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ANALYSIS OF THE DIAMETER OF MICROBUBBLES FORMED IN A CROSS FLOW MICROCHANNEL

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ABSTRACT

Two microfluidic devices for generating microbubbles are considered in the study presented in this paper. The first device consists of a liquid channel and a gas channel that is perpendicular to each other. In this device, the microbubble diameter varies inversely with the liquid flow rate (i.e. with flow velocity) but at the expense of high pressure drop. This device is modified by introducing a solid structure in front of the orifice to become the second device. This modification causes an increase in fluid velocity only in front of the orifice. In this paper a model is developed for determining the diameter of microbubbles generated in both of these devices. The model is developed by balancing the forces acting on the microbubble during growth and at the moment of detachment from the orifice. The detachment of the microbubble is assumed to happen when the sum of all detaching forces equals the net attaching force acting on the microbubble during the growth. Non-slip boundary condition is assumed on the walls of the channels in this model. Based on these assumptions a mathematical model is developed and solved numerically for different values of liquid and gas flow rate to obtain the microbubble diameter at the moment of detachment. A MATLAB® code is developed for solving the force balance equation. The superiority of the second device over the first one is validated by comparing the results obtained from the models in both cases.

KEYWORDS

Microbubble, bubble generation, cross flow channels.

INTRODUCTION

The generation of microbubbles in microchannels has received special attention in the lab on chip applications and medical applications in recent years [1, 2]. The study of bubble generation in stationary water began in late 1960's, one of the first analysis in this field done by Ramakrishnan, Kumar and Kuloor in 1969 [3]. The study of bubble generation in cross flowing liquid began in 1990's and analysis on the bubble formation and bubble detachment has been examined by various research studies. Marshall and Chudacek proposed a mathematical model in 1993 which studied the bubble formation from an orifice in cross flowing liquid [4]. The model results were compared with the experimental results and the theory was validated using the results obtained from the experimental data. Nahra and Kamotani studied the bubble formation in cross current liquid flow under reduced gravity conditions in 2000 [5]. They developed a simple mathematical model to predict the bubble formation under reduced gravity and experiments were conducted to support the mathematical model. Later in 2002 they came up with an improved model for the prediction of bubble growth and detachment diameter, and experiments were conducted under both the normal and reduced gravity conditions [6]. The bubble formation in co-flow configuration was experimentally examined in 1998 by Bhunia, Pais, Kamotani, and Kim [7]. All the studies discussed in the above papers were carried out at the macro level and the bubbles generated in the experiments varied from few millimeters to few centimeters. In 2007, Xiong, Mo Bai and Chung studied bubble generation in a co-flowing microchannel [8]. They demonstrated the bubble formation in microchannel with co-flowing liquid and gas using both simulated and experimental results, and both the results showed good agreement with each other. In this paper, a mathematical model is developed for predicting the bubble formation and bubble diameter from an orifice of diameter below 50 micrometer in a microchannel with cross current liquid flow. Additionally, a new technique is introduced in the paper which can help in the generation of smaller bubbles compared to the earlier models at a given flow rate of liquid and gas.

NOMENCLATURE

ρ_l : density of liquid
ρ_g : density of gas
σ : surface tension
A_l : cross sectional area of the liquid channel
A_{lm} : modified cross sectional area at orifice
A_{eff} : effective diameter of the microbubble
C_D : drag coefficient
C_{MD} : modified drag coefficient
D : diameter of the microbubble
D_o : diameter of the orifice
D_h : hydraulic diameter of the liquid channel
g : gravity
Q_0 : gas flow rate
R : radius of the microbubble
$\dot{R}$: rate of change of microbubble diameter
R_e : Reynolds number
U_l : average velocity of the liquid
U_{lm} : modified velocity around the obstacle
${U_l}^*$: effective liquid velocity during microbubble growth
V_B : volume of the microbubble

THEORY

There are different forces acting on the microbubble during its formation from the orfice. Some of these forces help the microbubble formation and some of them resist the microbubble formation. A mathematical model for the microbubble formation and detachment from the orifice in a microchannel is developed by considering each force acting on the microbubble during its growth separately and balancing them. In order to keep the model simpler to solve some general assumptions are taken during the development of this model and those assumptions are given below.

1. Gas behaves as an ideal gas.
2. Cross flowing liquid is uniform, isothermal, and inviscid.
3. There is no energy exchange or mass transfer between the gas - liquid interface.
4. Constant gas flow condition is assumed throughout the model.

