Large Extra Dimensions

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Abstract

The possibility of extra dimensions has been considered for a long time. In the 1920's, Kaluza and Klein sought to unify electromagnetism with gravity by adding an extra dimension that was curled up on the order of the Planck length (10^{-35} m.) Then, string theory sought to unify quantum mechanics and general relativity. Although it is a major contender for a quantum theory of gravity, it requires 6 or 7 additional spatial dimensions that are compactified. Extensions of string theory led to membranes (or branes) as objects in a higher dimensional space. It is possible that all of the particles and fields of the Standard Model are trapped on this brane, providing an explanation for why we have never observed more than our 3 spatial dimensions. On the other hand, gravity would be able to travel in (4 + n) dimensions, where n is the number of extra dimensions. Since we are constrained to the brane, extra dimensions need not be compactified, leading to theories with large extra dimensions. One of the first theories was developed by Arkani-Hamed, Dimopoulos, and Dvali. They proposed that for n = 2, the extra dimensions could be as large as a millimeter. In addition to this possibility, they found that their model could also describe the hierarchy between the Planck mass and the electroweak symmetry breaking scale in terms of the large size of the extra dimensions. Randall and Sundrum proposed a different brane world in which the hierarchy was due to an exponential warped factor caused by branes with tension in an Anti-de Sitter five-dimensional geometry. In their model, the extra dimension could even be of infinite size and still reproduce our four-dimensional gravity. Thus, it was found that large extra dimensions were not only allowed theoretically, but they provided an explanation for the hierarchy problem that has been a long-standing problem in particle physics.

It is commonly believed that we live in a universe with three spatial dimensions and one dimension of time. We say that d = 3 + 1, where d is the dimensionality of space-time. This belief is so ingrained that it seems unfathomable that our universe could be any other way. Even Aristotle thought about this idea. He concluded from physical experience that there are no physical objects described on manifolds of dimension greater than three. It was not until the 1850's that the formalism to describe geometry of arbitrary dimension manifolds was created by Bernhard Riemann. Since that time, the possibility of there being extra dimensions has received more attention. However, if there are extra dimensions, why do we not observe them? One belief is that the extra dimensions are so incredibly small (compactified to nearly the Planck length of 10^{-35} m) that we have never been able to probe them [1]. On the other hand, it has recently been shown theoretically that the extra dimensions could be as large as 1 mm [2], or they could even infinite in extent [4], and still have a universe that is consistent with our observations that space-time appears 4-dimensional. As an added bonus, extra dimensions are not only permissible, but they can also resolve some of the puzzles present in 4 dimensions.

Before investigating the possibility of large extra dimensions further, it is important to understand what we mean by a dimension. In terms of spatial dimensions, the number of dimensions is just a measure of the number of translational degrees of freedom available for movement in space. For example, in our everyday experience, we can move in any of three dimensions. We can move north/south, east/west, or up/down. Similarly, the dimensionality can be thought of as the number of points necessary to specify a location of an event or object (x, y, z in Cartesian coordinates). Mathematically, it is not difficult to add an extra dimension. Instead of using (x, y, z) to specify a point, you need to have the coordinates (x, y, z, w). In essence, you are introducing an extra degree of freedom. In addition to the three spatial dimensions, Einstein's Theory of Relativity taught us to think of time as a dimension. Therefore, time can be thought of as another degree of freedom. Not only must one specify the spatial location of an event, but they must also specify the time at which that event will occur (x, y, z, t) in space-time. Special relativity told us that the separation between two events in space-time is given by

$$ds^2 = -c^2 dt^2 + dx_1^2 + dx_2^2 + dx_3^2 \tag{1}$$

or, written in tensor notation,

$$ds^2 = \eta_{\mu\nu} dx^{\mu} dx^{\upsilon} \tag{2}$$

where ds is the differential of distance in this flat, 4-dimensional Minkowski space, $\eta_{\mu\nu}$ is the metric for this space, and μ, ν each run from 0 to 3 and label the coordinates. Equation (2) gives the metric for M_4 (4-d Minkowski). The importance of the metric when discussing large extra dimensions will become apparent later in this discussion.

