Interaction balance in symmetrical factorial designs with generalized minimum aberration

Mingyao Ai and Shuyuan He
Department of Probability and Statistics, School of Mathematical Sciences, Peking University, Beijing 100871, P. R. China
E-mail address: myai@eyou.com (M. Ai), syhe@math.pku.edu.cn (S. He)

Abstract: In this paper, the issue of balance pattern of the interaction columns of a symmetrical design is considered according to orthogonal components system. The minimum interaction unbalance criterion is proposed for ranking and comparing s-level factorial designs, where s is any a prime or a prime power. It is further showed that the interaction unbalance pattern is just the generalized wordlength pattern defined by Xu & Wu (2001) from the point of view of linear-quadratic system based on the ANOVA model, and consequently the two criteria, minimum interaction unbalance and generalized minimum aberration, coincide with each other for symmetrical factorial designs, although ground on two different systems of parameterization.

Key words and phrases: Coding theory; fractional factorial design; generalized minimum aberration; generalized wordlength pattern; interaction unbalance pattern; minimum interaction unbalance; nonregular; symmetrical

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1 Introduction

The fractional factorial designs, as we all know, can be broadly classified into two categories: regular fractional factorials and nonregular fractional factorials. A regular fractional factorial is determined by its defining relation and has a simple aliasing structure in that any two effects are either orthogonal or fully aliased. For these designs, the minimum aberration (MA) criterion proposed by Fries & Hunter (1980) has become the standard criterion for optimal factorial designs. For nonregular designs, Deng & Tang (1999) proposed the generalized resolution and minimum G-aberration criterion for ranking the different two-level designs. Subsequently, Tang & Deng (1999) further suggested a relaxed variant of minimum G-aberration, that is minimum G_2-aberration. Their discussion is restricted to the class of two-level factorial designs.

Since a regular symmetrical design in design theory is exactly a linear code in coding theory and the defining contrast subgroup justly corresponds to the dual code, the connection between the wordlength pattern and the distance distribution of a regular design can be established based on the MacWilliams identities. Ma & Fang (2001) generalized this relationship to adapt to nonregular symmetrical designs and defined their generalized wordlength patterns, and moreover proposed the minimum generalized aberration (MGA) criterion for ranking nonregular symmetrical designs. See Chapter 2 of Fang & Ma (2001) for the above details.

As we know, there are two systems of contrast parameterization in analyzing more than two-level factorial designs, that is orthogonal components system and linear-quadratic system. The former can only deals with the symmetrical designs, for the operations between two arguments from different Galios fields is still not clear, while the latter, in contrast, can flexibly adapts to general asymmetrical factorial designs. For details refer to Chapter 5 of Wu & Hamada (2000).

From the point of view of linear-quadratic system base on the ANOVA model, Xu & Wu (2001) proposed an efficient and systematic method, i.e., generalized minimum aberration (GMA) criterion for comparing and selecting general asymmetrical fractional factorial designs. It covers both the MA and the minimum G_2-aberration criteria as two special cases. It is also showed that for nonregular symmetrical designs the MGA criterion suggested by Ma & Fang (2001) is equivalent to the GMA criterion. By introducing the concept of the number of coincidence among runs, Xu (2003) further suggested minimum moment aberration (MMA) criterion, which is a good surrogate with tremendous computational advantages for the GMA criterion owing to their equivalence for symmetrical designs and weakly equivalence for asymmetrical designs.

In order to study the projection properties of a design, Tang (2001) and Ai & Zhang (2002) showed that sequentially minimizing the weighted projection variance vector is equivalent to the GMA criterion. On the other hand, Lu et al (2002) defined the projection balance pattern (BP) of a design and obtained some new BP-optimal designs via resolvable balanced incomplete block designs. Recently, Hickernell & Liu (2002) further showed that the generalized wordlength pattern of Xu & Wu (2001) can be expressed as the projection discrepancy pattern with some specified reproducing kernel, and thus established the connection between aberration and discrepancy for general designs.