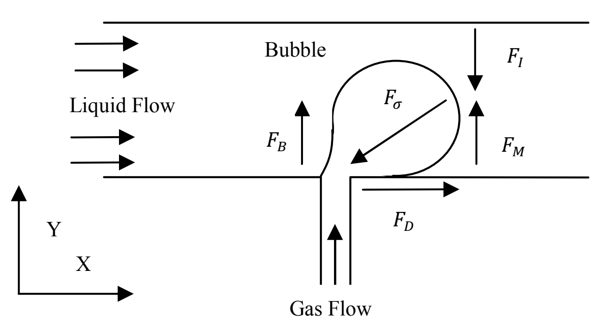

Figure 1: Schematic figure showing direction of different forces acting on the microbubble during its growth.

The forces acting on the microbubble during the microbubble formation are classified into either attaching or detaching forces. Each of the forces acting on the microbubbles and the direction of each force is represented in the figure 1. The attaching forces provide resistance to the microbubble expansion and the detaching forces assists the microbubble expansion. The dominating attaching forces are the surface tension that acts towards the orifice and inertia force that acts in the negative y direction. The drag force is a detaching force and it is considered to act in the direction of the liquid flow. Since the bubble formation is in the microchannel the drag forces in the y direction is considered negligible compared to the drag force in the x direction. The momentum force and the buoyancy force are the two other dominating forces and both of them are considered as detaching force and act in the positive y direction. Even though the momentum force is the dominating force when compared with the buoyancy force, the latter is also affecting the diameter of the microbubble at the moment of detachment and is considered in this study. Each of the forces considered in this study is explained below and the microbubble diameter prediction model is developed by balancing those equations.

Surface tension force:

$$F_\sigma = \pi D_o \sigma \tag{1}$$

Inertia force:

$$F_I = \frac{d}{dt}\left(M' \frac{ds}{dt}\right) \tag{2}$$

Where M' is given as

$$M' = \frac{11}{16}\rho_g V_B$$ (Davidson et. at. 1960)

Here $\frac{ds}{dt}$ is the bubble velocity at the center of the bubble.

Drag Force:

$$F_D = C_D \frac{1}{2}\rho_l {U_l}^2 A_{eff} \tag{3}$$

Where

$$C_D \approx \frac{24}{R_e} + \frac{6}{1+\sqrt{R_e}} + 0.4 \qquad 0 \le R_e \le 2e5$$

(White 1991)

When the bubble starts expanding the drag coefficient acting on the bubble and the effective liquid velocity around the region of the bubble will start changing. In order to compensate for this effect a modified drag coefficient and a new effective velocity is introduced in the current model.

$$C_{MD} = \frac{C_D}{1-\frac{2R}{D_h}} \quad (4)$$

$$U_l^{\ *} = \frac{U_l A_l}{A_l - \pi R^2} \quad (5)$$

Momentum force:

$$F_m = \frac{\rho_g 4 Q_o^{\ 2}}{\pi D_o^{\ 2}} \quad (6)$$

Buoyancy Force:

$$F_B = \frac{4}{3}\pi R^3 g(\rho_l - \rho_g) \quad (7)$$

The microbubble detachment condition is derived by balancing all the forces together, when the resultant of the detaching forces equals the attaching forces the microbubble will detach from the orifice. The detachment condition is obtained by applying the trigonometric relation to the different forces acting on the microbubble according to their direction.

$$F_\sigma = \sqrt{(F_M + F_B - F_I)^2 + F_D^{\ 2}} \quad (8)$$

The resultant of the momentum force, buoyancy force, and the inertia force forms the altitude of the right triangle, the drag force is taken as the base and the surface tension is considered as the hypotenuse as shown in figure 1. The resultant equation is obtained as below.

$$\pi D_o \sigma = \left\{ \left[\frac{\rho_g 4 Q_o^{\ 2}}{\pi D_o^{\ 2}} + \frac{4}{3}\pi R^3 g\left(\rho_l - \rho_g\right) - \frac{d}{dt}(M' \frac{ds}{dt}) \right]^2 + \frac{1}{2}\rho_l \frac{C_D}{1-\frac{2R}{D_h}} (\frac{U_l A_l}{A_l - \pi R^2})^2 \, \pi R^2]^2 \right\}^{\frac{1}{2}} \quad (9)$$

Radius of the microbubble at the moment of detachment is obtained by solving this equation for R. Since the constant gas flow condition is assumed in the model, the rate of change in volume of the microbubble with time can be used to obtain the change in radius of the microbubble with time.

$$\frac{d}{dt}\frac{4}{3}\pi R^3 = Q_o \quad (10)$$

$$\dot{R} = \frac{Q_o}{4\pi R^2} \quad (11)$$

A modified technique, which produces smaller microbubble at a given flow rate of liquid, with a semi circle structure in the liquid channel in front of the orifice is introduced in the section below.

