# Speed and Shape of Electrostatic Waves in Dust-Ion Plasma

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Nonlinear dust acoustic waves are studied in a magnetized plasma. Quasineutrality is considered. The existence of a soliton solution is determined by a pseudo-potential approach. Sagdeev's potential is obtained in terms of $U(=\alpha u_{\mathrm{dx}}+\gamma u_{\mathrm{dz}})$, the component of the dust-ion velocity in the direction of the propagation of the wave. It is shown that there exists a critical value of $U$, beyond which the solitary waves cease to exist.

*Key words:* Electrostatic Waves; Soliton; Pseudopotential; Dusty Plasma.

## 1. Introduction

Dusty plasma is a rapidly growing area in plasma science. It plays a significant roles in space plasma, astrophysical plasma, laboratory plasma and environment. The presence of dusty plasmas in cometary tails, asteroid zones, planetary rings, intersteller medium, lower part of the earth's ionosphere and magnetosphere [1 – 9] makes this subject increasingly important. Dusty plasmas also play a vital role in low temperature physics, radio frequency plasma discharge [10], coating and etching of thin films [11], plasma crystal [12] etc. Such plasmas are also investigated in laboratory experiments [13, 14].

Several authors have studied the wave phenomena and associated nonlinear structures such as soliton, socks and vortices in dusty plasmas. It began with the work of Bliokh and Yarashenko [15] who first theoretically observed the waves in such environment while dealing with waves in Saturn's ring. The discovery of dust acoustic wave (DAW) [16, 17] and dust-ion acoustic wave (DIAW) [18, 19] gave a new impetus to the study of waves in dusty plasmas. Later it was found that the dust grain dynamics also introduced few new eigen modes like Dust-Berstain-Greene-Kruskal (DBGK) mode, dust-lattice (DL) mode [20, 21], Shukla-Verma mode [22], dust-drift mode [23].

The arbitrary amplitude dust acoustic solitary waves in the one-dimensional and unmagnetized plasma have been rigorously investigated by a number of authors. The properties of dusty plasma waves in a magnetized plasma were also studied in different modes. Shukla and Rehman [24] studied the existence of lower-hybrid and dust-cyclotron waves. Chowe and Rosenberg [25, 26] also studied the instability of electrostatic ion-cyclotron waves in magnetized plasmas. Rao [27] studied the electrostatic waves and instabilities in non-ideal magnetized dusty plasmas. Most of the studies on waves in a magnetized dusty plasma, discussed up to now, have dealt with linear theories. However, there are many dusty plasma situations where the excitation mechanism gives rise to large amplitude waves, and as a result nonlinear effects become important. One of the most interesting topics concerning such nonlinear effects is the formation of solitary waves, particularly the dust acoustic solitary waves. Recently, Kotsarenko et al. [28] and Mamun [29, 30] have studied the nonlinear propagation of the dust acoustic waves in a magnetized dusty plasma by means of the reductive peturbation technique (RPT), which is valide for small amplitude waves only. A few years ago Malfliet and Wieers [31] reviewed the studies on solitary waves and found that RPT is based on the smallness of the amplitude. More recently Johnston and Epstein [32] derived Sagdeev's potential [33] in terms of $u$, the ion-acoustic speed instead of $\phi$, the electric potential. They observed that a very small change in the initial conditions destroys the oscillatory behaviour of the wave. Chatterjee and Das [34] also observed the effect of electron inertia on the critical value of $u$, the ion speed of the waves, for which the oscillatory behaviour is destroyed. Maitra and Roychoudhury [35] studied dust acoustic solitary waves by the same technique, considering the dust dynamics in a dusty plasma consisting of warm dust particles and Boltzmann-distributed electrons and ions.

