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ON THE ADAPTIVE CONTROL OF MECHANICAL SYSTEMS WITH TIME-VARYING PARAMETERS AND DISTURBANCES

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ABSTRACT

In this paper, we propose and investigate a new adaptive control algorithm for mechanical systems with time-varying parameters and/or time-varying disturbances. The proposed method does not assume any structure to the time-varying parameter or disturbance. The idea is based on the expansion of the time-varying parameter/disturbance using the Taylor series expansion; this facilitates expanding a time-varying function as a finite length polynomial and a bounded residue; the coefficients of the finite length polynomial are estimated in a time interval small enough so that they can be assumed to be constant within that interval. A novel experiment is designed using a two-link mechanical manipulator to investigate the proposed algorithm experimentally. Simulation and experimental results validate the proposed new adaptive control algorithm; we discuss these results and also give some future research directions.

1 Introduction

Broadly, adaptive control refers to control of partially known systems. It is rare that the control designer knows the true parameters of the system being controlled. It is this uncertainty in the knowledge of the system parameters that has led to continued strong interest in adaptive control research; adaptive control has been one of the most important research activity among control researchers since the 1950's. Significant amount of the published research has focused on unknown parameters being constant. Limited amount of research has been done in adaptive control of nonlinear systems with time-varying parameters. The amount of research in adaptive control assuming that the uncertain parameters are constant far overshadows research in adaptive control of systems with time-varying parameters. As pointed out in (1), one of the compelling reasons for considering adaptive methods in practical applications is to compensate for large variations in plant parameter values. The focus of this paper is on the design of a practical adaptive control algorithm for time-varying mechanical systems; which are an important practical class of nonlinear systems with time-varying parameters.

High performance tracking control of mechanical systems is essential in a number of industrial applications; examples include, material handling and part assembly. In many of the industrial applications the mechanical system dynamics is time-varying due to a time-varying payload and/or time-varying disturbances. Examples of such applications include pouring and filling operations using robots. Also, time-varying disturbances are common in a large-number of mechanical system applications. There has been an increase in recent research activity in adaptive control of time-varying systems. But most of this research has focused on assuming worst case bounds for time-varying parameters and/or their derivatives; an amalgam of adaptive control and robust control have been used in the control designs with the gains of the controller chosen based on the worst case bounds. The resultant controllers, although stable, gave rise to large and at times practically unbounded control inputs.

In (2), a robust switching controller is designed a the time-varying parameter model of the robot manipulators performing path tracking tasks. Properties of the element by element product

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of matrices is used to isolate the time-varying parameters from the inertia matrix. A robust adaptive control for robot manipulators consisting of slowly time-varying parameters is presented in (3). A smooth robust adaptive sliding mode controller is given in (4). A robust adaptive control algorithm subject to bounded disturbances and bounded and (possibly) time-varying parameters is given in (5); it is shown that the proposed controller achieves asymptotic tracking if the disturbances vanish and the parameters are constant. In (6), a new adaptive controller for time-varying mechanical systems is proposed based on the assumption that the time-varying parameters are given by a group of known bounded time functions and unknown constants. A time-scaling technique of mapping one cycle period of the desired trajectory into a unit interval is proposed to provide robustness to the parameter adaptation algorithms. A novel experimental platform consisting of a two-link manipulator with time-varying payload that mimics filling and pouring operations was built to verify the proposed adaptation algorithm experimentally.

In this paper, we do not assume any particular structure to the time-varying parameters and disturbances; that is, we consider the unknown time-varying parameters and disturbances to be general unknown time-varying functions. We express the general time-varying function as a finite ($(p-1)$-th order) length polynomial in time and a residue based on a result related to Taylor's formula(7). We compute the bound on the residue by assuming that a bound on the p-th derivative of the function is available. In the proposed new adaptive controller, the coefficients of the polynomial are estimated in a small time interval so that they can be assumed to be constant; and a robustness term is used in the controller to compensate for the unknown residue; the robustness term is much smaller than what is generally used in robust control literature and is proportional to the choice of the time interval chosen for estimation. To validate the proposed adaptive controller, an experiment is designed on a two-link manipulator platform. We use the second-link of the two-link planar manipulate to generate time-varying disturbances in the first link. We also show simulation results of the proposed adaptive controller and compare it with a classical robust control algorithm that is available in literature.

2 Time-Varying Dynamics

The dynamic equations of an n degree-of-freedom mechanical system with time-varying parameters and disturbances are (6) given by

$$M(q,\phi)\ddot{q}+C(q,\dot{q},\phi)\dot{q}+F(q,\dot{\phi})\dot{q}+g(q,\phi)=\tau+d(t) \qquad (1)$$

where $q \in \mathbb{R}^n$ are generalized coordinates, $M(q) \in \mathbb{R}^{n\times n}$ is the positive definite inertia matrix, $C(q,\dot{q}) \in \mathbb{R}^{n\times n}$ is the matrix composed of Coriolis and centrifugal terms, $g(q) \in \mathbb{R}^n$ is the gravity vector, $F(q,\dot{\phi}) \in \mathbb{R}^{n\times n}$ is a symmetric matrix, $\phi \in \mathbb{R}^m$ is the vector of constant and/or time-varying parameters, $\tau \in \mathbb{R}^n$ is the vector of control inputs, and $d(t) \in \mathbb{R}^n$ is the vector of time-varying disturbances. The properties of the dynamic model (1) are:

Property I: The inertia matrix, $M(q,\phi)$, of the time-varying mechanical system is a symmetric positive definite matrix. Assuming $\phi(t)$ is bounded, $M(q,\phi)$ is bounded from above and below for all system configurations.

