

Numerical Modeling and Analytical Validation for Transient Temperature Distribution in a Heterogeneous Geothermal Reservoir due to Cold-Water Reinjection

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ABSTRACT

Reinjection of cooled geothermal fluid after extraction of heat is a common practice in order to maintain the geothermal reservoir pressure, which gradually declines due to continuous extraction of geothermal fluid. Reinjection of geothermal fluid into the geothermal reservoir ensures its safe disposal and enhances the heat recovery the efficiency of the geothermal reservoir for extracting heat energy. But since the injected geothermal fluid is cooler than the geothermal reservoir it generates a cold front near the injection well which propagates through the reservoir domain. Heterogeneity of the geothermal aquifer is also an important factor to consider since homogeneous medium is practically very rare in nature and the thermo-hydrogeological properties of the medium varies in an aquifer.

The present study deals with the modeling of the transient temperature distribution in a heterogeneous geothermal reservoir in response to injection of cold geothermal water. The heterogeneous geothermal aquifer considered here is a confined aquifer with homogeneous layers of finite length and overlain and underlain by impermeable rock media. All the different layers in the aquifer and the overlying and underlying rocks are of different thermo-hydrogeological properties. The numerical modeling for the transient temperature distribution in the porous aquifer is modeled here using a software code DuMu^x. The heat transport modes considered are the advection, conduction and the heat loss to the confining rock media. Results show that heterogeneity plays a very significant role in determining the transient temperature distribution and controlling the advancement of the thermal front in the reservoir. The numerical model developed here is validated in this study using an analytical model. Temperature distribution derived by both methods match with each other quite well.

1. INTRODUCTION

In a geothermal power plant the heat energy of the geothermal water is extracted for power production. The waste-water which is produced after heat extraction is then reinjected back into the geothermal reservoir. One purpose of the reinjection is safe disposal of the thermal wastewater which otherwise could have created thermal pollution if disposed on surface. Moreover reinjection helps in keeping the reservoir pressure intact which gradually declines due to continuous extraction of geothermal fluid. Also according to Bodvarsson and Stefansson (1989) in most of the cases a very small fraction of the thermal energy present in the reservoir can be recovered without the reinjection of geothermal fluid. In spite of having several benefits there lies the possibility of cooling of the production well due to premature breakthrough of the cold-water thermal front generated by the reinjection since the reinjected fluid is much colder than the geothermal reservoir environment. The breakthrough affects the reservoir efficiency to produce power severely. Hence to maintain the reservoir efficiency and longer life of the reservoir, the injection-production well scheme is to be properly designed and injection and extraction rates are to be properly fixed.

Few researchers have studied the transient heat transport in porous media resulting from the thermal injection into it. A mathematical model was developed by Lauwerier (1954) to analyze the movement of the thermal front generated due to the injection of hot-water into an oil reservoir. Bodvarsson (1969) derived analytical models of temperature distribution for flow through a single fracture and laminated solids. The analytical solutions developed by him include constant mass flow input with time-varying temperature and weakly oscillating mass flow. Bodvarsson (1972) presented a simple mathematical model for transient temperature field in the reinjection zone for flow through both porous and fractured media. The paper also discusses about some practical issues related with the siting of reinjection wells. The reservoir lifetime and energy extraction efficiency depends a lot on reinjection of heat depleted water. The heat recovery factor and aquifer production potential is enhanced highly by reinjection process (Gringarten, 1978). Bodvarsson and Tsang (1982) derived analytical solutions to study the thermal behavior into a fractured reservoir with equally spaced horizontal fractures due to cold-water injection. Chen and Reddell (1983) developed another analytical model for injection into a confined aquifer; they derived two unsteady-state solutions, one for long time periods and another for short time periods. Stefansson's (1997) addressed practical issues like thermal breakthrough, silica scaling in surface equipment, siting of reinjection wells and energy recovery from the resource based on the experience gained by reinjection experiments in 44 geothermal fields. Shook (2001) performed tracer test experiments to predict the thermal breakthrough of single-phase fluid in porous media. Stopa and Wojnarowski (2006) examined the propagation of the cold-water thermal front by developing an analytical model where they considered heat capacity, density of rock and water to be functions of the temperature. Li et al. (2010a) did experiments to investigate the effects of temperature and pressure on in situ water saturation due to water injection in a geothermal reservoir. Yang and Yeh (2008), Li et al. (2010b) and Yeh et al. (2012) developed semi-analytical solutions to model the temperature distribution in an aquifer thermal energy storage (ATES) system for a confined aquifer.

