% and 0.85, respectively, for light of wavelength 30 nm,[5] and has no more effect in the SXR region.

Recently, multilayer mirrors for SXR have been extensively developed. Their application to SXR polarizers has been proposed,[6] because they have high throughput and high polarizance in the vicinity of an angle of incidence of 45° for SXR. For example, throughput and polarizance of our Ru/Si multilayer are ~60% and 0.97 for light of wavelength 12.8 nm,[7] respectively. Accuracy in the polarization measurement depends largely on the polarizance of the analyzer. Therefore, multilayer polarizers would provide more accurate results in the SXR region than conventional polarizers. Gluskin *et al.*[8] constructed a SXR polarimeter with multilayer mirrors and tested it with SR from the VEPP-2M ring. Khandar and Dhez[9] estimated P_L of SR of wavelength 15.4 nm from the ACO ring to be 0.73 using a Hf/Si multilayer polarizer. Kimura *et al.*[7] evaluated P_L of SR of wavelength 12.8 nm emergent from a 2-m grasshopper monochromator at the Photon Factory using a Ru/Si multilayer analyzer.

In order to realize a full polarization measurement in the SXR region, we have made a first attempt at polarization measurement for SR from the grasshopper monochromator using two multilayers, one for a phase shifter and the other for an analyzer. We have evaluated the ellipticity angle including the handedness and the azimuth of the major axis of the polarization ellipse. In this paper we describe the analytical method in Sec. II and the measurement and the results in the following sections.

II. ANALYTICAL METHOD

We will present here the aspect of a theory based on the Stokes vector and the Mueller matrix formalism, and will transform the results of the state of polarization to parameters of polarization ellipse for easy understanding.

Figure 1 schematically shows the side view of experimental arrangement in our polarization measurement, where P is a phase shifter and A and D is an analyzer-detector assembly, both are installed in a vacuum chamber. The chamber and the analyzer assembly A are rotatable about the incident beam from a monochromator and the reflected beam from P, respectively. The azimuthal angles χ of P and η of A are measured counterclockwise from the respective positions shown in Fig. 1.

The Stokes vector $S'(\chi,\eta)$ of the light reflected by P and A and reaching D is a function of χ and η. It is expressed using Mueller matrices as

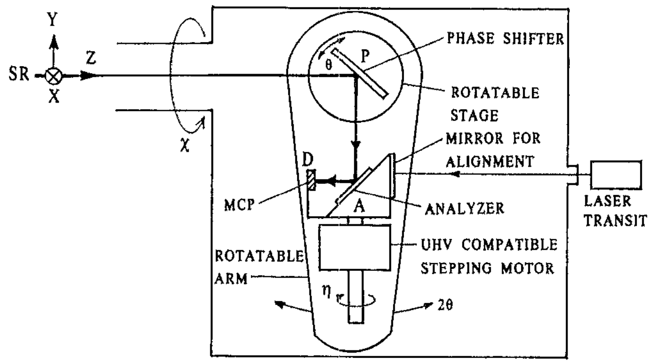

FIG. 1. Side view of the setup of the polarization measurement system.

$$\begin{aligned} S'(\chi,\eta) &= [M_A(0,\beta,\Delta')R(-\eta)R(-\chi)] \\ &\quad\times[R(\chi)M_P(0,\alpha,\Delta)R(-\chi)]S \\ &= M_A(0,\beta,\Delta')R(-\eta)M_P(0,\alpha,\Delta)R(-\chi)S \\ &= M_A(\eta,\beta,\Delta')M_P(\chi,\alpha,\Delta)S, \end{aligned} \tag{1}$$

where $M_P(\chi,\alpha,\Delta)$ is a Mueller matrix of P at χ:

$$M_P(\chi,\alpha,\Delta) = \frac{r_p^2}{2}\begin{pmatrix} \alpha^2+1 & \alpha^2-1 & 0 & 0 \\ \alpha^2-1 & \alpha^2+1 & 0 & 0 \\ 0 & 0 & 2\alpha\cos\Delta & -2\alpha\sin\Delta \\ 0 & 0 & 2\alpha\sin\Delta & 2\alpha\cos\Delta \end{pmatrix} \cdot R(-\chi),$$

and α is a ratio of amplitude reflectance of P for the s to p component, r_s/r_p, Δ is a retardation angle upon reflection for the two components, $\Delta_p-\Delta_s$. R is a Mueller matrix of an azimuth rotator:

$$R(\theta)=\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos 2\theta & -\sin 2\theta & 0 \\ 0 & \sin 2\theta & \cos 2\theta & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