In this paper, we consider the issue of balance pattern of the interaction columns of a symmetrical design according to orthogonal components system. Some reviews on GMA criterion are first presented in Section 2. Subsequently, an interaction unbalance pattern is defined in Section 3 for an s-level symmetrical factorial design, where s is any a prime or a prime power. And the minimum interaction unbalance (MIU) criterion is also proposed for ranking and comparing symmetrical factorial designs. It is further proved in Section 4 that the interaction unbalance pattern is just the generalized wordlength pattern defined by Xu & Wu (2001) from the point of view of linear-quadratic system, and consequently the two criteria, MIU and GMA, coincide with each other for symmetrical factorial designs, although ground on two different systems of parameterization.

The rest of this section is devoted to notations and definitions. Let $wt(u)$ be the weight of a vector $u = (u_1, \ldots, u_m)$, i.e., the number of nonzero elements of u, $R_s = \{0, 1, \ldots, s-1\}$ be the integer ring with modulus s, $|S|$ be the number of elements of a set S. For any two vectors $u = (u_1, \ldots, u_m)$ and $x = (x_1, \ldots, x_m)$, $\delta_{u,x}$ is the Kronecker delta which equals 1 if $u = x$ and 0 otherwise, and the Hamming distance $d_H(u, x)$ is the number of places where they differ.

A mixed-level (or asymmetrical) design of n runs and m factors with $s_1, \ldots, s_m$ levels, denoted by $(n, s_1 \cdots s_m)$, is a set of n row vectors (or points) in $R = R_{s_1} \times \cdots \times R_{s_m}$, or a set of m column vectors of length n, or an $n \times m$ matrix in which each row represents a run, each column represents a factor and the jth column takes values from a set of s_j symbols, say, R_{s_j}. In particular, an (n, s^m)-design is symmetrical. Two designs are called isomorphic if one can be obtained from the other through permutations of rows, columns and symbols in each column.

2 Reviews on generalized minimum aberration criterion

In this section, we give some reviews on generalized minimum aberration criterion based on the ANOVA model. For details see Xu & Wu (2001).

Let $\{\chi_{u_i}^{(i)}, u_i \in R_{s_i}\}$ be the orthonormal contrast coefficients for the ith factor which has s_i levels, that is,

$$\sum_{x_i \in R_{s_i}} \chi_{u_i}^{(i)}(x_i)\overline{\chi_{v_i}^{(i)}(x_i)} = s_i \delta_{u_i,v_i}, \text{ for any } u_i, v_i \in R_{s_i}, \tag{1}$$

where $\overline{\chi_{v_i}^{(i)}(\cdot)}$ is the complex conjugate of $\chi_{v_i}^{(i)}(\cdot)$. Let $\chi_0^{(i)}(x_i) = 1$, for any $x_i \in R_{s_i}$. As often done in practice, we only consider the contrast coefficients defined by tensor products:

$$\chi_u(x) = \prod_{i=1}^{m} \chi_{u_i}^{(i)}(x_i), \text{for} u = (u_1, \ldots, u_m) \in R \text{ and } x = (x_1, \ldots, x_m) \in R. \tag{2}$$

It is easily verified that $\{\chi_u, u \in R\}$ are the orthonormal contrast coefficients, i.e., $\sum_{x \in R} \chi_u(x)\overline{\chi_v(x)} = (s_1 \cdots s_m)\delta_{u,v}$ for any $u, v \in R$.

There are two classes of contrasts used usually. The first is from orthogonal polynomials. The second is the complex contrasts, which are of no meaning in practice but of great use in theory. For an s-level factor, the complex contrasts are defined to be $\chi_u(x) = \xi^{u \cdot x}$ for any $u, x \in R_s$, where $\xi = e^{2\pi i/s}$. Refer to Bailey (1982) for details of complex contrasts.