Recently Chatterjee and Jana [36] studied the speed and shape of dust acoustic solitary waves in the presence of dust streaming by the same technique used in [32, 34, 35]. But in both studies in [35] and [36] dust acoustic solitary waves in the one-dimensional and unmagnetized plasma were investigated. In this paper, we consider a magnetized ion-dust plasma and study the propagation of the coupled nonlinear dust acoustic waves by the technique used in [32, 34–36]. Here the dynamics of magnetized dust grains are governed by the continuity and momentum fluid equations. The Boltzmann distribution for unmagnetized ions is also considered. We also consider the quasineutrality condition so that the dust number density is localized. We have found here that there exists a critical value of $U = U_0$ beyond which solitary waves cease to exist. Criteria for the existence of such solitary waves are discussed. The effect of Mack number, obliqueness of wave propagation and $B_0$, the external electric field, on the existence of solitary waves is also discussed.

The organization of this paper is as follows. In Section 2 basic equations are written for dusty plasmas. The governing $2^{\mathrm{nd}}$ order ODE is derived. Conditions for the existence of soliton solutions and results are given in Section 3. Section 4 is kept for conclusions.

## 2. Basic Equations

A two-component dusty plasma is considered consisting of Boltzmann-distributed ions and negatively charged dust grains which are both magnetized. This model corresponds to a situation when most of the electrons from the ambient plasma are attached to the dust grain surface. Hence we have $n_{\mathrm{e}0} \ll Z_{\mathrm{d}} n_{\mathrm{d}0}$, where $n_{\mathrm{e}0}$ and $n_{\mathrm{d}0}$ are the unperturbed electron and dust partical number densities, respectively. $Z_{\mathrm{d}}$ is the number of electrons residing onto the dust grain surface and so the depletion of the electrons cannot be complete. As the grain surface potential approaches zero, the minimum value of the ratio between the electron and ion number densities turns out to be $(m_{\mathrm{e}}/m_{\mathrm{i}})^{1/2}$ where $m_{\mathrm{e}}$ $(m_{\mathrm{i}})$ is the electron (ion) mass. We consider the dusty plasma as a two-component plasma composed of negatively charged dust grains and ions. The later shield the dust grains. This model is relevant to planetary ring systems (e. g. Saturn's F-ring) and in comets (e.g. Halley's comet). Here, the situation is considered when $Z_{\mathrm{d}} \gg n_{\mathrm{e}0}/n_{\mathrm{d}0}$. This model is valid because for such a situation we have $\frac{n_{\mathrm{e}0}}{n_{\mathrm{i}0}} \simeq (m_{\mathrm{e}}/m_{\mathrm{i}})^{1/2}$ and $m_{\mathrm{e}}/m_{\mathrm{i}} \ll 1$ where $n_{\mathrm{i}0}$ is the unperturbed ion particle number density. Thus, at equilibrium we have $n_{\mathrm{i}0} \ll Z_{\mathrm{d}} n_{\mathrm{d}0}$. We also consider the Boltzmann distribution for ions in an external magnetic field as the parameters are so chosen that the wave length is shorter than the ion gyroradius. We assume that the dusty plasma is embedded in a uniform external magnetic field $B_0 = \hat{z} B_0$ where $\hat{z}$ is the unit vector along the $z$ axis. We also assume that the grain size is much smaller than the dusty plasma Debye radius. The dust ion wave frequency is assumed much lower than the dust grain charging time scale and so the effect of dust charge variation is negligible and the dust charge is assumed to be constant.

The basic equations are:

$$\frac{\partial n_{\mathrm{d}}}{\partial t} + \nabla \cdot (n_{\mathrm{d}} v_{\mathrm{d}}) = 0, \tag{1}$$

$$\frac{\partial v_{\mathrm{d}}}{\partial t} + v_{\mathrm{d}} \cdot \nabla v_{\mathrm{d}} = \frac{z_{\mathrm{d}} e}{m_{\mathrm{d}}} \nabla \phi - w_{\mathrm{cd}} v_{\mathrm{d}} \times \vec{z}, \tag{2}$$

$$n_{\mathrm{i}} = n_{\mathrm{i}0} \exp\left(-\frac{e\phi}{k_{\mathrm{b}} T_{\mathrm{i}}}\right), \tag{3}$$