Although our perceptions seem to tell us that there are (3 + 1) dimensions, there are other reasons why this has long been held true. One of the strongest sources of evidence for three spatial dimensions comes from the inverse square laws observed in physics (e.g. electric Coulomb force and Newton's gravitational force.) Both of these major force laws have $F \propto 1/r^2$. In general, these laws can be derived from Gauss's law. For example, take Gauss's law for electric charges and flux. The law can be written as

$$\Phi = \oint \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\varepsilon_0} \tag{3}$$

where Φ is the electric flux, **E** is the electric field, d**A** is the area of a differential square on a Gaussian surface of area A with an outward facing surface normal, and Q_{enc} is the amount of charge enclosed within the surface. Even though (3) was written as if in three spatial dimensions, Gauss's law can be generalized to n spatial dimensions. Assuming that the electric field is isotropic and choosing an n-dimensional sphere for a Gaussian surface, the electric field can be factored out of the integral. The area of the n-sphere is given by

$$\oint d\vec{A}_n = A_n \propto r^{n-1}. \tag{4}$$

Combining (3) and (4) yields the result that $E \propto 1/r^{n-1}$. Therefore, for a universe with n = 3, we recover the inverse square law that we expected (since electric field is proportional to force.) So, tests of the inverse square law are also tests of the dimensionality of our universe. For the case of gravity, table top experiments performed here on earth and observations of the celestial bodies around one another all provide evidence that suggests there are (3 + 1) dimensions.

With all of this evidence pointing to (3 + 1) dimensions, why would anyone want to propose that there existed additional dimensions? One such motivation was unification. In 1921, Theodor Kaluza discovered that if general relativity were extended to a five-dimensional space-time, then the equations could be separated out into normal 4-dimensional space-time and an additional set. This additional set is equivalent to Maxwell's equations for the electromagnetic field plus an additional scalar field known as the dilaton [5]. Under this framework, gravity and electromagnetism were unified by the addition of one extra dimension. The situation was analogous to when electricity and magnetism were unified into electro-magnetism by considering these two fields as components of a larger object: the $F^{\mu\upsilon}$ tensor. The immediate problem of Kaluza's theory was that he provided no explanation as to why none of the fields varied over the extra dimension. It was merely taken as an *ad hoc* assumption. In 1926, Oskar Klein provided an explanation for Kaluza's assumption. He proposed that the extra dimension is curled up into a circle with a radius on the order of the Planck length (10^{-35} m.) [6].

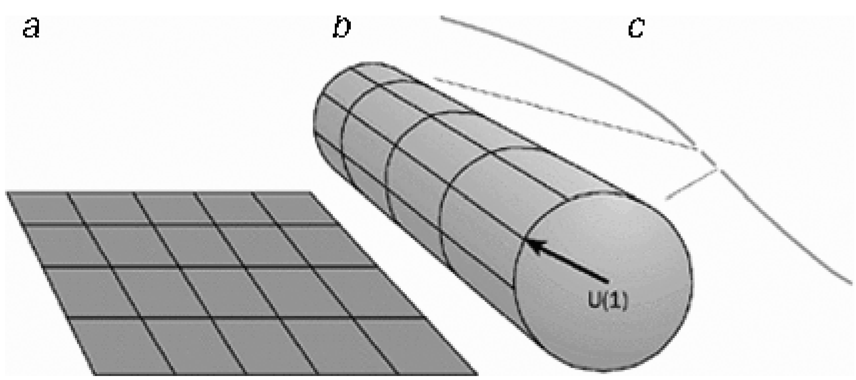

Figure 1. Compactification of a dimension. Initially, the space is 2-dimensional (a). In (b), one of the dimensions is curled up into a circle. If the radius is sufficiently small, then the space will appear 1-dimensional at large distances (c) [16].

