

# Strength Prediction of Notched Woven Composite Plates using a Cohesive Zone Approach

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**Abstract.** The present paper is concerned with modelling damage and fracture in notched woven fabric composites. Previous experimental work has shown that damage at a notch in a variety of GFRP and CFRP composites based on woven fabric reinforcement comprises matrix damage and fibre tow fracture along the plane of maximum stress. It is these experimental observations that inform the failure modelling developed here, in which a cohesive zone approach is used within a 2-D finite element framework. The cohesive zone parameters are based on previously reported experimental measurements for the strength and toughness of the woven fabric materials under investigation. The approach is shown to provide predictions of notched strength that are in very good agreement with experimental results from the literature for a range of GFRP and CFRP woven fabric systems.

## Introduction

The failure of composite materials from a stress raiser, in particular an open hole has been researched extensively over the years. Early closed-form models (of which the most well-known are perhaps the point and average stress criteria of Whitney and Nuismer, (1974)) have been superseded by finite element based approaches of varying degrees of complexity. Some of the latter models treat the composites as a homogeneous material, while others takes a fully three-dimensional approach and model the damage on a ply-by-ply basis. Many approaches are semi-empirical in nature and this limits their general applicability, in particular because the sub-critical damage that develops at notches (and which needs in some way to be represented within any model) varies according to the details of the composite under investigation. The challenge for researchers therefore is to develop physically realistic failure models. Indeed it is only through the use of physically based models that there is the possibility of a failure theory being applicable to more than one class of problem without extensive recalibration of the model.

In a notched woven fabric GFRP composite subjected to tension, it has been shown experimentally using a model (transparent) GFRP system that prior to failure an intense zone of damage develops in the region of maximum tensile stress (Belmonte et. al, 2001). Damage within that zone comprises matrix cracking and splitting, but also fiber tow fracture. In that sense the damage resembles stable self-similar crack growth, suggesting that a fracture mechanics-based model is a physically reasonable one to use. A similar study was carried by Belmonte et al. (2004) on woven quasi-isotropic and cross-ply CFRP. Although the opaque nature of the CFRP means that direct observations of damage are less straightforward, there was evidence of tow failure before specimen failure. Other valuable data concerning notched strength of woven composites are presented in the papers by Kim and Kim, (1995).

The previous paragraph suggests that an appropriate failure model for notched woven fabric composites would consider the damage growth explicitly. There are a number of essentially similar techniques that treat the problem in this way. A particular stress-displacement relation is assumed within the damage zone and the energy absorbed in separating the crack faces corresponds to the material toughness,  $G_c$ . One of the first models is that of Eriksson and Aronsson (1986) who assumed a constant cohesive stress-displacement relationship within the damage zone. An appropriate analytical "critical damage growth" model was presented by Hitchen et. al (1994), building on earlier work by Soutis and Fleck (1991), this was applied subsequently to woven composites (Belmonte et. al, 2001, 2004). Afaghi-Khatibi et. al. (1996) presented an "equivalent crack growth model" (ECGM) which assumed a linear decrease of cohesive stress with displacement in the damage zone, which they applied successfully to the data of Kim and Kim.

The present work explores further the applicability of this type of approach, referred to currently as cohesive zone modelling (CZM). A limitation of the previous studies is that the material parameters within the models have generally been calibrated against experimental data rather than determined independently. Here we use independent measurements of unnotched strength and toughness to develop the numerical models for woven GFRP and CFRP systems. In the next section details of the numerical procedure are provided. Model predictions are compared with experimental data in subsequent sections.

## Finite Element Method

**Notched Plate Model.** The (two-dimensional) finite element models were implemented using ABAQUS CAE Version 6.9.1. Details of specimen geometry and material properties were taken from the relevant experimental studies. For most of the geometries reported by Belmonte et. al (2001,2004) the plates were 130 mm long and 25 mm wide with notch sizes (hole diameters),  $d$ , of 2.5, 5.0 and 10.0 mm giving  $d/W$  ratios of 0.1, 0.2 and 0.4 respectively. The plate symmetry means that only half of the coupon needs to be modelled, reducing computational cost and time during processing stage. The meshes are refined in the vicinity of notch edge, while away from the notch the mesh can be made coarser.

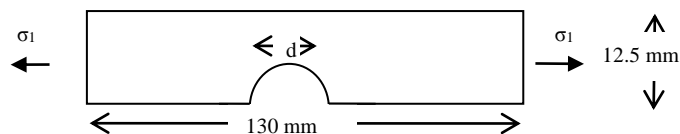


Fig. 1. Geometry dimensions of notched composite plate (after Belmonte et. al, 2001, 2004).

**Material Properties.** Belmonte (2001, 2004) carried out experiments on two different types of woven composite systems, i.e., GFRP and CFRP. The GFRP systems comprised two stacking sequences of a four-layer quasi-isotropic composite [Table 1]. The woven CFRP systems investigated were of plain weave with two different lay-ups (cross-ply, PX and quasi-isotropic, PQ ) and three different plate thickness for each giving 12 systems in total [Table 2]. The measured material properties required for the 2-D FEA are shown in Tables 1 and 2.