where $n_{\mathrm{d}}$ and $v_{\mathrm{d}}$ are the dust number density and dust fluid velocity, respectively, $\omega_{\mathrm{cd}}$ is the dust cyclotron frequency, $e$ the magnitude of the electric charge, $n_{\mathrm{i}}$ the ion number density, $T_{\mathrm{i}}$ the ion temperature, $\phi$ the electrostatic potential and $k_{\mathrm{b}}$ the Boltzmann constant. To obtain the dispersion relation we first consider $n_{\mathrm{d}} = n_{\mathrm{d}0} + n_{\mathrm{d}1}$, $v_{\mathrm{d}} = 0 + v_{\mathrm{d}}$ and $\phi = 0 + \phi$ and (1)–(3) reduces to the single equation

$$\left[\left(\frac{\partial^2}{\partial t^2} + \omega_{\mathrm{cd}}^2 - c_{\mathrm{d}}^2 \nabla^2\right) \frac{\partial^2}{\partial t^2} - \omega_{\mathrm{cd}}^2 c_{\mathrm{d}}^2 \frac{\partial^2}{\partial z^2}\right] = 0, \tag{4}$$

where $c_{\mathrm{d}} = (Z_{\mathrm{d}} k_{\mathrm{b}} T_{\mathrm{i}}/m_{\mathrm{d}})^{\frac{1}{2}}$. Now considering $\phi$ is proportional to $\mathrm{e}^{\mathrm{i}(kr-\omega t)}$ we obtain the dispersion relation

$$\omega_{\pm}^2 = \frac{1}{2}[\omega_{\mathrm{cd}}^2 + k^2 c_{\mathrm{d}}^2 \pm \sqrt{(\omega_{\mathrm{cd}}^2 + k^2 c_{\mathrm{d}}^2)^2 - 4\omega_{\mathrm{cd}}^2 k_z^2 c_{\mathrm{d}}^2}]. \tag{5}$$

Hence we obtain two types of obliquely propagating waves named dust acoustic wave and dust cyclotron wave for the branches $\omega = \omega_-$ and $\omega = \omega_+$, respectively. For details see [16, 37, 38].

Now we express (1)–(3) in terms of normalized variables, where $n_{\mathrm{d}}$ is normalized by $n_{\mathrm{d}0}$, $n_{\mathrm{i}}$ is normalized by $n_{\mathrm{i}0}$, $v_{\mathrm{d}}$ is normalized by $c_{\mathrm{d}}$, $\phi$ by $(k_{\mathrm{b}} T_{\mathrm{i}}/e)$, $t$ is normalized by $\omega_{\mathrm{cd}}^{-1}$, where $\omega_{\mathrm{cd}} = \frac{qB_0}{m_{\mathrm{d}}}$ and $r$ $(= xi + yj)$ is normalized by $\rho_{\mathrm{d}} = \frac{c_{\mathrm{d}}}{\omega_{\mathrm{cd}}}$ and the new variables are termed as $N_\mathrm{d}$, $N_\mathrm{i}$, $u_\mathrm{d}$, $\psi$, $\tau$ and $R(=Xi+Yj)$, respectively.

To investigate the properties of solitary wave solutions of (1)–(5), we assume that all the dependent variables depend on a single independent variable

$$\xi=\frac{1}{M}(\alpha X+\gamma Z-M\tau)\left(=\frac{\omega_\mathrm{cd}}{\omega}(k_x x+k_z z-\omega t)\right), \tag{6}$$

where $\xi$ is the special co-ordinate in the co-ordinate system moving with the solitary wave velocity. Also $k=(k_x,0,k_z)$, $\alpha=\frac{k_x}{|k|}=\sin\theta$, $\gamma=\frac{k_z}{|k|}=\cos\theta$. $|k|=\sqrt{k_x^2+k_y^2}$ and $\theta$ is the angle between $k$ and $B_0$. $M=v_\mathrm{p}/c_\mathrm{d}$, where $v_\mathrm{p}=\omega/|k|$ is the phase speed of the wave. Therefore,

$$\frac{\partial}{\partial t}=w_\mathrm{cd}\frac{\partial}{\partial\tau}=-w_\mathrm{cd}\frac{\mathrm{d}}{\mathrm{d}\xi}, \tag{7}$$

$$\frac{\partial}{\partial x}=\frac{w_\mathrm{cd}}{c_\mathrm{d}}\frac{\partial}{\partial X}=\frac{w_\mathrm{cd}}{c_\mathrm{d}}\cdot\frac{\alpha}{M}\frac{\mathrm{d}}{\mathrm{d}\xi}, \tag{8}$$

$$\frac{\partial}{\partial z}=\frac{w_\mathrm{cd}}{c_\mathrm{d}}\frac{\partial}{\partial Z}=\frac{w_\mathrm{cd}}{c_\mathrm{d}}\cdot\frac{\gamma}{M}\frac{\mathrm{d}}{\mathrm{d}\xi}. \tag{9}$$