Since the fifth dimension is periodic, a particle moving in the extra dimension would return to where it began. Due to the incredibly small size of the extra dimension, we cannot observe distance scales that small, and any field that began in a constant state in the extra dimension will remain that way. Unfortunately, there were problems and inconsistencies in the Kaluza-Klein theory that caused it to fall into disfavor. For instance, the theory predicted a unit of charge that was much smaller than the charge of the electron. In addition, the theory could not incorporate quantum mechanics in its original form, and it was at this time that quantum mechanics was beginning to gain much interest and activity. Although the possibility of unifying gravity and electromagnetism had looked promising at first, it quickly became little more than a curiosity. It was not until the 1980's that people started taking the ideas of Kaluza and Klein seriously again.

With the advent of quantum mechanics, it began to become apparent that the theory was incompatible with the theory of general relativity. The two theories described completely disparate realms. General relativity describes large scales and cosmology with a classical background. Quantum mechanics describes things at small distances with a background comprising of particles appearing from, and then disappearing into, the vacuum. At this time, a new theory called superstring theory (or string theory at the time) proposed that fundamental objects were strings and not point-particles. According to this theory, particles arose from excitations in the string. It was discovered that a particular excitation of the string is a particle with zero mass and two units of spin [7]. These are exactly the properties of the proposed graviton particle: the mediator of the gravitational force. In addition, since the strings were of finite size and not point-like, the strings collide at a small but finite distance. This kept the graviton from interacting at zero distance (as in usual particle interactions), and the mathematics of the behavior of the graviton do not give nonsensical results as they were before. It seemed as if string theory could be a theory of quantum gravity. However, there came a price for accepting this new paradigm: superstring theory is only consistent in (9 + 1) or (10 + 1) dimensions [8]. Therefore, there had to be 6 or 7 additional dimensions for the strings in which they could vibrate. Suddenly, the Kaluza-Klein theory of compactified dimensions became more attractive. If string theory was to be a theory of quantum gravity, then the average size of the string should be near the length scale of quantum gravity. This length scale is the Planck length, L_{Pl},

$$L_{Pl} = \sqrt{\frac{\hbar G_N}{c^3}} \approx 10^{-35}\, m. \tag{5}$$

with Planck's constant ħ, the speed of light c, and Newton's gravitational constant G_N. By this measure, strings are far too small to be seen by current or any future particle physics technology. The most powerful particle accelerator will be the LHC set to begin operation in 2007, and it will probe energies up to around 10 TeV. The Planck mass,

$$M_{Pl} = \sqrt{\frac{\hbar c}{G_N}} \approx 10^{16}\, TeV \tag{6}$$

given in units with c = 1, is a measure of the energy necessary to probe distance scales of the strings. We are 15 orders of magnitude short in energy, making an investigation of the Planck scale practically impossible in this paradigm.

The next step in string theory was to posit the existence of branes, because they are precisely formulated extended objects that are necessary for the consistency of string theory [9]. The term "brane" is derived from "membrane". These membranes are objects that exist in a higher dimensional space. It had been theorized as early as 1983 that it was possible that space-time has (4 + n) dimensions [10]. In this scenario, the Standard Model (SM) particles and forces (not including gravity) are confined to a four-dimensional subspace known as a 3-brane within a

(4 + n)-dimensional space-time. The realization of a mechanism for the localization of the SM fields is non-trivial. One possible mechanism is the trapping of zero modes on topological defects [10]. Here there was believed to be some potential that was flat in our 3 spatial dimensions and narrow in the extra n dimensions. Another possibility comes from string theory. It could be that our 3-brane is a D-brane. In this case, the ends of open strings are localized on the brane. In fact, at least one the open ends must terminate on a brane, as in Figure 2.