No study from the abovementioned ones, considered heterogeneity of the geothermal aquifer. Homogeneity is an idealistic concept and there is always some heterogeneity present in the porous media practically. The objective of the present study is to develop a

two-dimensional numerical model for the transient temperature distribution and the propagation of the thermal front generated due to thermal injection in a heterogeneous geothermal reservoir system. The numerical modeling has been performed using a software code DuMu^x which is capable of modeling single and multiphase, non-isothermal and isothermal flows through porous media. The heat transfer processes taken into account were the advection, conduction and heat transport to the confining rocks. The aquifer considered here is a confined one and is consisted of three vertical layers with different thermo-geological properties which take into account the heterogeneity of the medium. A simpler version of the numerical model for transient temperature distribution in a one-dimensional homogeneous aquifer was compared with a one-dimensional analytical model with an aim to validate the numerical model.

2. MATHEMATICAL MODELING

The mathematical model of heat transport in a geothermal reservoir system is given by a second order partial differential equation for energy conservation in the aquifer domain. The equation is well known form literature and given by

$$\frac{\partial}{\partial t} \left\{ (1-\phi_{1,2,3})\rho_r c_r T(x,y,t) + \phi_{1,2,3}\rho_w c_w T(x,y,t) \right\} + \frac{\partial}{\partial x} \left\{ u_x \rho_w c_w T(x,y,t) \right\} + \frac{\partial}{\partial y} \left\{ u_y \rho_w c_w T(x,y,t) \right\} + q_1 - q_2 = \lambda_{x,1,2,3} \frac{\partial^2 T(x,y,t)}{\partial x^2} + \lambda_{y,1,2,3} \frac{\partial^2 T(x,y,t)}{\partial y^2} \quad (1)$$

where the subscripts 1,2,3 stand for the three layers of the geothermal aquifer, T is the temperature (in K); c_r and c_w are the specific heats of rock and water, respectively (in J/Kg·K); ϕ is the porosity of the aquifer; ρ_r and ρ_w are the densities of the rock and water, respectively (in kg/m³); u_w is the velocity of groundwater (in m/s); λ_x and λ_y are the thermal conductivities of the aquifer in longitudinal and vertical directions, respectively (in W/m·K); t is injection time (in seconds); x and y represents the distances in longitudinal and vertical direction, respectively (in m) and q_1 and q_2 are the heat transfer to the overlying and underlying rocks, respectively. The above heat transport equation coupled with the two-dimensional groundwater flow equation given by Darcy's law, is solved in the model to derive the transient temperature distributions in the model domain.

3. NUMERICAL MODELING

As mentioned earlier the numerical modeling of this study has been performed by software code DuMu^x which is capable of handling both isothermal and nonisothermal single and multiphase flow through porous and fractured media. DuMu^x is written in C++ language and requires knowledge of the language to code specific problems in the software environment. A schematic of the two-dimensional geothermal reservoir with overlying and underlying rocks and the injection-production wells is shown in Fig. 1. The model domain considered here is a confined aquifer of dimensions 600 m×30 m, consisting of three vertical layers of length 265 m, 90 m and 240 m, respectively. The aquifer is underlain and overlain by rock media of thickness 90 m and 80 m, respectively. Cold-water is considered being injected by an injection well at a distance 200 m away from the left end of the aquifer and hot-water extracted by a production well at a distance 200 m from the injection well. The rectangular aquifer domain is discretized by 1200 elements in horizontal direction and 400 elements in vertical direction. The domain is open from in the longitudinal (x) direction i.e. it allows regional groundwater flow in that direction. The overlying and underlying rock media are of low permeability and heat loss occurs from the aquifer by only mode of heat conduction due to the temperature gradient between the aquifer and the rock media. Initial temperature of the aquifer is 80°C. Cold-water is injected at a rate of 300 m³/day, through the injection well having a temperature of 20°C which is assumed as constant throughout the injection time. All the physical and thermal properties used for the modeling study are listed in Table 1.