Equation (1) now reduces to

$$\frac{\mathrm{d}}{\mathrm{d}\xi}[L_u N_\mathrm{d}]=0, \tag{10}$$

where

$$L_u=\alpha u_\mathrm{dx}+\gamma u_\mathrm{dz}-M. \tag{11}$$

Equation (2) reduces to

$$\begin{aligned}&-c_\mathrm{d}w_\mathrm{cd}\frac{\mathrm{d}(\vec{u}_\mathrm{d})}{\mathrm{d}\xi}+c_\mathrm{d}^2\left(u_\mathrm{dx}\frac{\partial}{\partial x}+u_\mathrm{dy}\frac{\partial}{\partial y}+u_\mathrm{dz}\frac{\partial}{\partial z}\right)(\vec{u}_\mathrm{d})\\&\quad=c_\mathrm{d}^2\vec{\nabla}\psi-w_\mathrm{cd}c_\mathrm{d}(u_\mathrm{dy}-u_\mathrm{dx}).\end{aligned} \tag{12}$$

Taking the component in $x$ direction, we have

$$L_u\frac{\mathrm{d}u_\mathrm{dx}}{\mathrm{d}\xi}=\alpha\frac{\partial\psi}{\partial\xi}-Mu_\mathrm{dy},$$

taking the component in $y$ direction, we get

$$L_u\frac{\mathrm{d}u_\mathrm{dy}}{\mathrm{d}\xi}=Mu_\mathrm{dx},$$

and taking the component in $z$ direction, we get

$$L_u\frac{\mathrm{d}u_\mathrm{dz}}{\mathrm{d}\xi}=\gamma\frac{\partial\psi}{\partial\xi}.$$

Again, considering the quasineutrality condition, we have

$$N_\mathrm{d}=\exp(-\psi)=N_\mathrm{i}. \tag{13}$$

To solve the above set of differential equations, the following boundary conditions are used: $\phi$, $\frac{\partial\phi}{\partial\xi}$, $u\to 0$, $n_\mathrm{d}\to 1$ as $|\xi|\to\infty$.

From (7) we get,

$$N_\mathrm{d}=\frac{M}{M-u}, \tag{14}$$

where $L_u=U-M$ and $U=\alpha u_\mathrm{dx}+\gamma u_\mathrm{dz}$. Now using (9) and (12) in (11) we get

$$\frac{\mathrm{d}^2U}{\mathrm{d}\xi^2}=\frac{\partial V}{\partial U}, \tag{15}$$

where

$$\begin{aligned}V=&\frac{M^2}{(1-(M-U)^2)(M-U)^2}\\&\cdot\left[\gamma^2M\left(-U+M\log\frac{M}{M-U}\right)\right.\\&-M^2\left(1-\frac{M}{M-U}+\log\frac{M}{M-U}\right)\\&\left.-\left(\frac{M^2}{2}+\frac{\gamma^2}{2(M-U)^2}\right)U^2\right].\end{aligned} \tag{16}$$

Thus

$$\begin{aligned}\frac{\mathrm{d}^2U}{\mathrm{d}\xi^2}=&\frac{(M-U)^2}{1-(M-U)^2}\left(\frac{M\gamma^2U}{M-U}+\frac{M^2U}{(M-U)^2}\right.\\&\left.-\frac{U^2\gamma^2}{(M-U)^3}-U\left(M^2+\frac{\gamma^2}{(M-U)^2}\right)\right)\\&+\frac{4(M-U)^3+2(M-U)(1-(M-U))}{(1-(M-U)^2)^3}\\&\cdot\left(-U^2\left(\frac{M^2}{2}+\frac{\gamma^2}{2(M-U)^2}\right)\right.\\&-M^2\left(1-\frac{M}{M-U}+\log\frac{M}{M-U}\right)\\&\left.-M\gamma^2(M-U)\left(1-\frac{M}{M-U}+\frac{M}{M-U}\log\frac{M}{M-U}\right)\right).\end{aligned} \tag{17}$$