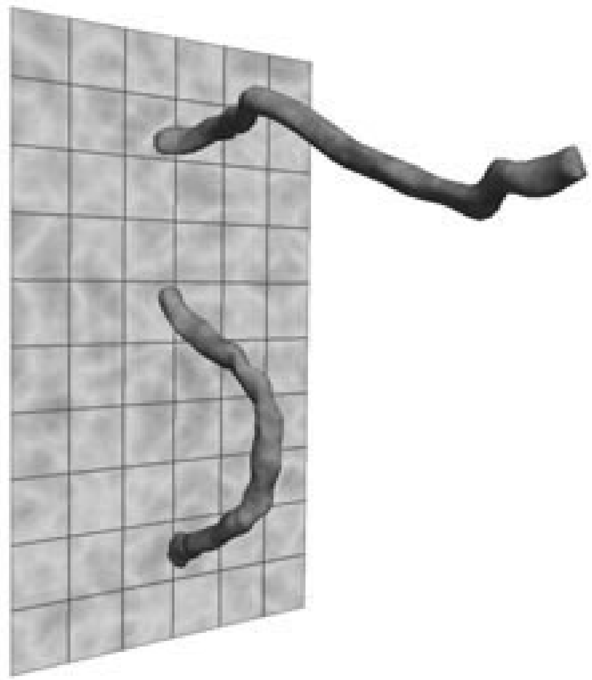

Figure 2. If we do live on a D-brane, then open strings must have at least one end on the brane, confining them to it [17].

Therefore, most SM particles and fields are trapped to the brane, and they cannot explore the extra dimensions. As far as the SM particles and fields are concerned, there are only 4 dimensions. The graviton, on the other hand, is a closed string. Since it does not have to end on a brane, the graviton is free to traverse the extra n dimensions. An alternative way of thinking about this is that the graviton is connected to the full space-time geometry. It is coupled to energy everywhere in the "bulk" (the full space-time) [11].

The concepts of branes opened up a new pathway for thinking about the sizes of the extra dimensions. Through the use of particle accelerators, distances on the order of 10^{-19} m have been probed, and there has been no evidence of extra dimensions. These extra dimensions would manifest themselves in high energy experiments in the form of Kaluza-Klein (KK) excitations that appear as particles of mass ~ R^{-1}, where R is the size of the extra dimension. This comes from the fact that momentum would be quantized in the compact dimensions. Therefore, there was previously an upper bound of 10^{-19} m on the size of any extra dimension. However, with the idea of all SM particles and fields trapped on the 3-brane, those high energy particles did not have a large enough energy to escape the brane. Gravity would be the only way to probe extra dimensions. As of 10 years ago, gravity had only been measured in the 1 cm range. Now, there existed the possibility that any extra dimensions could be large, even as large as 1 mm, and we would have never seen them in any experiment! This was the proposal of Nima Arkani-Hamed, Savas Dimopoulos, and Georgi Dvali (hereafter referred to as the ADD model) [2]. With this idea, something amazing happened. The believed fundamental scale of nature where gravity becomes as strong as the gauge interactions, the Planck scale $M_{Pl} = G_N^{-1/2} \approx 10^{16}$ TeV (6), turns out to not be fundamental at all. In fact, the real fundamental Planck scale in (4 + n) dimensions could be as small as the electroweak scale, $m_{ew} \approx 1$ TeV, where electromagnetism is the same strength as the weak interaction.

What began as the exploration of the extra dimensions led to a solution of a predicament that had been present in particle physics for some time: the hierarchy problem. The hierarchy problem was a question as to why there was such a huge disparity (believed to be around 15 orders of magnitude) between two fundamental quantities with the same units. Specifically, the hierarchy problem addresses the enormous gap between the energy scale associated with the electroweak symmetry breaking and the Planck energy. This problem is also a question about the hypothetical Higgs boson that is thought to give mass to the W and Z bosons of the weak interaction, thus breaking the symmetry between the weak and electromagnetic interactions (with its massless photon as the carrier of the force). Why is it that the Higgs boson is so much lighter than the Planck mass when large quantum contributions to the Higgs mass should make it massive, comparable to the Planck mass? Previously, the most popular theory to solve the hierarchy problem was supersymmetry. Supersymmetry theory makes the assertion that there is a fundamental symmetry between fermions and bosons. Every fermion has a superpartner that is a boson, and every boson has a superpartner that is a fermion. It is a vital part of many theories that go beyond the SM, including string theory. However, supersymmetry only protects the tiny Higgs mass from quantum corrections, it does not account for it being small and nonzero in the first place.