4. ANALYTICAL SOLUTION

The numerical model here is validated using a one-dimensional analytical solution derived by Ganguly and Mohan Kumar (2014), as two-dimensional analytical solution is unavailable in literature. The analytical solution for transient temperature distribution in a porous geothermal reservoir due to cold-water injection including the convective and conductive modes of heat transfer as well as the heat transfer to the underlying and overlying rocks is given by

$$\begin{aligned} T = T_0 - \frac{2}{\pi^{1/2}} (T_0 - T_m) \exp\left(\frac{Ux}{2\lambda}\right) \int_0^\infty \exp\left(-\zeta^2 - \frac{U^2 x^2}{16\lambda^2 \zeta^2}\right) \operatorname{erfc}\left\{\frac{\alpha x^2}{8\lambda \zeta^2 \left(t - \frac{Cx^2}{4\lambda \zeta^2}\right)^{1/2}}\right\} d\zeta \\ - \frac{2}{\pi^{1/2}} \frac{(\omega - \alpha T_0)}{\alpha} \exp\left(\frac{Ux}{2\lambda}\right) \int_0^\infty \exp\left(-\zeta^2 - \frac{U^2 x^2}{16\lambda^2 \zeta^2}\right) \cdot \left[\operatorname{erfc}\left\{\frac{\alpha x^2}{8\lambda \zeta^2 \left(t - \frac{Cx^2}{4\lambda^2 \zeta^2}\right)^{1/2}}\right\} \right. \\ \left. - \exp\left\{\frac{\alpha^2 x^2}{4\lambda^2 \zeta^2 C} + \frac{\alpha^2}{C^2} \left(t - \frac{Cx^2}{4\lambda^2 \zeta^2}\right)\right\} \cdot \operatorname{erfc}\left\{\frac{\alpha x^2}{8\lambda^2 \zeta^2 \left(t - \frac{Cx^2}{4\lambda^2 \zeta^2}\right)^{1/2}} + \frac{\alpha}{C} \left(t - \frac{Cx^2}{4\lambda^2 \zeta^2}\right)^{1/2}\right\} \right] d\zeta \\ + \frac{(\omega - \alpha T_0)}{\alpha} \left\{ 1 - \exp\left(\frac{\alpha^2}{C^2} t\right) \cdot \operatorname{erfc}\left(\frac{\alpha}{C} t^{1/2}\right) \right\} \end{aligned} \quad (2)$$

Where, T_m and T_0 are the injection water temperature and the temperature of the geothermal aquifer prior to cold-water injection, respectively; $C=(1-\phi)\rho_r c_r + \phi\rho_w c_w$; $U=\rho_w c_w u_w$; $\alpha=(C_1\lambda_1)^{1/2} + (C_2\lambda_2)^{1/2}$ and $\omega=(C_1\lambda_1)^{1/2}T_{01} + (C_2\lambda_2)^{1/2}T_{02}$. The lower limit of the integration in Eq. (1) is given by

$$l = \frac{x}{2} \left(\frac{C}{\lambda t} \right)^{1/2} \quad (3)$$

Evidently the second term of right hand side in Eq. (1) emerges due to the difference in initial temperature of the aquifer and the injection water temperature and has greater contribution to the solution. The third and fourth term contributes to the solution due to the difference in temperature between the geothermal reservoir.