Property II: $F(q,\dot{\phi})$ is a symmetric matrix, which is a consequence of the symmetry of the inertia matrix.

Property III: The matrix $\dot{M}(q,\phi)-2C(q,\dot{q},\phi)-F(q,\dot{\phi})$ is skew-symmetric. Notice that the skew-symmetry property for the time-varying case is different from that of the time-invariant case.

Property IV: The dynamic equation (1) is linear in the unknown parameters, i.e.,

$$\begin{aligned} M(q,\phi)\ddot{q}+C(q,\dot{q},\phi)\dot{q}+F(q,\dot{\phi})\dot{q}+g(q,\phi) \\ =Y_1(q,\dot{q},\ddot{q})\phi+Y_2(q,\dot{q})\dot{\phi} \end{aligned} \qquad (2)$$

where $Y_1(q,\dot{q},\ddot{q})$ and $Y_2(q,\dot{q})$ are the regressor matrices corresponding to $\phi(t)$ and $\dot{\phi}(t)$, respectively.

3 Representation of Time-Varying Functions

To represent a general time-varying function, we invoke the following result (7):

Lemma 1. *Let I be an open interval in $\mathbb{R}$, and f be a p-times continuously differentiable function of I into $\mathbb{R}$; then, for any pair of points t_0, t in I*

$$\begin{aligned} f(t)=&f(t_0)+\frac{(t-t_0)}{1!}f^{(1)}(t_0)+\cdots+\frac{(t-t_0)^{p-1}}{(p-1)!}f^{(p)}(t_0) \\ &+\int_{t_0}^{t}\frac{(t-\xi)^{p-1}}{(p-1)!}f^{(p)}(\xi)d\xi \end{aligned} \qquad (3)$$

where $f^{(p)}(\cdot)$ stands for the p-th derivative of the function $f(\cdot)$.

With lemma 1, we can represent an arbitrary time-varying parameter, $\phi(t)$, as follows:

$$\phi(t)=\sum_{i=0}^{p-1}k_i(t-t_0)^i+\delta_\phi \qquad (4)$$

where

$$k_i = \frac{\phi^{(i)}(t_0)}{i!} \quad \text{and} \quad \delta_\phi = \int_{t_0}^{t} \frac{(t-\xi)^{p-1}}{(p-1)!}\phi^{(p)}(\xi)d\xi.$$

Notice that k_i and δ_ϕ are vectors of the same size as that of ϕ. Suppose that the p-th derivative of $\phi(t)$ is bounded, i.e., $\sup_t \|\phi^{(p)}(t)\| \leq c_p$, we can bound δ_ϕ as

$$\|\delta_\phi\| \leq \frac{c_p(t-t_0)^p}{p!}. \tag{5}$$

The motivation for the representation of $\phi(t)$ as given in (4) is as follows. If the interval $(t-t_0)$ is chosen small, then the coefficients k_i can be assumed to be constant and can be estimated on-line during that time-interval. The bound on δ_ϕ given by (5) depends on c_p, $(t-t_0)$, and p. The bound on δ_ϕ can be made small by either choosing a small $(t-t_0)$ or a large p even in the case when c_p is large.

The time-derivative of $\delta_\phi(t)$ can be obtained by using Leibnitz rule [1] of differentiating an integral with variable limits, and is given by

$$\dot{\delta}_\phi = \int_{t_0}^{t} \frac{(t-\xi)^{p-2}}{(p-2)!}\phi^{(p)}(\xi)d\xi. \tag{6}$$

With the bound on $\phi^{(p)}(t)$ we can obtain a bound on $\dot{\delta}_\phi$ as

$$\|\dot{\delta}_\phi\| \leq \frac{c_p(t-t_0)^{p-1}}{(p-1)!}. \tag{7}$$

In a similar fashion, one can represent the time-varying disturbance $d(t)$ as follows:

$$d(t) = \sum_{i=0}^{r-1} l_i(t-t_0)^i + \delta_d. \tag{8}$$

If we assume that $\sup_t \|d^{(p)}(t)\| \leq c_r$ then

$$\|\delta_d\| \leq \frac{c_r(t-t_0)^r}{r!}. \tag{9}$$

[1] Leibnitz rule

$$\frac{d}{dt}\left[\int_{\theta(t)}^{\psi(t)} f(x,t)dx\right] = \int_{\theta(t)}^{\psi(t)} \frac{\partial f(x,t)}{\partial t}dx - \frac{d\theta(t)}{dt}f(\theta(t),t) + \frac{d\psi(t)}{dt}f(\psi(t),t)$$

It should be observed that the bounds on δ_θ and δ_d depend on the choice of the time interval $(t-t_0)$ and the number of terms in the finite length polynomial.

4 Adaptive Control Design

We consider the trajectory tracking problem for the mechanical system with time-varying parameters. Let $q_d(t)$ be the desired trajectory. It is assumed that $q_d(t)$ is twice continuously differentiable. Let $e = q(t) - q_d(t)$ be the joint tracking error, and $e_v = \dot{e} + \Lambda e$ be the reference velocity error. Consider the control law, τ, given by

$$\begin{aligned}\tau = &- K_v e_v + Y_1(q,\dot{q},\dot{q}_r,\ddot{q}_r)\hat{k}_0 - \sum_{i=0}^{r-1}\hat{l}_i(t-t_0)^i \\ &+ \sum_{i=1}^{p-1}(t-t_0)^{i-1}\Big[(t-t_0)Y_1(q,\dot{q},\dot{q}_r,\ddot{q}_r) \\ &+ \frac{i}{2}Y_2(q,\dot{q}+\dot{q}_r)\Big]\hat{k}_i - \Big(\|Y_1(q,\dot{q},\dot{q}_r,\ddot{q}_r)\|\|\delta_\phi\| \\ &+ \frac{1}{2}\|Y_2(q,\dot{q}+\dot{q}_r)\|\|\dot{\delta}_\phi\| + \|\delta_d\|\Big)\mathrm{sgn}(e_v)\end{aligned} \tag{10}$$