For a fractional factorial $(n, s_1 \cdots s_m)$-design D, define

$$\chi_u(D) = \sum_{x \in D} \chi_u(x), \text{ for any } u \in R, \tag{3}$$

and

$$A_j(D) = n^{-2} \sum_{u \in R, wt(u)=j} |\chi_u(D)|^2 \qquad \text{for} \quad j = 1, \ldots, m. \tag{4}$$

Obviously, $\chi_{(0,\ldots,0)}(D) = n$. Xu & Wu (2001) showed that $A_j(D)$ values are independent of the choice of orthonormal contrasts. The vector $(A_1(D), \ldots, A_m(D))$ is called the generalized wordlength pattern of design D. Thus, The GMA criterion is to sequentially minimize $A_j(D)$ in (4) for $j = 1, \ldots, m$. The isomorphic designs are equivalent under GMA. It should be noted that the GMA reduces to the minimum aberration for regular designs and the minimum G_2-aberration for two-level nonregular designs.

In particular, A symmetrical (n, s^m)-design D is a set of n row vectors of length m. The distance distribution of D is the vector $(E_0(D), \ldots, E_m(D))$, where

$$E_j(D) = n^{-1}|\{(a, b) : d_H(a, b) = j, a \in D, b \in D\}| \qquad \text{for} \quad j = 0, \ldots, m. \tag{5}$$

The MacWilliams transforms of the distance distribution are defined as

$$E'_j(D) = n^{-1} \sum_{i=0}^{m} E_i(D) P_j(i; m, s) \qquad \text{for} \quad j = 0, \ldots, m, \tag{6}$$

where $P_j(x; m, s) = \sum_{i=0}^{j} (-1)^i (s-1)^{j-i} \binom{x}{i} \binom{m-x}{j-i}$ are the Krawtchouk polynomials. Let $d_{i,j}(D)$ be the Hamming distance between the ith and jth rows of D. Then it can be easily shown that $\sum_{i=1}^{n} \sum_{j=1}^{n} d_{i,j}(D) = n \sum_{i=0}^{m} i E_i(D)$. Xu & Wu (2001) showed that for an (n, s^m)-design D, the generalized wordlength pattern is exactly the MacWilliams transform of the distance distribution, that is

$$A_j(D) = E'_j(D) \qquad \text{for} \quad j = 1, \ldots, m. \tag{7}$$

Throughout this paper, we extend the definition of $\binom{n}{i}$ to allow n and i to be any integers:

$$\binom{n}{i} = \begin{cases} \frac{n(n-1)\cdots(n-s+1)}{i(i-1)\cdots 1} & \text{for } 0 < i \le n, \\ 1 & \text{for } i = 0 \text{ and } n \ge 0, \\ 0 & \text{otherwise.} \end{cases}$$

3 Interaction unbalance pattern

In this section, an (n, s^m)-design D is regarded as a set of m columns $D = \{d_1, \ldots, d_m\}$ or as an $n \times m$ matrix $D = (d_{ij})$, depending on our convenience. For $1 \le k \le m$ and any k-subset $S = \{d_{j_1}, \cdots, d_{j_k}\}$ of D, the set of k-factor interactions of S based on orthogonal components system of parameterization is

$$I(S) = \{w : w = \sum_{t=1}^{k} c_t d_{j_t} \pmod s, c_1 = 1, 1 \le c_t \le s-1, t = 2, \ldots, k.\}. \tag{8}$$

For any $w \in I(S)$, let $n_i(w)$ be the number of level i in the interaction column w. The magnitude $\sum_{0 \le i < j \le s-1} [n_i(w) - n_j(w)]^2$ measures the discrepancy among the frequencies all $s-1$ levels appear in the column w, that is the unbalance of the interaction w. Then the k-factor interaction unbalance of S can be defined as

$$J_k^2(S) = \sum_{w \in I(S)} \sum_{0 \le i < j \le s-1} [n_i(w) - n_j(w)]^2. \tag{9}$$

For two-level factorial design D, if we replace its two levels 0 and 1 by $+1$ and -1, respectively, then $I(S) = \{w = d_{j_1} \cdots d_{j_k}\}$, one element set, and $J_k(S) = |n_{-1}(w) - n_{+1}(w)| = |\sum_{i=1}^{n} d_{ij_1} \cdots d_{ij_k}|$, which is exactly J-characteristics $J_k(S)$ defined by Tang & Deng (1999). So our definition is a generalization of their $J_k(S)$ values. Following their concept, the $J_k(S)$ values are also called the J-characteristics of design D. These $J_k(S)$ values play an instrumental role in our development of minimum interaction unbalance criteria.