The ADD model provides a framework for solving the hierarchy problem that does not rely on supersymmetry. They propose that there exists $n \geq 2$ new compact spatial dimensions. According to this model, the Planck scale is not fundamental; its enormity is due to the large size of the new dimensions [2]. Only the electroweak scale is fundamental; it even sets the scale for the strength of the gravitational interaction. SM fields must be localized to the 4-dimensional submanifold (the brane), while the graviton is free to propagate into the (4 + n) bulk. Since it is assumed that m_{ew} is the only short-distance scale in the theory, our brane should have a "thickness" m_{ew}^{-1} in the extra n dimensions. It is easy to see how the usual strength of gravitation can arise in such a framework with n extra compact dimensions of radius R. The gravitational potential between two test masses m_1 and m_2 placed within a distance $r << R$ is given by Gauss's law in (4 + n) dimensions

$$V(r) = -\frac{m_1 m_2}{M_{Pl(4+n)}^{n+2}} \frac{1}{r^{n+1}}, \qquad (r << R) \tag{7}$$

in units where $\hbar = c = 1$ and where $M_{Pl(4+n)}$ is the Planck scale in the (4 + n) theory. For masses placed a distance $r >> R$ apart, their gravitational flux lines do not penetrate the extra dimensions; so, Gauss's law obtains

$$V(r) = -\frac{m_1 m_2}{M_{Pl(4+n)}^{n+2} R^n} \frac{1}{r}, \qquad (r >> R) \tag{8}$$

which is the expected 1/r potential. From this, we can get a relation between the effect M_{Pl} in our 4 dimensions and the fundamental unification scale $M_{Pl(4+n)}$:

$$M_{Pl}^{2} = M_{Pl(4+n)}^{n+2} R^{n} . \tag{9}$$

If we set the fundamental scale $M_{Pl(4+n)} \approx m_{ew}$ and demand that R reproduces the observed Planck mass, then we get

$$R \approx 10^{\frac{30}{n}-17} \times \left(\frac{1\,TeV}{m_{ew}} \right)^{1+\frac{2}{n}} cm . \tag{10}$$

Assuming that $m_{ew} \approx 1$ TeV, we find that this explains the hierarchy problem and describes the expected size of the extra dimensions. For n = 1, R ≈ 10^{13} cm. This implies deviations from Newtonian gravity over solar system distances. Thus, n = 1 is excluded by empirical observations. However, for n = 2, R ≈ 100 μm which is exactly at the range at which experiments are currently being done to look for deviations from Newtonian gravity.

One of tests for deviations was performed at the University of Washington [12]. For their experiment, they suspended a torsion pendulum with 10-fold symmetry from a tungsten wire.

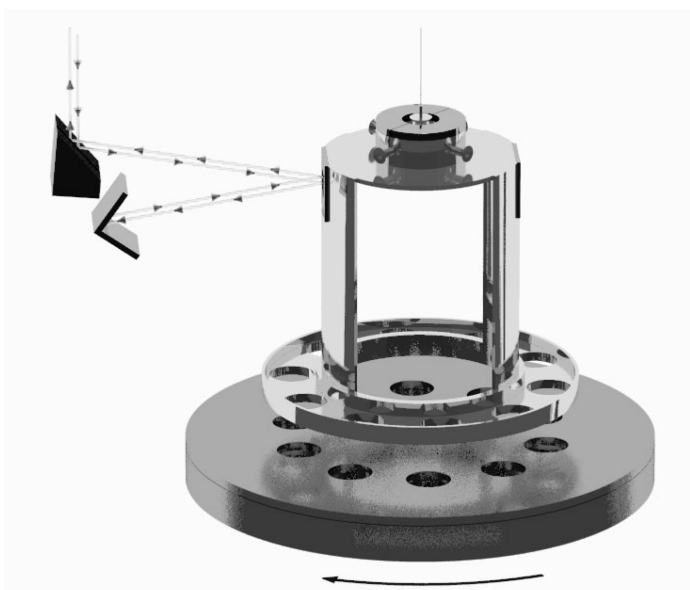

Figure 3. Scale drawing of torsion pendulum and rotating attractor. A large separation of 1.5 cm is shown for clarity. The BeCu membrane has also been omitted [13].