5. RESULTS AND DISCUSSION

The thermo-geological properties of the three layers of the heterogeneous porous aquifer and that of the fluid are enlisted in Table 1. At the right boundary of the domain a pressure of 30.0 kPa and a temperature of 80°C (293 K) are considered as boundary conditions whereas a pressure of 31.2 kPa and the same temperature of 80°C are considered to be those for the left boundary. Hence there is a regional groundwater flow driven by the gradient from left to right existing prior to the injection. It is to be noticed that the temperature of the right and left boundaries are kept equal to the initial temperature of the geothermal aquifer which means the conditions are valid until the cold-water thermal front reaches those boundaries. After that the temperature of those boundaries starts increasing. The cold-water at a temperature of 20°C is considered being injected at a rate of 300 m³/day through an injection well situated 200 m away from the left end. Hot water is extracted by a production well at 400 m from the left end. Injection and extraction wells are considered to be fully penetrating.

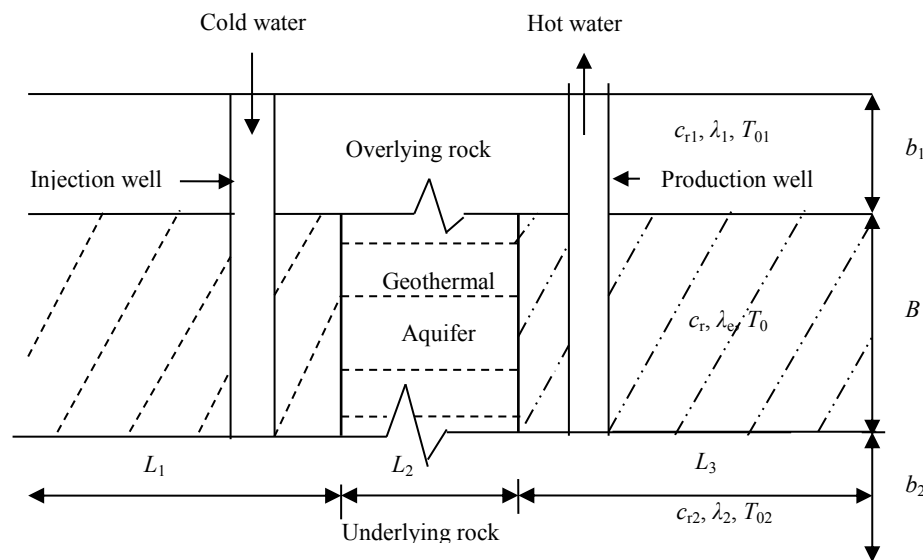


Fig. 1 Schematic representation of the geothermal reservoir with two wells installed

The temperature distribution plots in the heterogeneous aquifer domain at different injection times have been shown in Fig. 2. The two-dimensional plots show that owing to continuous injection of cold-water into hot aquifer environment a thermal interface or a thermal front is set up which propagates through the aquifer with time in both the directions. The temperature of the aquifer also decreases gradually with the passage of injection time due to the advancement of the thermal front, the effect of which is most pronounced around the injection well. Sharp changes in the temperature plots are visible when the thermal front enters the second and third layers. The change in trend of the temperature distribution is triggered by the change in thermo-hydrogeological properties of different layers present in the geothermal aquifer, which implies the properties and thus heterogeneity have profound effect on the temperature distribution and movement of the thermal front. The temperature plots also show that the cold-water thermal front reaches the production well at 63 days, which implies the occurrence of thermal-breakthrough. After the breakthrough the temperature of the geothermal fluid at the production well decreases and the geothermal reservoir loses its efficiency for power production.