Table 1. Material properties for woven GFRP from Belmonte et. al (2001)

Stacking sequence	t (mm)	$E_x$ (GPa)	$E_y$ (GPa)	$\nu_{xy}$	$G_{xy}$ (GPa)	$\sigma_o$ (MPa)	$G_c$ (kJ/m <sup>2</sup> )
(0/90/+45/-45) <sub>s</sub>	1.25	15.85	15.85	0.3	6.096	291	20.25
(90/0/+45/-45) <sub>s</sub>	1.25	16.0	16.0	0.3	6.096	291	20.25

Table 2: Material properties all woven CFRP from Belmonte et. al (2004)

Series	Laminate code	t (mm)	E <sub>x</sub> (GPa)	E <sub>y</sub> (GPa)	ν <sub>xy</sub>	G <sub>xy</sub> (GPa)	σ <sub>o</sub> (MPa)	G <sub>c</sub> (kJ/m <sup>2</sup> )
Plain weave cross-ply (PX)	PX2	0.51	50.36	50.36	0.1026	4.42	481	26.0
	PX4	1.03	51.40	51.40	0.0924	4.42	527	27.7
	PX8	2.03	53.14	53.14	0.0830	4.42	538	22.7
Plain weave quasi-isotropic (PQ)	PQ4	1.02	37.19	37.19	0.3528	13.75	390	21.6
	PQ8	2.03	36.80	36.80	0.3276	13.86	428	17.9
	PQ12	3.17	35.17	35.17	0.2965	13.56	372	18.3

**Damage Growth and Fracture.** As indicated earlier, in the notched plate under increasing tensile load, damage is assumed to grow in a self-similar manner along the plane of maximum tensile stress. Hence only crack opening (mode 1) needs to be considered in the CZM and the form assumed is shown in Figure 2. The work of Belmonte et. al (2001, 2004) provides the unnotched strength (σ<sub>o</sub>) and toughness (G<sub>c</sub>) parameters for each system. The strength was measured using plain rectangular coupons while the toughness was obtained following the ASTM Standard E 399-90 for single-edge notch (SEN) test; the values are given in Tables 1 and 2. [This is in contrast to other approaches where the notched experimental data are used to calibrate models to determine an apparent fracture energy.]

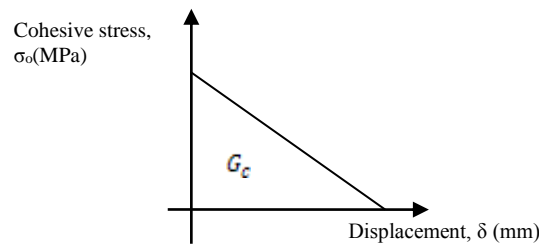


Fig. 2. Physically-based constitutive model used in the current analysis.

To implement the CZM for the open hole problem is straightforward because the failure plane is known a priori. The cohesive zone elements are therefore located along net-section plane. The elements are required to be thin (0.01 mm was used in the current work) and were assigned a high stiffness. The model was made fine enough to assure non-dependency on strength prediction upon mesh refinement. The model was assumed to be in plane stress throughout the study, cohesive elements of COH3D8 were used within cohesive zone region and quadratic elements C3D8 used in un-cracked region.

### Comparison of Strength Prediction with Experimental Data and Closed Forms Approaches

**Quasi-isotropic woven GFRP.** As expected, all of the models showed failure along the net-section plane where the CZM elements were located. Figure 3 shows that at the maximum load in a GFRP specimen containing a 5 mm diameter hole, the CZM elements had been activated and taken to their failure displacement over a distance of about one hole radius from the edge of the hole. Typical load-displacement plots from implementing CZM are shown in Figure 4, in this plot three hole sizes of (0/90/+45/-45)<sub>s</sub> lay-up woven GFRP plates are displayed. The predicted notched strengths are compared with experimental data and various analytical approaches used by Belmonte (2001) in Table 3. It is clear that the CZM gives very reasonable agreement with experimental data, to within less than 10% for most samples. For the (0/90/+45/-45)<sub>s</sub> lay-up the Whitney-Nuismer point and average stress criterion (PSC and ASC) (Whitney and Nuismer, 1974), give excellent agreement as expected (these models are calibrated at one of the notched strength results). The critical damage growth model (CDG) conducted by Belmonte (2001), which is an appropriate analytical approach using the unnotched strength and toughness data as input parameters, also gives good agreement. The (90/0/+45/-45)<sub>s</sub> predictions are reasonable. Note that for this stacking

sequence, the same strength and toughness values were assigned as for the other stacking sequence. The results perhaps suggest that this stacking sequence has a lower strength and/or toughness.