Define

$$B_k(D) = n^{-2} \sum_{|S|=k, S \subseteq D} J_k^2(S) \qquad \text{for} \quad k = 1, \ldots, m, \tag{10}$$

which measures the k-factor interactions unbalance of design D. The vector $(B_1(D), \ldots, B_m(D))$ is called the interaction unbalance pattern of design D. For the orthogonal designs, $B_1(D) = B_2(D) = 0$. For two designs D_1 and D_2, let r be the smallest integer such that $B_r(D_1) \neq B_r(D_2)$. If $B_r(D_1) < B_r(D_2)$, then we say that D_1 has less interaction unbalance than D_2. If no other design has less interaction unbalance than D_1, then D_1 is said to have minimum interaction unbalance (MIU). If D is regular, then $B_k(D) = (s-1)A_k(D)$, where $A_k(D)$ is the number of words of length k in the defining contrast subgroup of D, which implies that the MIU criterion is equivalent to minimum aberration for regular designs. It is also obvious that the MIU criterion is reduced to minimum G_2-aberration for special two-level designs.

As for the statistical justification of this criterion, we can follow Tang & Deng (1999) and get the same conclusion that the MIU is equivalent to a criterion that sequentially minimizes the contamination of nonnegligible interactions on the estimation of main effects, in the order of importance given by the hierarchical assumption. Furthermore, if the original design is regular, then the minimum aberration design sequentially minimizes the number of interactions of order j confounded with the main effects in the order given by $j = 2, \cdots, m$.

4 Coincidence between GMA and MIU criteria

Although the generalized wordlength pattern and interaction unbalance pattern are defined from two completely different systems of parameterization, it is proved in this section that they are in fact the same, and so the GMA and MIU criteria coincide with each other for symmetrical factorial designs.

For a symmetrical (n, s^m)-design D, let $D^{(k)}$ be composed of all k-factor interaction columns of D, that is $D^{(k)} = \bigcup_{|S|=k, S\subseteq D} I(S)$. Obviously, $c_k = |D^{(k)}| = \binom{m}{k}(s-1)^{k-1}$. Then the interaction unbalance pattern defined in (10) can be expressed as the distance distribution of $D^{(k)}$, which is presented in the following theorem.

Theorem 1. *For a symmetrical (n, s^m)-design D, its interaction unbalance pattern can be expressed as follows:*

$$B_k(D) = \binom{m}{k}(s-1)^k - \frac{s}{n}\sum_{i=1}^{c_k} iE_i(D^{(k)}) \qquad \text{for} \quad k = 1, \ldots, m. \tag{11}$$

Proof. Firstly, $J_k^2(S)$ in (9) can be expressed as

$$\begin{aligned} J_k^2(S) &= 2^{-1}\sum_{w\in I(S)}\sum_{i=0}^{s-1}\sum_{j=0}^{s-1}\left\{[n_i(w)]^2 + [n_j(w)]^2 - 2n_i(w)n_j(w)\right\} \\ &= \sum_{w\in I(S)}\left\{s\sum_{i=0}^{s-1}[n_i(w)]^2 - \left[\sum_{i=0}^{s-1} n_i(w)\right]^2\right\} \\ &= s\sum_{w\in I(S)}\sum_{i=0}^{s-1}[n_i(w)]^2 - n^2(s-1)^{k-1}. \end{aligned}$$

Define

$$\tau_{(i),j}(w) = \begin{cases} 1, & \text{if the } j\text{th element of } w \text{ is } i, \\ 0, & \text{otherwise;} \end{cases}$$

$$\tau_{i,j}(w) = \begin{cases} 1, & \text{if the } i\text{th and } j\text{th element of } w \text{ are the same,} \\ 0, & \text{otherwise;} \end{cases}$$

and $d_{i,j}(w) = 1 - \tau_{i,j}(w)$. Then

$$\begin{aligned} \sum_{i=0}^{s-1}[n_i(w)]^2 &= \sum_{i=0}^{s-1}\left[\sum_{j=1}^{n}\tau_{(i),j}(w)\right]^2 = \sum_{i=0}^{s-1}\sum_{j,l=1}^{n}\tau_{(i),j}(w)\tau_{(i),l}(w) = \sum_{j,l=1}^{n}\sum_{i=0}^{s-1}\tau_{(i),j}(w)\tau_{(i),l}(w) \\ &= \sum_{j,l=1}^{n}\tau_{j,l}(w) = n^2 - \sum_{j,l=1}^{n} d_{j,l}(w). \end{aligned}$$