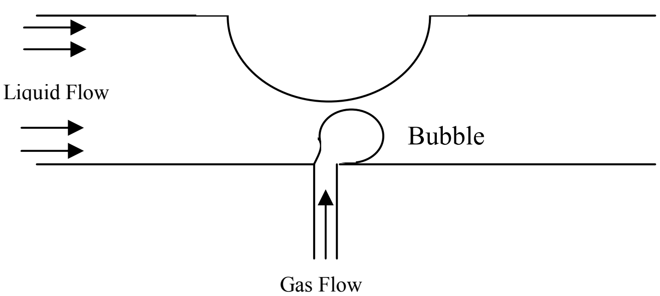

Figure 2: Schematic figure showing microbubble generation in a microchannel with a semi circle structure in front of the orifice.

Due to the presence of the semi circle inside the microchannel in front of the orifice, the velocity of the liquid flowing in the region increases and the pressure in that region decreases. These changes will result in the generation of smaller microbubbles at a higher frequency inside the microchannel. According to Bernoulli's equation, the pressure P_2 in front of the orfice can be calculated using the pressure P_1 at the inlet of the channel and the velocities V_1 and V_2 at the inlet and at the orifice respectively.

$$P_2 = P_1 - \rho \frac{(V_2^{\ 2} - V_1^{\ 2})}{2} \quad (12)$$

The modified liquid velocity due to the semi circle present in the microchannel can be derived as.

$$U_{lm} = \frac{U_l A_l}{A_{lm}} \quad (13)$$

The mathematical model (equation 9) developed for the first case can be modified by substituting modified liquid velocity U_{lm} in the place of initial liquid velocity U_l in the equation.

$$\pi D_o \sigma = \left\{ \left[\frac{\rho_g 4 Q_o^{\ 2}}{\pi D_o^{\ 2}} + \frac{4}{3}\pi R^3 g\left(\rho_l - \rho_g\right) - \frac{d}{dt}\left(M' \frac{ds}{dt}\right) \right]^2 + \frac{1}{2}\rho_l \frac{C_D}{1-\frac{2R}{D_h}} (\frac{U_l A_l}{(A_{lm} - \pi R^2)})^2 \, \pi R^2]^2 \right\}^{\frac{1}{2}} \quad (14)$$

Both the models are numerically solved for the radius of the microbubble using a MATLAB® programs developed for this application. The application of this model is limited only for the bubble generation where the size of the bubbles formed in the microchannel is less than the size of the liquid channels. When the size of the bubble is more than the size of the liquid channel in which it is formed, a slip occurs between the gas and the solid interface and the above developed model will not be sufficient for prediction of the bubble diameter in such cases.

RESULTS

No experimental results of microbubble generation in sub-250µm microchannels are available for the validation of the current model, so the diameter of the microbubbles generated using the current model is compared with some of the macro level existing models. The current model is validated with the experimental values of the normal gravity model proposed by Nahra and Kamotani in 2002. In Nahra and Kamotani's model, the bubble diameters produced in a square channel (2.54cm × 2.54cm) with an orifice diameter of 0.033 cm at two different liquid velocities of 1cm/sec and 4cm/sec and gas flow rates between 0.43cm^3/sec and 1.4cm^3/sec ranged between 3mm and 2.7 mm. In the current model under the same conditions, the bubble diameters obtained were between 2.4mm and 2.3mm respectively. When the bubble is being generated in the macro channel, the bubble will also experience a drag force in the y direction. This force is neglected in the microchannel because this force is considered negligible compared to the drag force in the x direction. The slight variation in the bubble diameter in both the models can be accounted to such forces that are neglected in the present model for the microchannels.

The comparison of two microbubble generators proposed in this paper is carried out by comparing the microbubble diameter generated by both the models at same flow rate of liquid and by maintaining the gas flow rate at a constant value. A 100 micrometer square channel is used as the liquid channel and the orifice diameter is kept as 20 micrometer. Air is used as the gas and deionized water is used as the liquid. The gas flow rate is kept constant a 0.4ml/hr and the liquid flow rate is varied from 20ml/hr to 120ml/hr. Two semicircles of radius 50µm and 70µm are introduced into the liquid channel in front of the orifice and the diameter of the microbubble generated by both of them are compared to the results obtained from the model without any obstacles in the liquid channel.

The comparison of the microbubble diameter in the plot below shows the advantage of including a semicircle structure in front of the orifice. It can be observed that as the semicircle is introduced into the micro liquid channel in front to the orifice the diameter of the microbubble generated is reduced. Smaller microbubbles at same liquid flow rate are obtained by introducing a semicircle in front of the orifice. The diameter of the microbubble generated is reduced in half when a semicircle structure of radius 70µm is introduced into the micro liquid channel. This proves the superiority of the newly proposed model over the simple cross current liquid flow microbubble generator which doesn't have any structures in it.