where $\dot{q}_r = \dot{q}_d - \Lambda e$, K_v and Λ are positive definite gain matrices, $\hat{k}_i$ and $\hat{l}_i$ are estimates of k and l, respectively, and $\mathrm{sgn}(e_v)$ represents the component-wise sign vector of e_v. Substitution of the control input (10) into the dynamic equation (1) and simplifying using the linear parameterization property, i.e., Property IV, we obtain the error dynamics in terms of e_v

$$\begin{aligned}&M(q,\phi)\dot{e}_v + C(q,\dot{q},\phi)e_v + \frac{1}{2}F(q,\dot{\phi})e_v + K_v e_v \\ =&Y_1(q,\dot{q},\dot{q}_r,\ddot{q}_r)\tilde{k}_0 + \sum_{i=1}^{p-1}W_i(q,\dot{q},\dot{q}_r,\ddot{q}_r,t,i)\tilde{k}_i \\ &- \sum_{i=0}^{r-1}\tilde{l}_i(t-t_0)^i - \Big(\|Y_1(q,\dot{q},\dot{q}_r,\ddot{q}_r)\|\|\delta_\phi\| \\ &+ \frac{1}{2}\|Y_2(q,\dot{q}+\dot{q}_r)\|\|\dot{\delta}_\phi\| + \|\delta_d\|\Big)\mathrm{sgn}(e_v) \\ &- Y_1(q,\dot{q},\dot{q}_r,\ddot{q}_r)\delta_\phi - \frac{1}{2}Y_2(q,\dot{q}+\dot{q}_r)\dot{\delta}_\phi + \delta_d\end{aligned} \tag{11}$$

where $\tilde{k}_i = \hat{k}_i - k_i$, $\tilde{l}_j = \hat{l}_j - l_j$, and

$$\begin{aligned}W_i(q,\dot{q},\dot{q}_r,\ddot{q}_r,t,i) = &(t-t_0)^i Y_1(q,\dot{q},\dot{q}_r,\ddot{q}_r) \\ &+ (t-t_0)^{i-1}\frac{i}{2}Y_2(q,\dot{q}+\dot{q}_r).\end{aligned}$$

To estimate the unknown parameters, k_i and l_i, we use the gradient projection algorithm given in (9; 8), which we briefly illustrate in the following. Consider the parameter set Π_i given by

$$\hat{k}_i = (\hat{k}_{i1}, \cdots, \hat{k}_{im})^T \in \Pi_i \iff |\hat{k}_{ij} - \rho_{ij}| < \sigma_{ij}, \ \forall j \in \{1, m\} \tag{12}$$

with ρ_{ij} and σ_{ij} some given real numbers. Consider the function

$$\mathcal{P}(\hat{k}_i) = \frac{2}{\varepsilon}\left[\sum_{j=1}^{m}\left|\frac{\hat{k}_{ij} - \rho_{ij}}{\sigma_{ij}}\right|^q - 1 + \varepsilon\right] \tag{13}$$

where $0 < \varepsilon < 1$ and $q \geq 2$. Now, consider the "smooth projection" Proj, which will be used to estimate $\hat{k}_i$ while maintaining it in Π_i:

$$Proj(p,y) = \begin{cases} y, & \text{if } \mathcal{P}(\hat{k}_i) \leq 0. \\ y, & \text{if } \mathcal{P}(\hat{k}_i) \geq 0 \quad \text{and} \\ & \quad \frac{\partial \mathcal{P}}{\partial \hat{k}_i}(\hat{k}_i) y \leq 0. \\ y - \dfrac{\mathcal{P}(\hat{k}_i)\frac{\partial \mathcal{P}}{\partial \hat{k}_i}(\hat{k}_i) y}{\left\|\frac{\partial \mathcal{P}}{\partial \hat{k}_i}(\hat{k}_i)\right\|^2}\left[\frac{\partial \mathcal{P}}{\partial \hat{k}_i}(\hat{k}_i)\right]^T, & \\ & \text{otherwise.} \end{cases} \tag{14}$$

where $\nabla \mathcal{P} = \dfrac{\partial \mathcal{P}(\hat{k}_i)}{\partial \hat{k}_i}$. Based on the smooth projection defined above, the following estimation algorithms are designed for $\hat{k}_i$ and $\hat{l}_i$:

$$\dot{\hat{k}}_0 = Proj(\hat{k}_0, -\Gamma_0 Y_1^T(q, \dot{q}, \dot{q}_r, \ddot{q}_r)e_v), \tag{15}$$

$$\dot{\hat{k}}_i = Proj(\hat{k}_i, -\Gamma_i W_i^T(q, \dot{q}, \dot{q}_r, \ddot{q}_r, t, i)e_v), \quad \forall\, i \in \{1, p-1\}, \tag{16}$$

$$\dot{\hat{l}}_j = Proj(\hat{l}_j, \Gamma_{dj}(t - t_0)^j e_v), \ \forall\, j \in \{0, r-1\}. \tag{17}$$

where $\Gamma_0, \Gamma_i,\ i = 1:m$, and $\Gamma_{dj},\ j = 1:n$, are symmetric positive definite gain matrices. The following theorem gives the stability of the closed-loop error dynamics.