$M_A(\eta,\beta,\Delta')$ is a Mueller matrix of A at η, $\beta = r_s'/r_p'$ where $r_s'(r_p')$ is an amplitude reflectance of A for the s (p) component, and $\Delta' = \Delta_p' - \Delta_s'$. S stands for the Stokes vector of the incident light,

$$S=\begin{pmatrix} S_0 \\ S_1 \\ S_2 \\ S_3 \end{pmatrix}.$$

The intensity $I(\chi,\eta)$ of the transmitted light is derived from Eq. (1),

$$I(\chi,\eta)\equiv S_0'(\chi,\eta)=\frac{r_p^2 r_p'^2}{4}[A\cdot\cos 2\eta + B\cdot\sin 2\eta + C], \tag{2}$$

where

$$\begin{aligned} A &= (\beta^2-1)[S_0(\alpha^2-1)+S_1(\alpha^2+1)\cos 2\chi \\ &\quad + S_2(\alpha^2+1)\sin 2\chi], \\ B &= 2\alpha(\beta^2-1)[-S_1\cdot\cos\Delta\cdot\sin 2\chi + S_2\cos\Delta\cdot\cos 2\chi \\ &\quad - S_3\sin\Delta], \\ C &= (\beta^2+1)[S_0(\alpha^2+1)+S_1(\alpha^2-1)\cos 2\chi \\ &\quad + S_2(\alpha^2-1)\sin 2\chi]. \end{aligned}$$

Substituting the equations

$$A=\rho\cos 2\tau, \quad B=\rho\sin 2\tau$$

into Eq. (2), we obtain a formula with a form showing the Malus law

$$I(\chi,\eta)=2\rho\cos^2(\eta-\tau)+C-\rho. \tag{3}$$

For a fixed azimuth χ^* of P, $I(\chi^*,\eta)$ has a minimum at $\eta=\eta^*$ satisfying the condition

$$\eta^*-\tau=(2m+1)\pi/2 \quad \text{for } m=\text{integer}.$$

At these minimum intensities, we have the relation

$$\tan 2\eta^* = \tan 2\tau = B/A = \frac{2\alpha(-S_1\cos\Delta\cdot\sin 2\chi^* + S_2\cos\Delta\cdot\cos 2\chi^* - S_3\sin\Delta}{S_0(\alpha^2-1)+S_1(\alpha^2+1)\cos 2\chi^* + S_2(\alpha^2+1)\sin 2\chi^*}. \tag{4}$$

The normalized Stokes parameters can be written with parameters of polarization ellipse and the degree of polarization V as

$$\begin{aligned} &S_0=1, \quad S_1=V\cdot\cos 2\chi\cdot\cos 2\delta, \\ &S_2=V\cdot\cos 2\epsilon\cdot\sin 2\delta, \quad S_3=V\cdot\sin 2\epsilon, \end{aligned} \tag{5}$$

where ϵ is an ellipticity angle ($\epsilon>0$ for right-handed, $\epsilon<0$ for left-handed), and δ is an azimuthal angle of the major axis of the polarization ellipse measured counterclockwise from the x axis as shown in Fig. 1.

Substituting Eqs. (5) into Eq. (4), we obtain

$$\tan 2\eta^* = \frac{2\alpha V[\cos 2\epsilon\cdot\cos\Delta\cdot\sin 2(\delta-\chi^*) - \sin 2\epsilon\cdot\sin\Delta]}{\alpha^2-1+V(\alpha^2+1)\cos 2\epsilon\cdot\cos 2(\delta-\chi^*)}. \tag{6}$$

By fitting Eq. (6) by means of a least-squares method to the measured data set of (χ^*,η^*), we can finally determine the parameters V, η, and δ for the incident SR and α and Δ for the phase-shifting polarizer.

In this method we do not need accurate intensities of the transmitted light but azimuthal angles where the intensity takes a minimum. Therefore, instability in incident beam intensity or nonuniformity in the sensitivity over the detector cathode does not have much influence on the accuracy of our method. Moreover, a polarization-sensitive characteristic of the detector does not have much effect on this measurement, because the detector rotates together with the analyzer receiving the constant polarization.