The pendulum was placed above an attractor with two coaxial disks that had 10-fold symmetry that slowly rotated about the vertical axis of the pendulum (Figure 3). A 20 μm beryllium-copper membrane was placed between the attractor and the pendulum to minimize torques caused by spurious electric forces (since any electric forces would completely dominate any gravitational forces.) Due to the 10-fold symmetry of the instrument, the torque varied at 10 times the rotation frequency of the attractor ω. The torque of the pendulum was measured by shining a laser beam off of a mirror mounted to the pendulum. Although both upper and lower disks of the attractor had 10 holes bored into them, the holes of the lower disk were larger and rotated 18° relative to those of the upper disk. This way, the holes of the lower disk lie half way between the holes of the upper disk. If there are n = 2 extra dimensions of size R, then one would expect that Newtonian gravity would go from $F \propto 1/r^2$ to $F \propto 1/r^4$. If the inverse square law is correct, then the lower holes should produce a twist on the ring that just cancels the twist induced by the upper disk. On the other hand, if gravity becomes stronger at smaller distances (e.g. $1/r^4$), then the twist induced by lower disk, which is further from the pendulum ring, will not cancel the twist from the upper disk. This will result in a strong signal in which the magnitude of the twist varies with the separation between the ring and the disks. Their experiment showed no deviation from the $1/r^2$ gravitational law down to a separation of 218 μm. Since the time of publication, they have further improved their setup and tested gravity at even shorter separations. Currently, their results set an upper bound of 200 μm on the size of the largest dimension, and they set an upper bound of 150 μm on the size for n = 2 large extra dimensions [13].

In addition to bounds on the extra dimensions due to this table top experiment, there are also restrictions that come from observations of supernova (SN). The largest constraint on extra dimensions comes from the observed duration of the SN 1987A neutrino burst. If there are large extra dimensions, then the 4 dimensional graviton is accompanied by a tower of KK states. These gravitons would be emitted from the SN core after collapse in a process that would compete with cooling by neutrinos, shortening the observed duration of neutrinos. From this duration, a conservative bound of $R \leq 0.9 \times 10^{-4}$ mm is made for n = 2 and $R \leq 1.9 \times 10^{-7}$ mm for n = 3 in the ADD model.

One of the problems with the ADD model is that while it does eliminate the hierarchy between the weak scale m_{ew} and the Planck scale M_{Pl}, it introduces a new hierarchy between the compactification scale $\mu_c \approx 1/R$ and m_{ew}. Indeed, there are different higher-dimensional mechanisms for solving the hierarchy problem. One such method was theorized by Lisa Randall and Raman Sundrum (hereafter referred to as the RS model) [3 - 4]. In fact, there are two RS models, RS1 and RS2, and both will be discussed. First, it is important to qualify some of the things that have been said previously. It is not necessarily true that the Planck scale is related to the higher dimensional scale of gravity, $M_{Pl(4+n)}$, by (9) with n extra compact dimensions. This property relies on a factorizable geometry. Recall that the metric is a way to measure distances in different space-time geometries. For flat, Minkowski space-time, the metric is given by (2). Now, suppose you wanted to add an extra dimension that was orthogonal to x_1, x_2, and x_3. The simplest way to do this is to put in an extra term *ad hoc*, such as

$$ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu + dy^2 \tag{11}$$

where y is the position in the fifth dimension. In this case, the metric of our four space-time dimensions is not dependent of the y coordinate. By remaining on a constant y-slice (a 3-brane), you recover the same metric, independent of where you are in the fifth dimension. However, in the presence of a non-factorizable geometry, the Planck scale is no longer determined by the size of the extra dimension; instead, it is determined by the higher dimensional curvature [4]. In this case, it is possible that there are (4 + n) non-compact dimensions, and this would still be compatible with all experiments performed thus far. The warped geometry hides the extra dimensions even more efficiently than ADD models of extra dimensions.