Theorem 4.1. *For the time-varying mechanical system given by* (1)*, the proposed adaptive control law given by* (10) *together with the parameter estimation algorithms given by* (15), (16), (17)*, and with the knowledge of the bounds given in Section 3, the errors e and $\dot{e}$ converge to zero asymptotically and the parameter estimates are bounded.*

Proof: Consider the Lyapunov function candidate:

$$V(e_v, \tilde{k}_i, \tilde{l}_i, t) = \frac{1}{2}e_v^T M(q,\phi)e_v + \frac{1}{2}\tilde{k}_0^T \Gamma_0^{-1}\tilde{k}_0 + \frac{1}{2}\sum_{i=1}^{p-1}\tilde{k}_i^T \Gamma_i^{-1}\tilde{k}_i + \frac{1}{2}\sum_{j=0}^{r-1}\tilde{l}_j^T \Gamma_{dj}^{-1}\tilde{l}_j. \tag{18}$$

Differentiating the Lyapunov function candidate along the trajectories of the closed-loop system (11), using Property III, and simplifying we obtain

$$\begin{aligned} \dot{V} = &- e_v^T K_v e_v + \Big(\tilde{k}_0^T Y_1^T(q,\dot{q},\dot{q}_r,\ddot{q}_r)e_v \\ &+ \tilde{k}_0^T \Gamma_0^{-1} Proj(\hat{k}_0, -\Gamma_0 Y_1^T(q,\dot{q},\dot{q}_r,\ddot{q}_r)e_v)\Big) \\ &+ \sum_{i=1}^{p-1}\Big(\tilde{k}_i^T W_i(q,\dot{q},\dot{q}_r,\ddot{q}_r,t,i)e_v \\ &+ \tilde{k}_i^T Proj(\hat{k}_i, -\Gamma_i W_i^T(q,\dot{q},\dot{q}_r,\ddot{q}_r,t,i)e_v)\Big) \\ &+ \sum_{j=0}^{r-1}\Big(-\tilde{l}_j^T(t-t_0)^j e_v + \tilde{l}_j^T \Gamma_{dj}^{-1} Proj(\hat{l}_j, \\ &\Gamma_{dj}(t-t_0)^j e_v)\Big) - \Big(\|Y_1(q,\dot{q},\dot{q}_r,\ddot{q}_r)\|\|\delta_\phi\| \\ &+ \frac{1}{2}\|Y_2(q,\dot{q}+\dot{q}_r)\|\|\dot{\delta}_\phi\| + \|\delta_d\|\Big)e_v^T \operatorname{sgn}(e_v) \\ &- e_v^T\Big(Y_1(q,\dot{q},\dot{q}_r,\ddot{q}_r)\delta_\phi + \frac{1}{2}Y_2(q,\dot{q}+\dot{q}_r)\dot{\delta}_\phi - \delta_d\Big) \end{aligned} \tag{19}$$

With the parameter estimation algorithms given by (15), (16), (17), the following are true using (14) (see (8)):

$$\begin{aligned} &\tilde{k}_0^T\big(Y_1^T(q,\dot{q},\dot{q}_r,\ddot{q}_r)e_v \\ &\quad + \Gamma_0^{-1}Proj(\hat{k}_0, -\Gamma_0 Y_1^T(q,\dot{q},\dot{q}_r,\ddot{q}_r)e_v)\big) \leq 0, \\ &\tilde{k}_i^T\big(W_i^T(q,\dot{q},\dot{q}_r,\ddot{q}_r,t,i)e_v \\ &\quad + \Gamma_i^{-1}Proj(\hat{k}_i, -\Gamma_i W_i^T(q,\dot{q},\dot{q}_r,\ddot{q}_r,t,i)e_v)\ \big) \leq 0, \\ &\tilde{l}_j^T\Big(-(t-t_0)^{j-1}e_v + \Gamma_{dj}^{-1}Proj(\hat{l}_j, \Gamma_{dj}(t-t_0)^j e_v)\Big) \leq 0. \end{aligned}$$

Hence, the derivative of the Lyapunov function candidate satisfies

$$\dot{V} \leq -e_v^T K_v e_v \tag{20}$$

This implies that $e_v, \tilde{k}_0, \tilde{k}_i, \tilde{l}_i \in L_\infty$, and $e_v \in L_2$. Further, from (11) $\dot{e}_v \in L_\infty$. Therefore, invoking Barbalat's lemma, e_v asymptotically converges to zero. Since $e_v = \dot{e} + \Lambda e$, both $e(t)$ and $\dot{e}(t)$ asymptotically converge to zero.

5 Simulation Results

In this section, the proposed adaptive controller is compared with a traditional robust control via computer simulations. We consider a single-link robot with the dynamics given by

$$I(t)\ddot{q}+\dot{I}(t)\dot{q}+f_c sgn(\dot{q})+f_v\dot{q}=\tau+d(t) \qquad (21)$$

where $I(t)$, q, f_c, f_v, τ, $d(t)$ are the moment of inertia, angular position, Coulomb friction coefficient, viscous friction coefficient, control effort and external disturbance, respectively. The objective is to control the link to track a desired trajectory $q_d = 0.5\sin(\pi t)$(rad) with $f_v = 0.01$(N-m-s), $f_c = 3$(N-m), $I(t) = 3+1.5\sin(4\pi t)$ (Kg-m^2), and $d(t) = 120\sin(3\pi t)$ (N-m).