So $B_k(D)$ in (10) can further be written as

$$\begin{aligned} B_k(D) &= s\binom{m}{k}(s-1)^{k-1} - sn^{-2}\sum_{j,l=1}^{n}\sum_{|S|=k,S\subset D}\sum_{w\in I(S)} d_{j,l}(w) - \binom{m}{k}(s-1)^{k-1} \\ &= \binom{m}{k}(s-1)^k - sn^{-2}\sum_{j,l=1}^{n} d_{j,l}(D^{(k)}) \\ &= \binom{m}{k}(s-1)^k - sn^{-1}\sum_{i=1}^{c_k} iE_i(D^{(k)}). \end{aligned}$$

This theorem is proved.

Next, we introduce some known properties of Krawtchouk polynomials to deduce the relationship between the generalized wordlength pattern and the interaction unbalance pattern [see MacWilliams & Sloane (1977) for details].

Lemma 1. *The Krawtchouk polynomials $P_j(i;m,s)$ have the following properties:*

(i) $P_j(0;m,s) = \binom{m}{j}(s-1)^j$ *for* $0 \le j \le m$,

(ii) $P_0(i;m,s) = 1$ *for* $0 \le i \le m$,

(iii) $\sum_{j=0}^{m} P_j(i;m,s) = s^m\delta_{i,0}$.

For any a k-subset S of D, let $d_{j,l}^{(k)}(S) = \sum_{w\in I(S)} d_{j,l}(w)$. Obviously, $d_{j,l}(D^{(k)}) = \sum_{|S|=k,S\subseteq D} d_{j,l}^{(k)}(S)$. It can be seen that the design $\bigcup_{S_1\subseteq S,S_1\neq\phi} I(S_1)$ consists of n rows justly coming from the saturated design with s^k runs, permitting some replicated rows. Since the Hamming distance between any two different runs in the saturated design with s^k runs is the constant s^{k-1} [Peterson & Weldon (1972), page 75], Then for $j,l = 1,\ldots,n$, if $d_{j,l}(S) > 0$, we have

$$d_{j,l}^{(k)}(S) = s^{k-1} - \sum_{t=1}^{k-1}\sum_{|S_1|=t,S_1\subseteq S} d_{j,l}^{(t)}(S_1). \tag{12}$$

It can be seen that the value $d_{j,l}^{(k)}(S)$ depends only on the parameters s, k and $d_{j,l}(S)$, but not on the choice of set S.

From this deductive formula, we can get the following Lemma 2, which plays an instrumental role in establishing the coincidence between the two criteria.

Lemma 2. *The values $d_{j,l}(D^{(k)})$ can be expressed in terms of the Krawtchouk polynomials as follows:*

$$d_{j,l}(D^{(k)}) = s^{-1}\binom{m}{k}(s-1)^k - s^{-1}P_k(x;m,s) \qquad \text{for} \quad k=1,\ldots,m;\ j,l=1,\ldots,n, \tag{13}$$

where $x = d_{j,l}(D)$.

Proof. The formula (13) is now being proved by using the inductive method for $k = 1,\ldots,m$.

Firstly, since $d_{j,l}(D^{(1)}) = d_{j,l}(D) = x$, and $P_1(x;m,s) = (s-1)(m-x) - x = m(s-1) - sx$, the formula (13) can easily be verified to hold for $k = 1$.

Secondly, from the formula (12), if $d_{j,l}(S) > 0$ for a two-subset $S \subseteq D$, then $d_{j,l}^{(2)}(S) = s - d_{j,l}(S)$. So $sd_{j,l}(D^{(2)}) = s\sum_{|S|=2,S\subseteq D} d_{j,l}^{(2)}(S) = s\sum_{y=1}^{2}\binom{x}{y}\binom{m-x}{2-y}(s-y) = s(s-1)x(m-x) - s(s-2)\binom{x}{2}$. Referring to the definition of $P_2(x;m,s)$, we can verify that the formula (13) also holds for $k = 2$.