Table 1. Parameters of the rock and fluid used

Parameter name	Symbol (unit)	Layer 1	Layer 2	Layer 3
Length	L (m)	265	95	245
Specific heats of rock	c_r (J/kg·K)	850	1560	1000
Density of the porous aquifer	ρ_r (kg/m ³)	2650	1047	1950
Thermal conductivity of the aquifer (longitudinal)	λ_x (W/ m·K)	2.8	0.6	1.2
Thermal conductivity of the aquifer (vertical)	λ_y (W/ m·K)	1.0	0.3	0.7
Porosity of the aquifer	ϕ	0.15	0.20	0.10
Density of the fluid	ρ_w (kg/m ³)	985	985	985
Specific heats of fluid	c_w (J/kg·K)	4180	4180	4180

Temperature distribution plots for a homogeneous aquifer, the properties of which is equal to the first layer of the heterogeneous aquifer (Fig. 1) are presented in Fig. 3. The plots show due to absence of layers of different properties, the movement of the thermal front is smooth and no change of trend is noticed. The movement of the thermal front in the heterogeneous aquifer also differs from that in the homogeneous. For instance at 30 days of injection times the thermal front reaches 132 m and 118 m for the heterogeneous and homogeneous aquifers, respectively. The thermal-breakthrough for the homogeneous geothermal reservoir occurs at 67 days compared to 63 days for the heterogeneous ones.

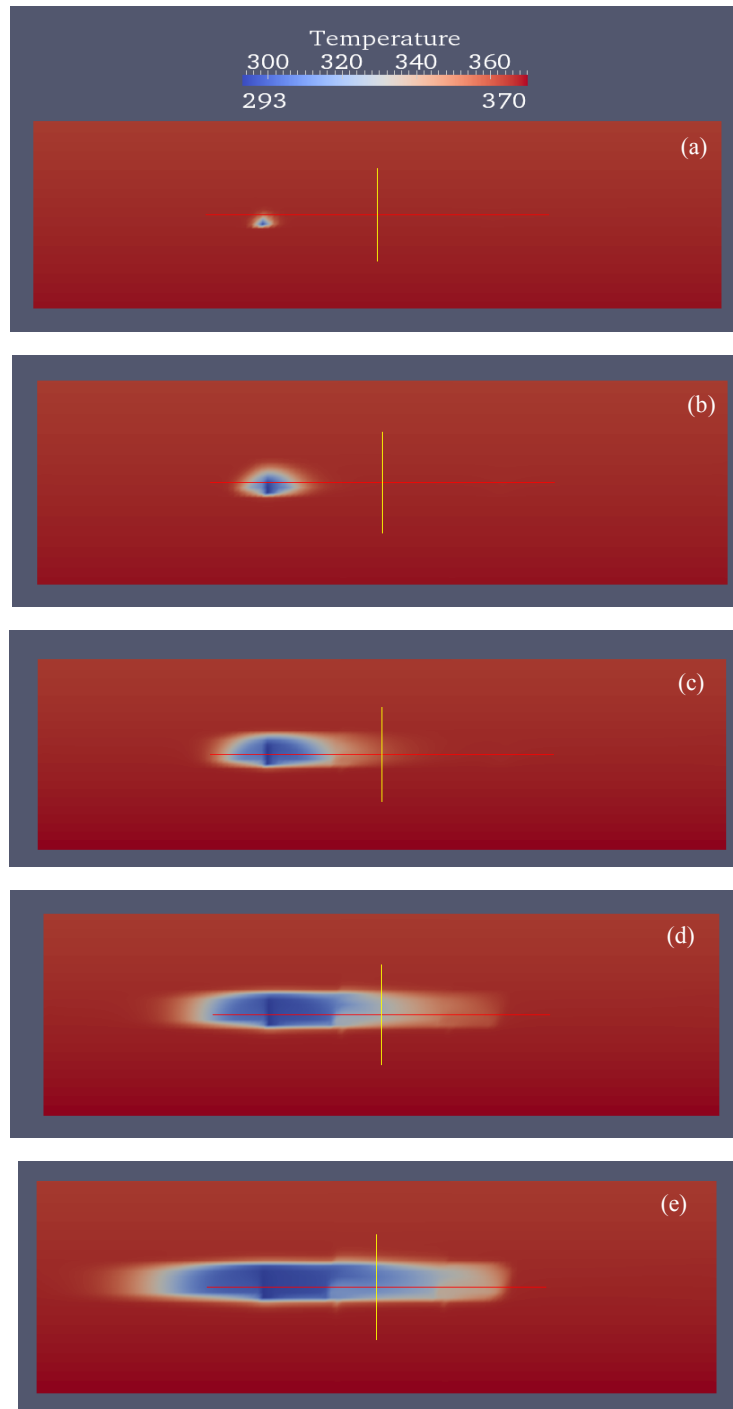


Fig. 2 Temperature distribution in the heterogeneous geothermal reservoir at (a) 1 day, (b) 10 days, (c) 30 days, (d) 60 days and (e) 100 days .