5.1 Robust control

Assuming that the friction coefficients are exactly known, we can choose the following robust control law

$$\begin{aligned}\tau &= \widehat{I}\ddot{q}_r+f_v\dot{q}+f_c sgn(\dot{q})+u_\delta-F_v e_v-U_\delta sgn(e_v),\\ U_\delta &\geq |\widetilde{I}||\ddot{q}_r|+|\dot{I}||\dot{q}|+|d|,\end{aligned} \qquad (22)$$

where $\widehat{I}$ is the estimate of I, and $\widetilde{I}=\widehat{I}-I$. The controlled system is asymptotically stable. However the robust control term U_δ is very large which will results in control chattering. If we choose $\widehat{I}=3$, U_δ should satisfy

$$U_\delta \geq 1.5|\ddot{q}_r|+18.8|\dot{q}|+120. \qquad (23)$$

The simulation results are shown in Figures 1, 2, and 3. Figure 1 shows the angular position, velocity and tracking error, respectively. The time-varying moment of inertia and disturbance are shown in Figure 2. Figure 3 displays the control effort by using the robust control law given by (22); notice that there is significant chattering, which makes practical implementation infeasible.

5.2 Adaptive control

Using the approach given in Section 3, and assuming $p=2$, the inertia and the disturbances, $I(t)$ and $d(t)$, can be expanded as

$$\begin{aligned}I(t) &= k_0+k_1(t-t_0)+\delta_I,\\ d(t) &= l_0+l_1(t-t_0)+\delta_d, \quad t\geq t_0\end{aligned} \qquad (24)$$

where $|\delta_I| \leq \frac{c_I}{2}(t-t_0)^2$, $|\dot{\delta}_I| \leq c_I(t-t_0)$, $|\delta_d| \leq \frac{c_d}{2}(t-t_0)^2$, $k_0 = I(t_0)$, $k_1 = I^{(1)}(t_0)$, $l_0 = d(t_0)$, $l_1 = d^{(1)}(t_0)$, $c_I = \sup_t I^{(2)}(t)$, and $c_d = \sup_t d^{(2)}(t)$.

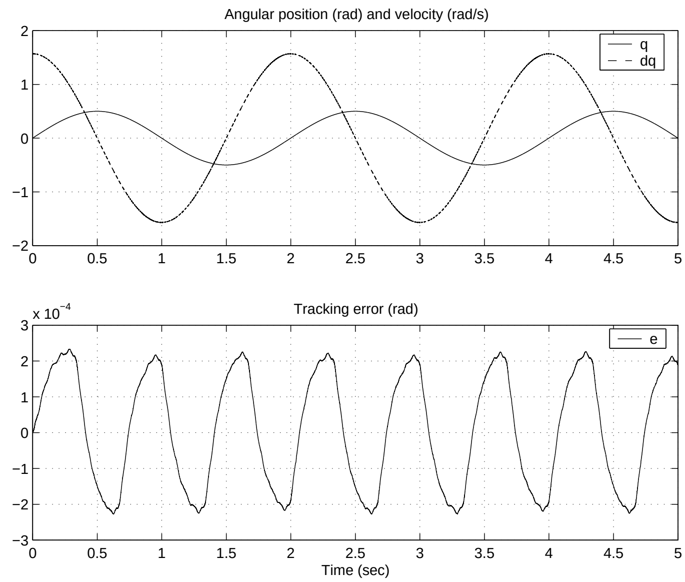

Figure 1. Robust control simulation results: angular position, velocity and tracking error

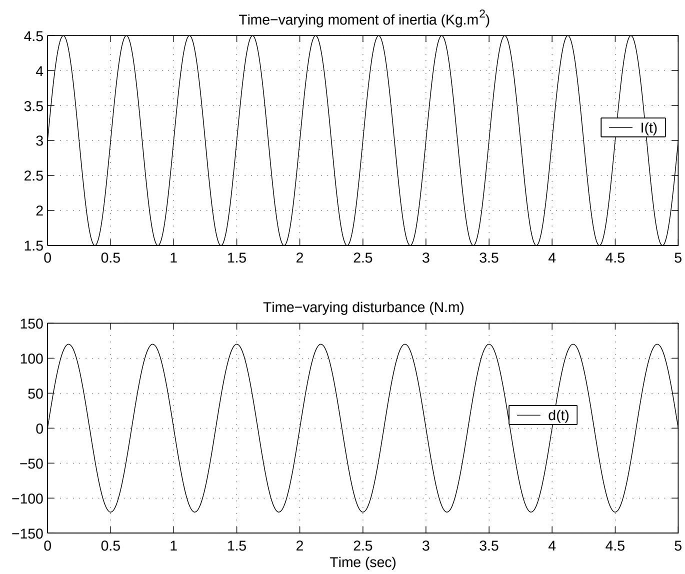

Figure 2. Robust control simulation results: time-varying moment of inertia and disturbance

Using the results from the previous section, we can choose

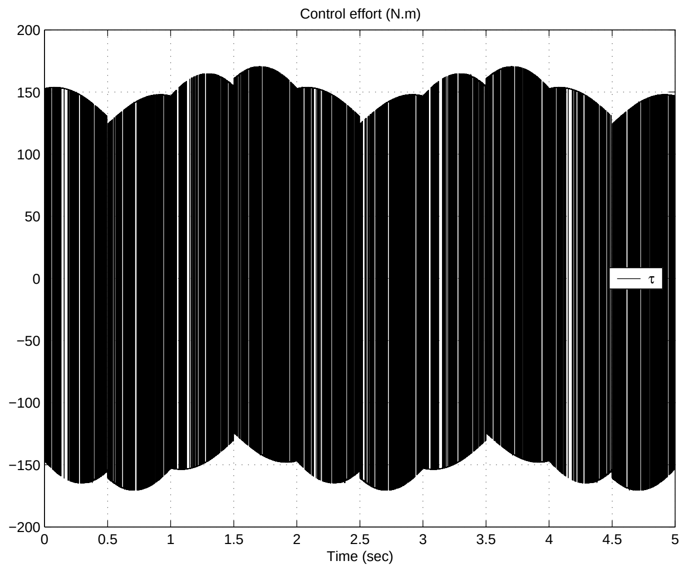

Figure 3. Robust control simulation results: control effort

the following control law and estimation law as

$$\tau = (\widehat{k}_0 + \widehat{k}_1(t-t_0))\ddot{q}_r + \widehat{k}_1(\dot{q}_r + \dot{q}) + \widehat{f}_v\dot{q} + \widehat{f}_c sgn(\dot{q}_r) - F_v e_v - \widehat{l}_0 - \widehat{l}_1(t-t_0) - Usgn(e_v), \tag{25}$$