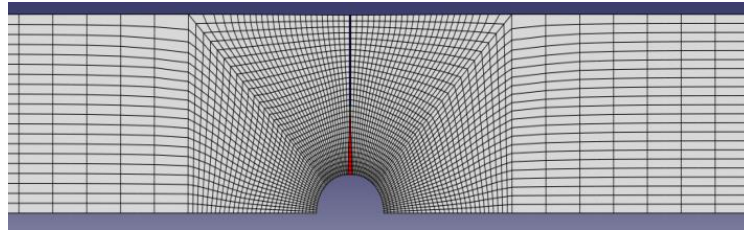


Fig. 3. Crack propagation from notch edge using CZM

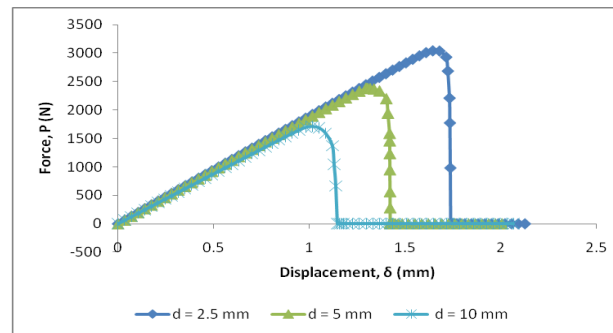


Fig. 4: Typical load-displacement plot implementing CZM approach of GFRP  $(0^\circ/90^\circ/\pm 45^\circ)_s$  lay-up

**Table Error! No text of specified style in document.:** Comparison of GFRP notched strength prediction using CZM with other closed-form solutions (Belmonte, 2001)

Laminate sequence	Notch diameter Size (mm)	d/W Ratio	Experimental Strength, $\sigma_0$ (MPa)	CZM (MPa)	Error (%)	PSC (MPa)	ASC (MPa)	CDG (MPa)
$(0^\circ/90^\circ/\pm 45^\circ)_s$	2.5	0.1	184	194	+5	201	190	194
	5.0	0.2	151	149	+1	152	152	156
	10.0	0.4	112	110	+2	105	108	114
$(90^\circ/0^\circ/\pm 45^\circ)_s$	2.5	0.1	169	188	+11	-	-	-
	5.0	0.2	141	152	+8	-	-	-
	10.0	0.2	103	106	+3	-	-	-

**Cross-ply and Quasi-isotropic woven CFRP.** The results for the cross-ply and quasi isotropic woven CFRP composite systems with different notch sizes are presented in a similar way in Table 4. Overall agreement is very good, within 10% for most systems (the range of error is 0.1% - 11%). Interestingly, the cross-ply systems show a slightly higher discrepancy than the quasi-isotropic systems.

Table 4: Comparison of CFRP notched strength prediction using current numerical approaches with experimental data and other closed-form solution (Belmonte, 2004).

Laminate	Notch diameter Size, d (mm)	d/W Ratio	Experimental Strength, $\sigma_0$ (MPa)	CZM (MPa)	Error (%)	PSC (MPa)	ASC (MPa)	CDG (MPa)
PX2	2.5	0.1	317	317	-0.1	334	340	288
	5.0	0.2	287	255	-11	252	274	225
	10.0	0.4	185	179	-3	175	193	162
PX4	2.5	0.1	342	335	-2	362	368	306
	5.0	0.2	294	271	-8	274	296	237
	10.0	0.4	211	191	-9	190	209	171
PX8	2.5	0.1	349	325	-7	355	362	291

PQ4	5.0	0.2	291	262	-10	269	290	224
	10.0	0.4	195	187	-4	189	206	163
	2.5	0.1	273	283	+4	306	287	275
	5.0	0.2	239	227	-5	235	234	226
	10.0	0.4	170	159	-6	158	165	165
PQ8	2.5	0.1	277	281	+1	299	284	275
	5.0	0.2	234	228	-3	226	227	220
	10.0	0.4	160	161	+1	157	162	162
PQ12	2.5	0.1	254	264	+4	280	263	253
	5.0	0.2	211	211	+0.1	213	213	205
	10.0	0.4	156	148	-5	145	150	150

As for the GFRP, the PSC and ASC from Whitney-Nuismer failure criterion gave the best predictions compared to other approaches, but these are calibrated against the experimental data. Overall the numerical results obtained here using CZM show better prediction than the critical damage growth (CDG) model implemented by Belmonte et. al (2004), especially for the cross-ply systems. The CDG model was formulated based on an isotropic analysis, which could only be corrected partially for the orthotropic cross-ply systems and so the discrepancies are perhaps not surprising.

### Concluding Remarks

2-D modelling of notched woven GFRP and CFRP composites plates has been implemented within FEA framework using ABAQUS CAE Version 6.9.1. A CZM gave very reasonable agreement with the experimental data for all the systems investigated. The attractions of the CZM approach in the present work are that it is physically based (and consistent with experimental observations of damage and fracture) and that it has been implemented using independently measured material properties.

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