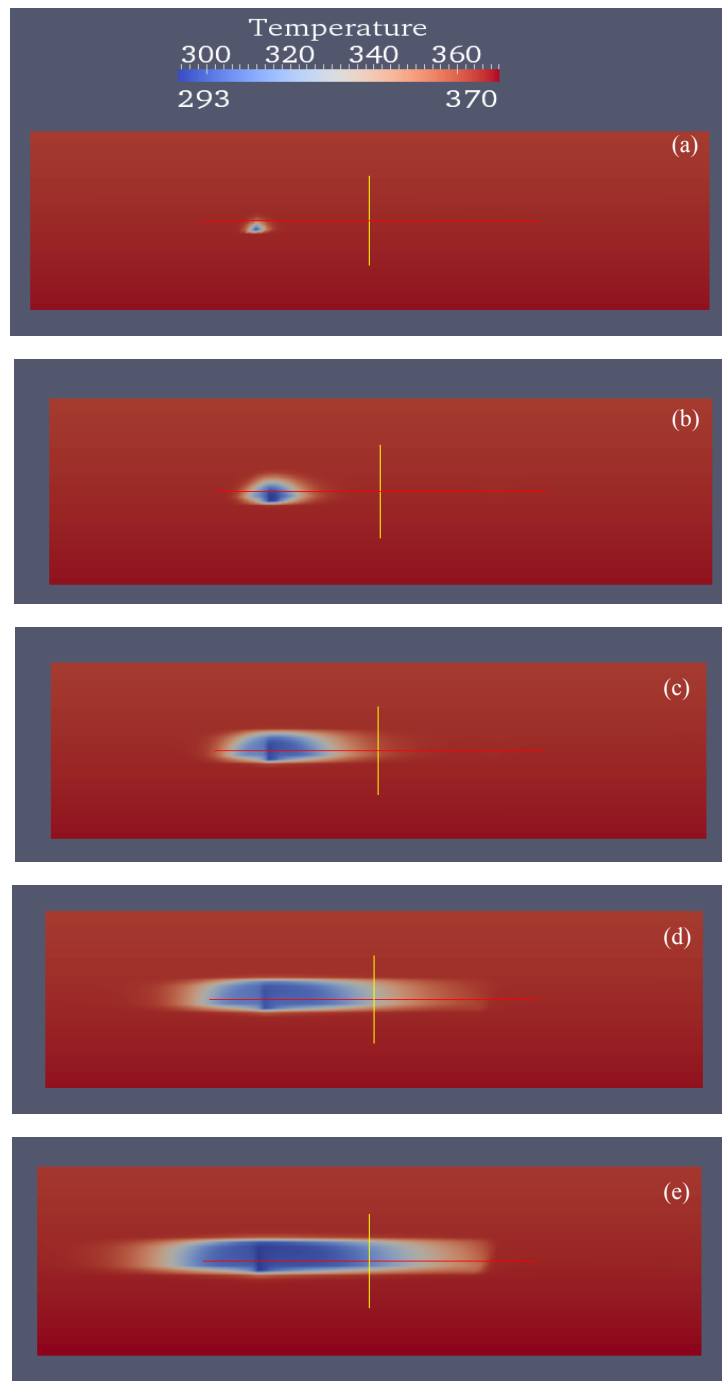


Fig. 3 Temperature distribution in the homogeneous geothermal reservoir at (a) 1 day, (b) 10 days, (c) 30 days, (d) 60 days and (e) 100 days .