III. EXPERIMENT

The polarization measurement unit was installed in the chamber of the apparatus for optical elements characterization.[10] They were arranged downstream from the 2-m grasshopper monochromator on the beam line 11A at the Photon Factory (see Fig. 2). We used 21-layer Ru/Si (3.95 nm/5.40 nm) multilayer mirrors for P and A. A microchannel plate (MCP) was used for D. We mounted them on a θ–2θ stage at $\sim 45°$ angle of incidence (Fig. 1). The polarization measurement was made for SXR of 12.8 nm. The azimuth χ of the chamber was set manually in a range of rotation of $-5° < \chi < 105°$. At each azimuth χ^* thus chosen, the analyzer A was rotated by a computer-controlled UHV-compatible stepping motor to obtain a full 360° data for Eq. (3), which gives accurate values of η^*. The η drive was always referred to a home position detected by means of autocollimation.

The observation angle for the emitted SR depends largely on the position of the first mirror M_0 of the beam line. That is, if the M_0 mirror is inclined upward, we are able to observe SR emitted under the plane parallel to the positron orbit. We, therefore, studied the polarization state by varying the vertical inclination angle ϕ of M_0, whose original point was arbitrarily chosen. Data sets of χ^* and η^* were measured for $\phi = -0.25°$, 0°, 0.25°, and 0.45°. At each position we realigned the beam line optics to optimize the throughput by adjusting the horizontal tilt of M_0 and the vertical tilt of the first mirror of the monochromator.

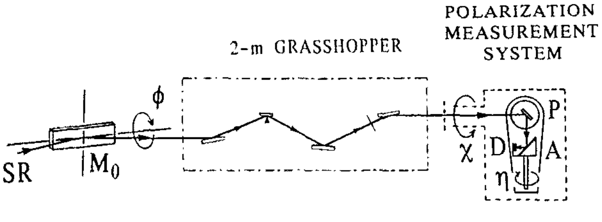

FIG. 2. Schematic diagram of the polarization measurement on BL 11A at the Photon Factory.

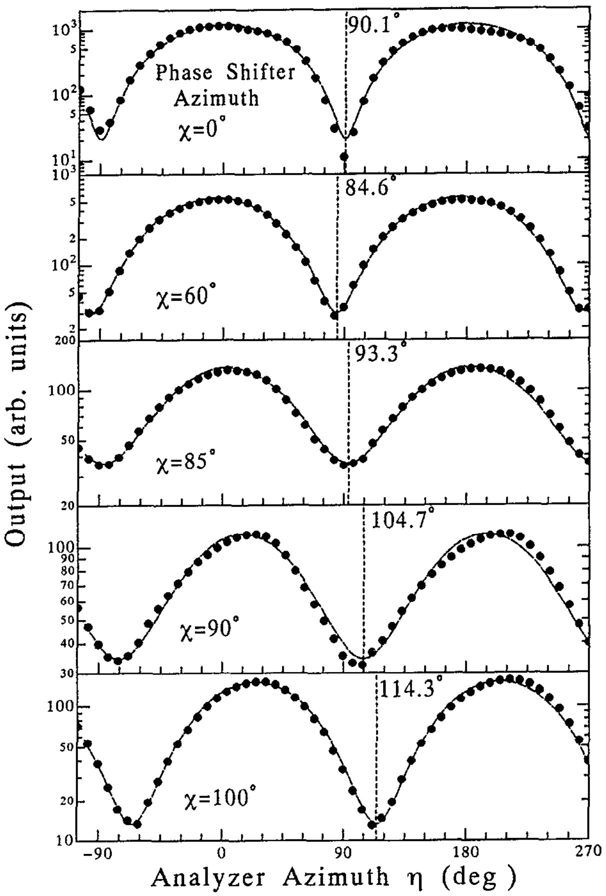

FIG. 3. Typical examples of detector outputs while keeping phase shifter at several angles, and inclination angle of M_0: $\phi = 0.25°$. The solid circles show the measured data and curves show the result of fitting. Dashed lines and values show minimum position.

Figure 3 shows typical examples of MCP output obtained by rotating A while keeping P at several fixed angles χ^* plotted in logarithmic scale. The data were taken at an M_0 attitude of $\phi = 0.25°$. Fitting $I = a \times \cos^2(\eta - \eta^*) + b$ [refer to Eq. (3)] to these data, we determine the azimuthal angles η^*, at which the intensity has a minimum value. The values of η^* are plotted against χ for individual inclination angles ϕ_0, as shown in Fig. 4. As described in Sec. II, we could determine parameters V, ϵ, δ, α, and Δ by fitting Eq. (6) to these data. However, the fitting was not