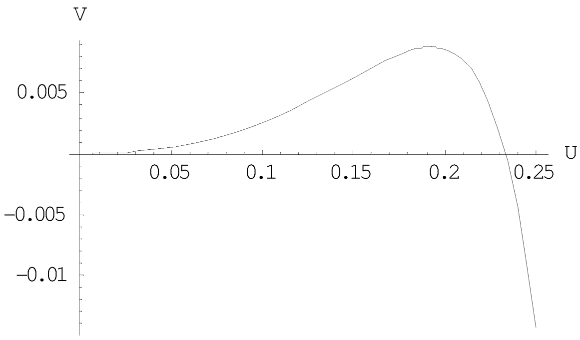


Fig. 1. Plot of $V$ vs. $U$ for $M = 0.5$ and $\alpha = 0.6$ ($\gamma = 0.4$).

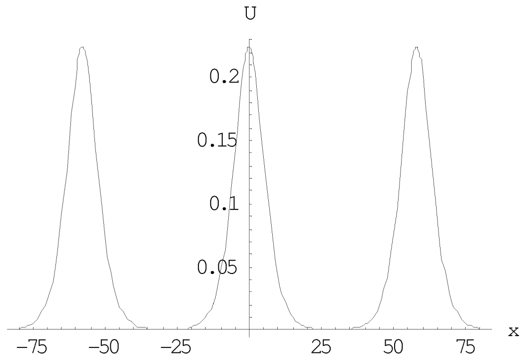


Fig 2. The soliton solution $U$ plotted against $x$ for $U_0 = 0.233464$. The other parameters are the same as those in Figure 1. (Read $\xi$ in place of $x$.)

One can also write

$$V = \frac{1}{2}\left(\frac{\mathrm{d}U}{\mathrm{d}\xi}\right)^2. \tag{18}$$

## 3. Results and Discussion

To find the region of existence of solitary waves one has to study the nature of the function $V(U)$ and $\phi_1(U)$, defined by

$$V(U) = \frac{(U')^2}{2}, \tag{19}$$

where

$$U'' = \frac{\mathrm{d}V}{\mathrm{d}U} = \phi_1(U). \tag{20}$$

For solitary waves see [37, 36], $\phi_1$ will have two roots, one being at $U = 0$ and the other at some point $U = U_1 (\geq 0)$. Also $\phi_1$ should be positive on the interval $(0, U_1)$ and negative in $(U_1, U_{\max})$,

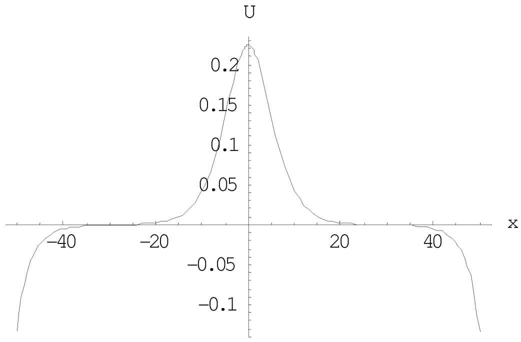


Fig. 3. The soliton solution $U$ is plotted against $x$ for $U_0 = 0.233465$. The other parameters are the same as those in Figure 1. (Read $\xi$ in place of $x$.)

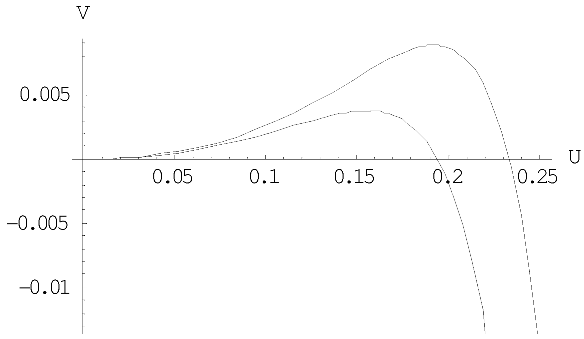


Fig. 4. Plot of $V$ vs. $U$ for $M = 0.5$ and 0.48. The other parameters are the same as those in Figure 1.