$$U \geq K_1|\ddot{q}_r| + \frac{1}{2}K_2|\dot{q} + \dot{q}_r| + K_3, \tag{26}$$

$$\dot{\widehat{\beta}} = Proj(\widehat{\beta}, y) \tag{27}$$

where $(\widehat{*})$ is the estimate of $(*)$, $K_1 \geq |\delta_I|$, $K_2 \geq |\dot{\delta}_I|$, $K_3 \geq |\delta_d|$, $\widehat{\beta} = [\widehat{f}_v,\ \widehat{f}_c,\ \widehat{k}_0,\ \widehat{k}_1,\ \widehat{l}_0,\ \widehat{l}_1]^T$, $y = -[\Gamma_{f_v}\dot{q},\ \Gamma_{f_c} sgn(\dot{q}_r),\ \Gamma_{k_0}\ddot{q}_r,\ \Gamma_{k_1}((t-t_0)\ddot{q}_r + \frac{1}{2}(\dot{q} + \dot{q}_r)),\ -\Gamma_{l_0},\ -\Gamma_{l_1}(t-t_0)]^T e_v$, $\Gamma_{f_v} > 0$, $\Gamma_{f_c} > 0$, $\Gamma_{k_0} > 0$, $\Gamma_{k_1} > 0$, $\Gamma_{l_0} > 0$ and $\Gamma_{l_1} > 0$. The robustness term U should satisfy

$$U \geq 0.002|\ddot{q}_r| + 0.95|\dot{q} + \dot{q}_r| + 0.085, \tag{28}$$

for $(t - t_0) \leq 2$ ms. Notice that the robustness term in the adaptive controller, U, is much smaller than the robustness term, U_δ used in the robust control.

The simulation results by using the control algorithm in Equation (26) are shown in Figure 4 and Figure 5. Figure 4 shows the angular position, velocity and tracking error the controlled system have. The control effort, shown in Figure 5, is much smoother than that achieved by robust control(see Figure 3). Almost no chattering is found.

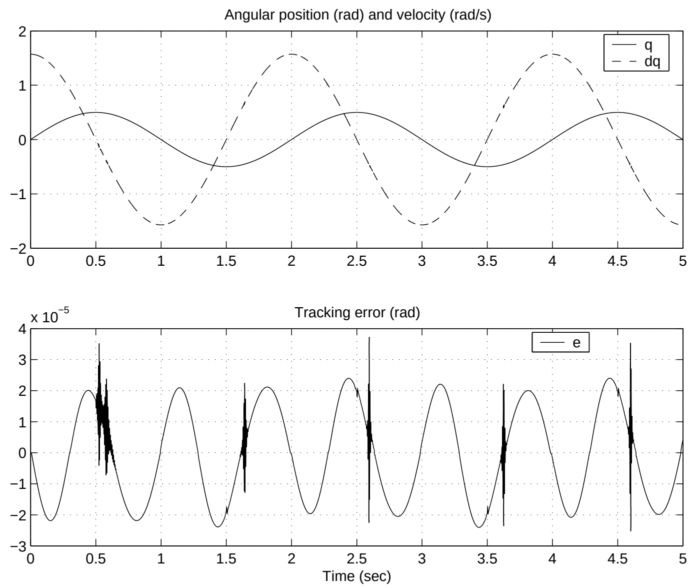

Figure 4. Adaptive control simulation results: angular position, velocity and tracking error

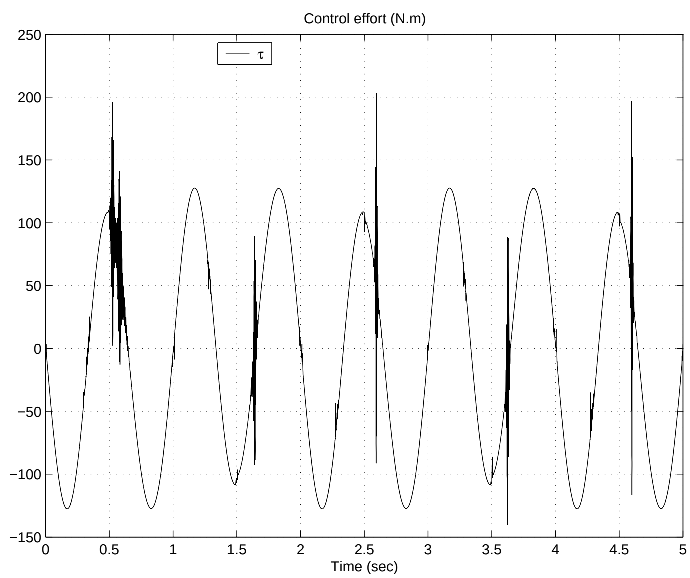

Figure 5. Adaptive control simulation results: control effort

6 Experiments

To further investigate the feasibility of the proposed control algorithm, a time-varying experiment designed for a two-link robot, which consists of a two-axis direct drive manipulator as shown in Figure 6. The direct drive manipulator operates in the absence of the undesirable factors of mechanical backlash an gear train compliance, eliminates the need for gear reduction, so