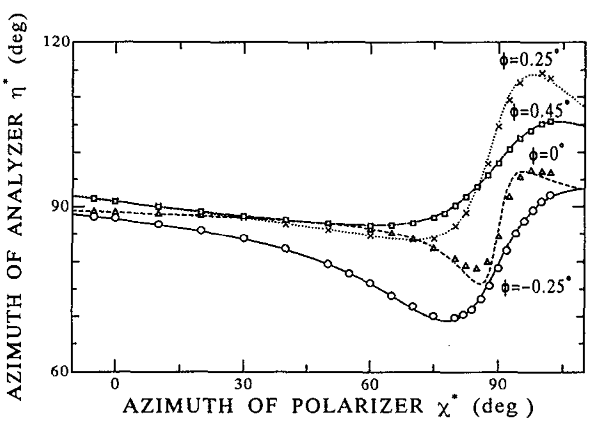

FIG. 4. The results of four sets of polarization measurements. The points show the measured data and the curves show the result of best fitting.

TABLE I. The best fitting parameters of the phase shifter and of the polarization state of light.

ϕ (M_0 vertical inclination)	Angle of incidence at P	α $\lvert r_s/r_p \rvert$	Δ (deg) (retardation $\Delta_p - \Delta_s$)	ϵ (deg) (ellipticity angle)	δ (deg) (azimuth of the ellipse)	Degree of linear polarization[a]
−0.25°	45°	7.74±0.72	35.9± 7.8	15.8±1.6	−4.86±2.50	0.85±0.03
0°	43°	31.7 ±2.8	21.4±12.3	4.5±1.7	−0.53±1.47	0.99±0.01
0.25°	45°	7.73±1.42	35.0± 6.7	−13.1±2.3	3.21±2.96	0.90±0.03
0.45°	45°	7.74±1.80	37.5± 8.8	−18.9±2.6	5.56±4.74	0.79±0.05

[a] Calculated by the ellipticity angle ϵ.

in most cases convergent. Equation (6) has five parameters, V, η, δ, α, and Δ, suggesting a fair chance to encounter a local minimum in the fitting procedure. To make the curve fitting simpler we should fix a few parameters to reasonable values. From our recent study V was found to be very close to 1.0. We therefore made the curve fitting with $V=1.0$.

IV. RESULTS AND DISCUSSION

The results of the polarization parameters thus obtained are listed in Table I. As is shown in Table I for the same incident angle of P, i.e., 45°, the parameters obtained for $\phi = -0.25°$, 0.25°, and 0.45° coincide with one another very well. This result means that our method is reliable. Figure 5 shows ϵ and δ as a function of ϕ. The polarization ellipses are also illustrated in Fig. 5. For $\phi = 0.05°$, ϵ and δ are both close to zero, which means that the incident light is expected to be horizontally linearly polarized. For $\phi = 0.25°$ and 0.45° the ϵ's are negative, i.e., the state of polarization is left-handed ellipse. While for $\phi = -0.25°$ ϵ is positive; the state of polarization is right-handed ellipse. Both ϵ and δ correspond exactly to the vertical observation angle with respect to the plane of the positron orbit.

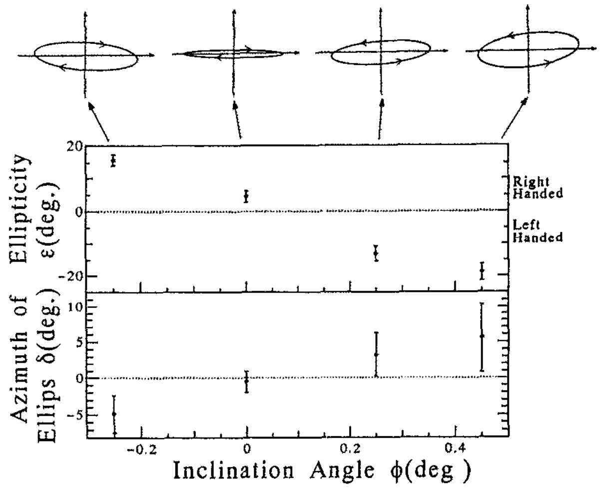

FIG. 5. Changes in ellipticity angle ϵ and azimuth of the major axis δ as a function of inclination angle ϕ of M_0. The inclination angle of ϕ has an arbitrary origin. For each inclination angle of M_0, the polarization ellipse is shown at the top of the figure.

V. SUMMARY

We have carried out the first attempt of full polarization measurements for soft x-ray synchrotron radiation with use of a multilayer phase shifter and a polarizer. By utilizing the phase information, we have successfully determined the state of polarization including handedness of the emerging light from the monochromator at the Photon Factory.

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