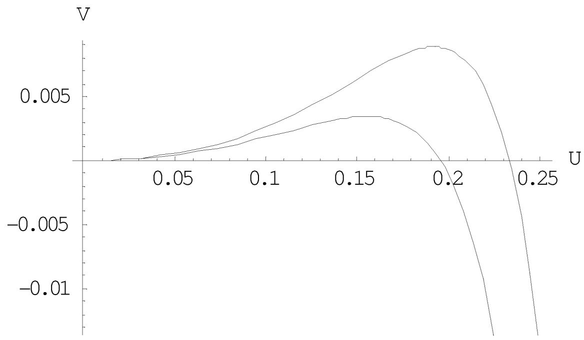


Fig. 5. Plot of $V$ vs. $U$ for $\alpha = 0.6$ and 0.58. The other parameters are the same as those in Figure 1.

where $U_{\max} = U_0$ is obtained from the nonzero root of $V(U)$.

Figure 1 shows the plot of $V$ vs. $U$ for $M = 0.5$ and $\alpha = 0.6$ ($\gamma = 0.4$). It is seen that $V(U)$ crosses the $U$ axis at $U = 0.233464$. Hence $U_{\max} = U_0 = 0.233464$ is the amplitude of the solitary wave. To get the shape of the travelling solitary wave one has to solve $U'' = \phi_1(U)$ numerically with $U_0 = 0.233464$ and $U'_0 = 0$

and Fig. 2 depicts the soliton solution $U(\xi)$ plotted against $\xi$. The other parameters are same as those in Figure 1. It is seen that $U_0 = 0.233464$ is the critical value for $U$. For $U > U_0$ the soliton solution ceases to exist; this is shown in Figure 3. In this figure $U_0$ is taken as 0.233465. Other parameters are same as those in Figure 2. It is seen that a small change (0.000001) in $U_0$ destroys the struture of the solitary wave. To see the effect of the parameters $M$ and $\alpha$ on the critical values, Figs. 4 and 5 are drawn. In Fig. 4 $V(U)$ is plotted against $U$ for $M = 0.5$ and 0.48. Other parameters are same as those in Figure 1. Here it is seen that the critical value increases with the increase of $M$. In Fig. 5 $V(U)$ is plotted against $U$ for $\alpha = 0.58 (\gamma = 0.42)$ and $0.6 (\gamma = 0.4)$. Other parameters are same as those in Figure 1. Here it is seen that the critical value increases with the increase of $\alpha$. But from (6) it is clear that the change in $\alpha$, the angle of propagation, directly reflects the change in cyclotron frequency and that $\alpha = M\frac{\omega_{cd}}{\omega} = \frac{k_x}{k} \cdot \frac{qB_0}{m_d c_d}$. Hence $\alpha$ varies directly with $B_0$, the external magnetic field. Hence the critical value increases with the increase of $B_0$. soliton velocity $M$, external magnetic field $B_0$ or the obliqueness parameters ($\alpha$ or $\gamma$), all play a significant role in the existence of solitary waves and forming and breaking of solitary waves.

## 4. Conclusion

Using the pseudo-potential approach we have studied the speed and shape of the dust acoustic solitary waves in homogeneous magnetized dust-ion-electron plasmas. The quasineutrality condition is also considered. Sagdeev's potential is obtained in terms of $U = \alpha u_{dx} + \gamma u_{dz}$, the component of dust fluid velocity in the direction of propagation of a solitary wave. It is seen that there exists a critical value of $U$, at which $U'^2 = 0$, beyond which the soliton solution does not exist. This critical value is extremely sensitive to other parameters and also depends on the soliton velocity. This technique can be extended to the study of a nonthermal distribution of electrons in magnetized plasmas. Work in this direction is in progress.

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