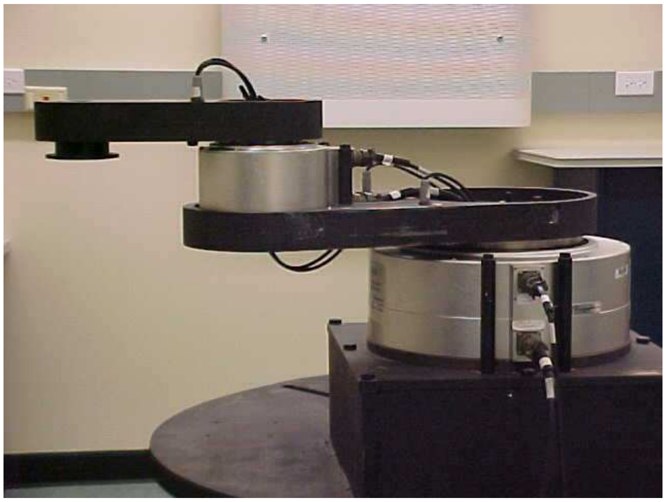

Figure 6. Picture of the two-link robot

repeatability is limited only the resolution of the position feedback. Each axis of the manipulator is driven by an NSK Megatorque direct drive servo-motor.

The NSK-Megatorque motor system consists of a high torque direct drive brushless actuator, a high-resolution brushless resolver, and a heavy duty precision bearing. The servo-motors are capable of up to 3 revolutions per second maximum velocity and position feedback resolution of up to 156,400 counts per revolution. The base motor delivers up to 240 N-m of torque output, and the elbow motor produces up to 40 N-m torque output. The real-time system associated with the direct drive manipulator consists of a host computer, a servo DSP card, and a DSP associated with the sensors. For a complete description of the experimental platform we refer the reader to (6).

The first link is designed to track a sinusoidal trajectory with an amplitude of 0.5 radians and a frequency of 0.5 Hz. The second link is used to generate a time-varying disturbance and time-varying moment of inertia to the first link. A constant torque of $4N.m$ is used as input to the second link; with this torque input the second link will run with an angular velocity of around $20rad/s$ after its velocity reaches the steady state. A control sampling period of 2 milli-seconds is chosen.

6.1 Dynamics of the two-link manipulator

The dynamics of the two-link manipulator is given by

$$M(q)\ddot{q}+C(q,\dot{q})\dot{q}=u \tag{29}$$

where

$$M(q)=\begin{bmatrix} p_1+2p_3c_2 & p_2+p_3c_2 \\ p_2+p_3c_2 & p_2 \end{bmatrix}, \tag{30}$$

$$C(q,\dot{q})=\begin{bmatrix} -p_3\dot{q}_2s_2 & -p_3(\dot{q}_1+\dot{q}_2)s_2 \\ p_3\dot{q}_1s_2 & 0 \end{bmatrix}, \tag{31}$$

and $u=[u_1\ u_2]^T$ is the vector of motor torques, $c_1=\cos(q_1), s_1=\sin(q_1), c_2=\cos(q_2)$ and $s_2=\sin(q_2)$, and p_1, p_2 and p_3 are coupled inertia parameters without any payload. The true values of the coupled inertial parameters are $p_1=3.4$, $p_2=0.4$ and $p_3=0.3$.

Solving (29) results in

$$(p_1p_2-p_2^2-p_3^2c_2^2)\ddot{q}_1-p_3(2p_2\dot{q}_1\dot{q}_2+p_2\dot{q}_1^2+p_2\dot{q}_2^2 + p_3c_2\dot{q}_1^2)s_2=p_2u_1-(p_2+p_3c_2)u_2. \tag{32}$$

(32) can be rewritten as

$$I(t)\ddot{q}_1+\dot{I}(t)\dot{q}_1=u_1+d \tag{33}$$

where

$$I(t)=p_1-p_2-\frac{p_3^2}{p_2}c_2^2 \tag{34}$$

$$d=p_3((\dot{q}_1+\dot{q}_2)^2+\frac{p_3}{p_2}c_2\dot{q}_1^2)s_2-(1+\frac{p_3}{p_2}c_2)u_2 \tag{35}$$

$$u_1=\tau_1-f_f \tag{36}$$

where τ_1 is the torque generated by the motor at the first link. $f_f=f_v\dot{q}_1+f_c sgn(\dot{q}_1)$ is the friction. System (33) has time-varying moment of inertia and time-varying external disturbance. By choosing u_2, we can introduce time-varying disturbances into the first link .

6.2 Experimental results

Figure 7 shows the tracking error of the first link, and angular velocity and input torques for both links. Notice that the peak tracking error of link 1 is below 0.04 radians even in the presence of time-varying inertia and very large time-varying disturbances. The disturbance and time-varying inertia computed from Equation (35) and (34) are shown in Figure 8. Notice that the time-varying disturbance is periodic with an amplitude of about 120 $N.m$ and frequency is about 3.1 Hz. The peak of the time