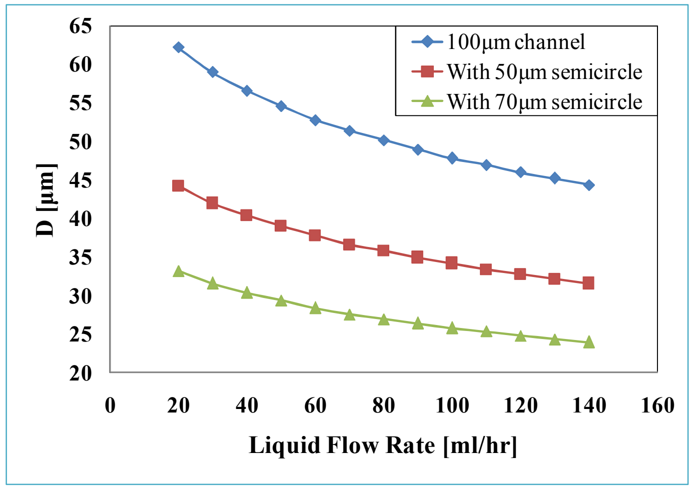

Figure 3: Comparison of the diameter of microbubble generated with three different models

Figure 4 shows the formation of the microbubbles in two different models, the first model without any structure in the liquid channel and the second model with a 50µm radius semicircle introduced in it. In both models, the liquid flow rate and the gas flow rate are kept constant at 50ml/hr and 0.4ml/hr respectively and the microbubble growth and detachment with respect to time is studied. As can be seen from the figure, Since the gas flow rate is maintained constant in both models the frequency of the microbubble generated is increased in the model with the 50µm semicircle inserted in the liquid channel. The microbubble grows very abruptly in the initial stage of the growth and slows down towards the detachment.

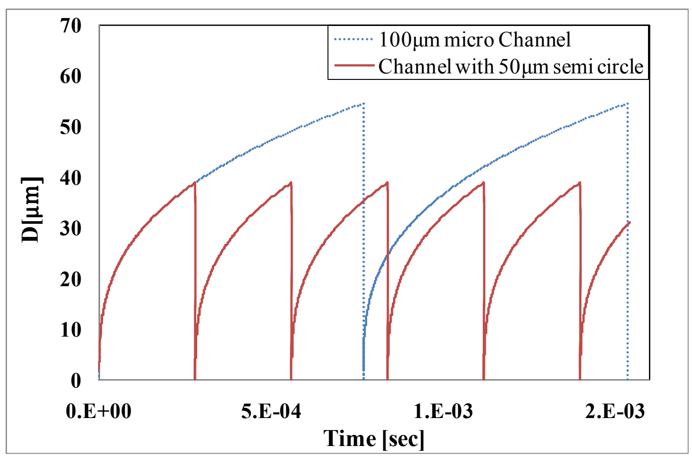

Figure 4: Comparison of the microbubble growth in two different models

The effect of the microbubble diameter in the dimension of the liquid channel is studied and the result is plotted in the figure 5. The microbubble diameter in micro channels of 5 different dimensions is represented in the figure. All microchannels show the same trend in the microbubble diameter variation with the increase in the liquid flow rate, and the size of the microbubble decreases as the size of the microchannel is reduced.

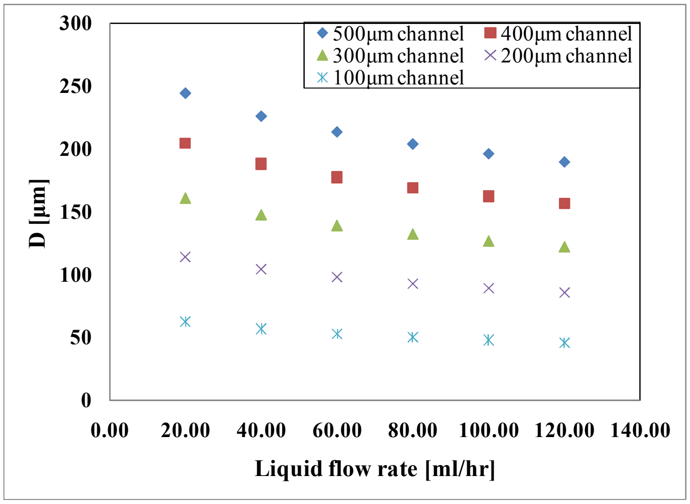

Figure 5: Comparison of the microbubble diameter for different channel dimensions.

CONCLUSION

Two mathematical models for microbubble generation were developed in the paper and solved numerically to obtain the microbubble diameter at the moment of detachment. The superiority of the second model with a semicircle structure in front of the orifice is validated by comparing the results of the both the models. The second model produces smaller microbubbles at higher frequency and the microbubble diameter decreases as the radius of the semi circle increases. The technique of introducing a semi circle in front of the orifice proved useful for the generation of smaller microbubbles.

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