The present numerical model is needed to be validated to check the efficiency and accuracy of the model in prediction of transient temperature distribution in the geothermal reservoir. As no two-dimensional analytical solutions are available in literature, a simpler version of the present model is validated here using one-dimensional analytical solution in Eq. (2). For this the geothermal aquifer is considered to be homogeneous and it has properties same as those of the first layer of the heterogeneous aquifer throughout the length and the thickness of the geothermal aquifer is considered negligible. The other geological conditions and boundary conditions are kept same as that of the model. The one-dimensional temperature plots are derived by both analytical (Eq. 2) and numerical model (DuMu^x) for two different injection times of 1 day and 30 days and shown in Fig. 6.8. The plots of temperature distribution derived by both the methods match very well with each other, implying that this model is suitable for determining the transient temperature distribution in a geothermal reservoir due to cold-water injection.

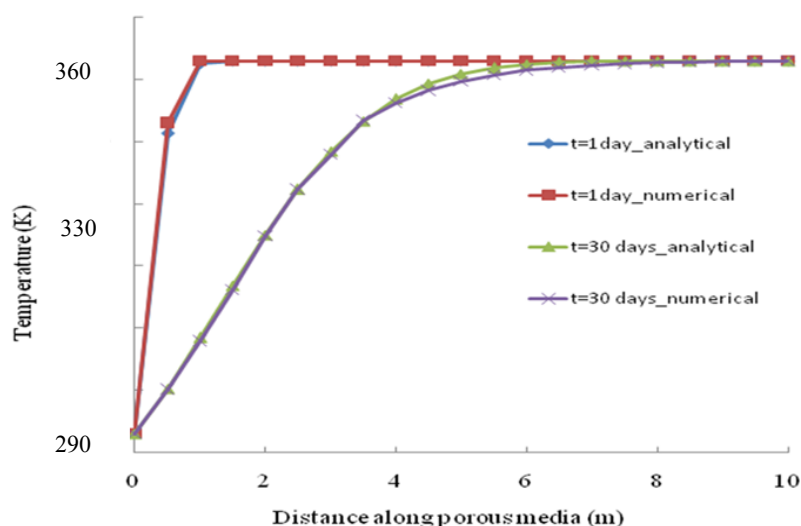


Fig. 4 Comparison between temperature distribution plots derived numerically (DuMu^x) and analytically (Eq. 2).

6. CONCLUSIONS

A two-dimensional numerical model for heat transport in a heterogeneous porous geothermal reservoir overlain and underlain by impermeable rocks is presented here. The primary target was to model the movement of the thermal front with time generated in the aquifer due to the cold-water injection in a heterogeneous geothermal reservoir consisted of layers of different thermo-hydrogeological properties. Temperature distribution for a homogeneous geothermal aquifer is also derived with an aim to compare the results with that of the heterogeneous one. The numerical code used here has been validated with an analytical solution for the heat transfer in a simple homogeneous aquifer. The conclusions which can be drawn from study are

1. With continuous injection of cold-water a thermal front is set up in the aquifer and proceeds with passage of injection time. The aquifer temperature thus increases gradually with time at a fixed distance and decreases with increasing longitudinal distance at a fixed injection time.
2. The thermo-hydrogeological properties in a heterogeneous porous aquifer influence the transient heat transfer phenomenon. The temperature distribution in a heterogeneous geothermal aquifer changes trend when the thermal front enters layers of different properties. Thus heterogeneity plays a vital role in controlling the movement of the thermal front and determination of the properties of different layers is very essential.
3. The temperature distribution in a homogeneous geothermal aquifer is smoother and shows no change of trend. The movement of the thermal front differs from that in the heterogeneous one.
4. The results for heat transfer in homogeneous aquifer due to thermal injection derived numerically agrees with the results derived from the analytical solution very well.

Lastly the present two-dimensional model gives some insight into the problem of transient heat transport phenomenon in a heterogeneous porous geothermal aquifer due to cold-water injection into it. The results presented here can be effectively used in design of the injection-production well scheme in a heterogeneous geothermal reservoir system. The model can also serve as a reference solution to more complex numerical models.

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