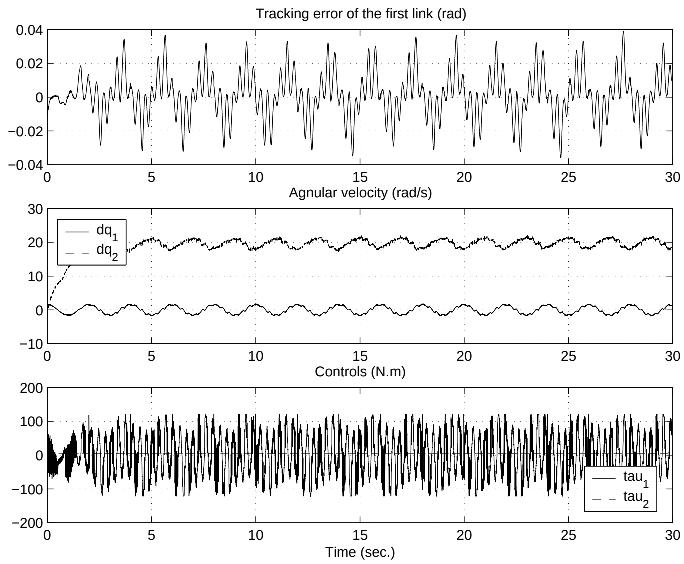

Figure 7. Tracking error, angular velocity and control effort

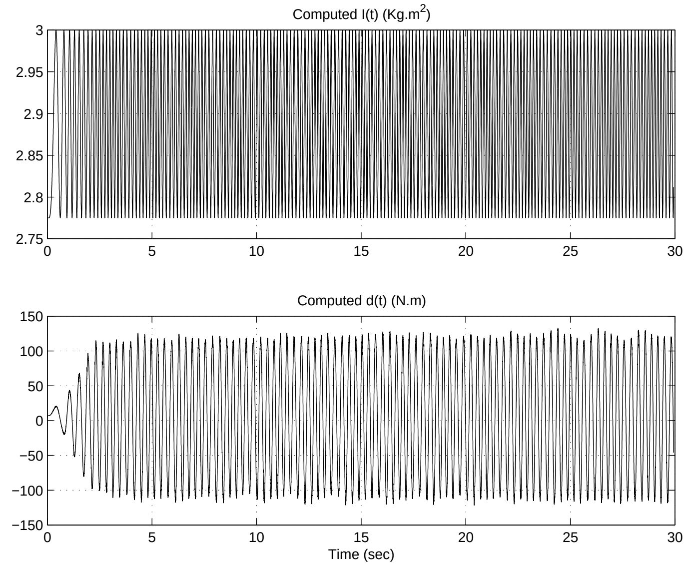

Figure 8. Computed $I(t)$ and $d(t)$

derivative of the disturbance is about 6000 $N.m/s$. The moment of inertia varies with a peak-to-peak value of 0.23 $Kg.m^2$ and a frequency of 6.2 Hz. The peak of the time derivative of the moment of inertia is about 4.8 $Kg.m^2/s$. The middle plot in Figure 7 shows the angular velocities of both links. The steady state velocity of the second link is about 20 rad/s, which is equivalent to 3.1 cycles/second. The input torque of each link is shown in the last plot of Figure 7. The second link is controlled with a constant torque of $4N.m$. Observe that the motor torque for the first link has almost the same amplitude and frequency as that of the time-varying disturbance; and some chattering can be observed due to the robustness switching term in the controller. Figure 9 shows the estimated parameters, estimated disturbance and estimated moment of inertia. It can be seen that all the estimated parameters are within the range defined in the projection algorithm.

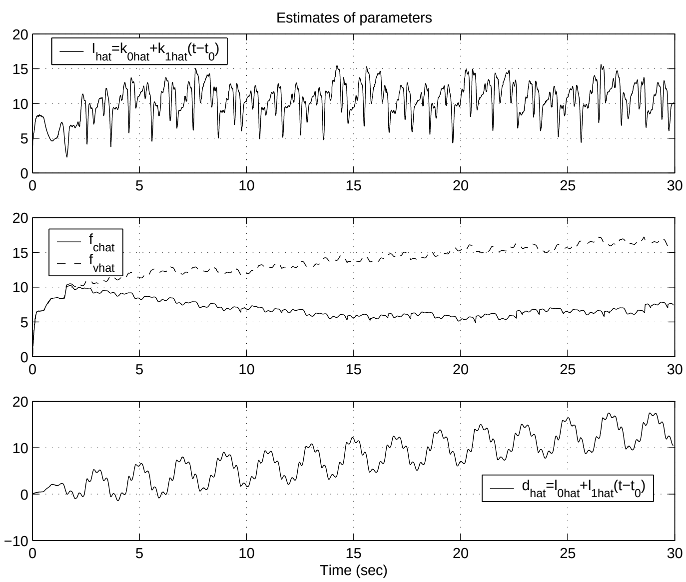

Figure 9. Estimated parameters

7 Conclusion

In this paper, a new adaptive controller for mechanical systems with time-varying parameters and disturbances is proposed. The time-varying parameter/disturbance is expanded as a finite length polynomial of time and a residue. The coefficients of the finite length polynomial are assumed to be constant in a small interval of time. Based on this expansion of time-varying parameter/disturbance, an adaptive controller is developed for trajectory tracking. The unknown coefficients were estimated using a gradient projection algorithm. Asymptotic convergence of the tracking errors with the proposed controller is shown. Experimental results using the proposed adaptive controller show good tracking performance in the presence of large time-varying disturbances. It is evident that the choice of the time interval discussed in Section 3 plays an important role in the control design. Since the time-varying function is estimated in each interval using a local approximation, the initial condition of the coefficient estimates at the beginning of each interval must be chosen appropriately such that the estimated function is smooth and the candidate Lyapunov function is non-increasing at each resetting point. Future research work should focus on ways to appropriately choose this time interval and the initial condition of the coefficient estimates at the beginning of each interval. Further, there is a need for development of an experimental platform in which it is possible to generate arbitrary time-varying parameters and time-varying disturbances.

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