Suppose that the formula (13) holds for all $1 \le t \le k-1$, and that S^* is a specified k-subset of D with $d_{j,l}(S^*) = y$, then combining with the formula (12) and Lemma 1, we have

$$\begin{aligned}
sd_{j,l}(D^{(k)}) &= s\sum_{|S|=k,S\subseteq D} d_{j,l}^{(k)}(S) = s\sum_{y=1}^{k}\binom{x}{y}\binom{m-x}{k-y}d_{j,l}^{(k)}(S^*)\\
&= s\sum_{y=1}^{k}\binom{x}{y}\binom{m-x}{k-y}\left[s^{k-1} - \sum_{t=1}^{k-1}\sum_{|S_1|=t,S_1\subseteq S^*} d_{j,l}^{(t)}(S_1)\right]\\
&= s^k\left[\binom{m}{k} - \binom{m-x}{k}\right] - \sum_{y=1}^{k}\binom{x}{y}\binom{m-x}{k-y}\sum_{t=1}^{k-1}\left[\binom{k}{t}(s-1)^t - P_t(y;k,s)\right]\\
&= s^k\left[\binom{m}{k} - \binom{m-x}{k}\right]\\
&\quad -\sum_{y=1}^{k}\binom{x}{y}\binom{m-x}{k-y}\left[\sum_{t=1}^{k-1}\binom{k}{t}(s-1)^t + P_k(y;k,s) + P_0(y;k,s)\right]\\
&= s^k\left[\binom{m}{k} - \binom{m-x}{k}\right] - \sum_{y=1}^{k}\binom{x}{y}\binom{m-x}{k-y}\left[s^k - (s-1)^k + P_k(y;k,s)\right]\\
&= (s-1)^k\left[\binom{m}{k} - \binom{m-x}{k}\right] - \sum_{y=1}^{k}\binom{x}{y}\binom{m-x}{k-y}P_k(y;k,s).
\end{aligned}$$

Since $P_k(y;k,s) = \sum_{i=0}^{k}(-1)^i(s-1)^{k-i}\binom{y}{i}\binom{k-y}{k-i} = (-1)^y(s-1)^{k-y}$, the above identity can further be simplified as

$$\begin{aligned} sd_{j,l}(D^{(k)}) &= (s-1)^k \left[\binom{m}{k} - \binom{m-x}{k} \right] - \sum_{y=1}^{k} \binom{x}{y} \binom{m-x}{k-y} (-1)^y (s-1)^{k-y} \\ &= (s-1)^k \binom{m}{k} - P_k(x; m, s). \end{aligned}$$

Thus the formula (13) holds for k, and the proof of this lemma is completed.

Substituting $d_{j,l}(D^{(k)})$ in Theorem 1 with the formula (13) and Noting that $\sum_{i=0}^{m} E_i(D) = n$, we have

$$\begin{aligned} B_k(D) &= \binom{m}{k}(s-1)^k n^{-1} \sum_{x=0}^{m} E_x(D) - n^{-1} \sum_{x=0}^{m} E_x(D) \left[\binom{m}{k}(s-1)^k - P_k(x; m, s) \right] \\ &= n^{-1} \sum_{x=0}^{m} E_x(D) P_k(x; m, s) = E'_k(D) = A_k(D). \end{aligned}$$

Thus we get the following theorem.

Theorem 2. *For a symmetrical (n, s^m)-design D, the interaction unbalance pattern is exactly the generalized wordlength pattern, that is, $B_k(D) = A_k(D)$ for all $k = 1, \ldots, m$, and so the MIU and GMA criteria coincide with each other.*

Remark. It can be concluded that the GMA designs balance the levels of the interactions as equally as possible sequentially from lower orders to higher orders. Therefore, the interaction balance justification of GMA criterion is established according to the other different system of parameterization, that is the orthogonal components system.

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