The first model to be described by Randall and Sundrum (RS1) involved two 3-branes [3]. They had one extra dimension of finite size r_c (which can be thought of as the compactification radius; space begins at one brane and ends at the location of the other, and the branes are separated by a distance πr_c.) The only assumptions made were the existence of two

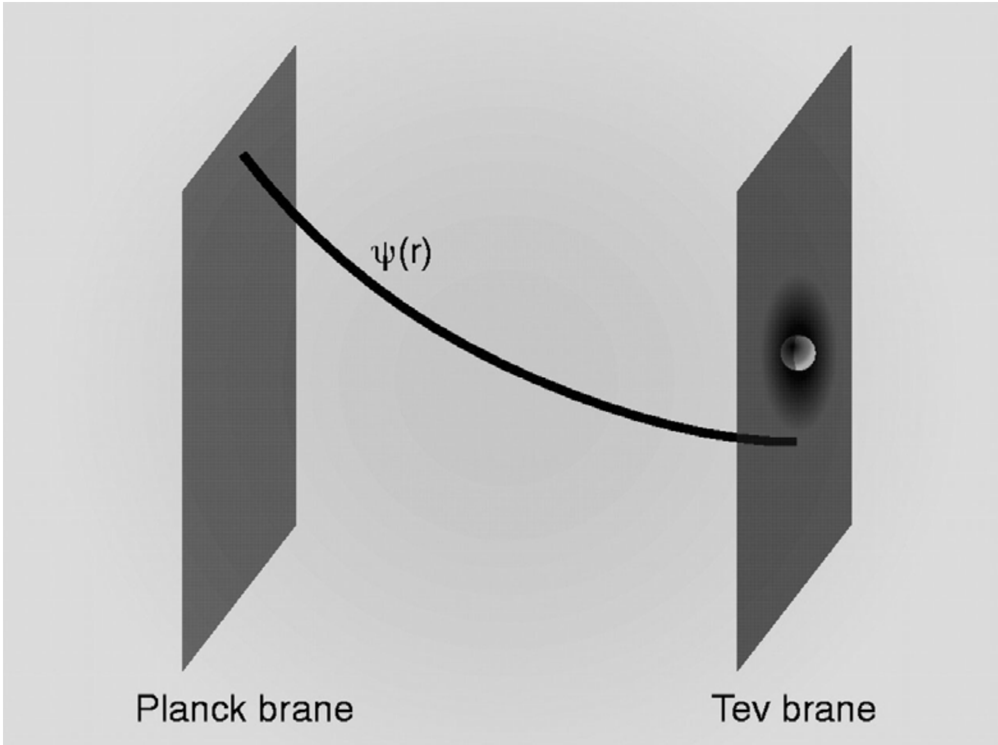

Figure 4. Basic setup for RS1. Gravity is trapped on the Planck brane, and it appears weak on our TeV brane due to the exponential decrease of the graviton wavefunction away from the Planck brane (shown as the black line) [11].

3-branes in five dimensions and their compatibility with four-dimensional Poincaré invariance in the x^{μ} directions. Solving the five-dimensional Einstein's equations for the action in this given scenario resulted in the five dimensional metric

$$ds^2 = e^{-2k|y|}\eta_{\mu\nu}dx^{\mu}dx^{\nu} + dy^2 \qquad (12)$$

where $0 \leq y \leq \pi\, r_c$ and k is a scale of the order of the Planck scale. It is the exponential factor in (12) that causes the large hierarchy between the observed Planck and electroweak scales. See Figure 4. Indeed, since this source is an exponential function of r_c, a large compactification radius is not necessary in generating the hierarchy. Here, it may be beneficial to point out the differences between RS1 and ADD. Although both seek to solve the hierarchy problem, using the two brane model of RS1 requires only one extra dimension (though there could be more) as opposed to $n \geq 2$ extra dimensions. While the ADD brane world had a tower of KK modes, there are no light KK modes in the RS1 brane world [3]. In this case, the excitations of the KK modes are set by the weak scale. They should be observable at high energy colliders as spin-2 resonances [4]. For RS1, there is the Planck brane and the TeV brane. The Planck scale is the fundamental scale, and gravity is trapped on the Planck brane. The gravitational force weakens away from this brane due to the exponential rescaling of masses [11]. Now, imagine that the SM is stuck on the TeV brane. The electroweak force sees only this brane, while gravity probes the bulk. Since the electroweak mass scale decreases exponentially from the brane that traps gravity, a relatively natural sized dimension is all that is required to explain the hierarchy. The 5-d Planck scale is related to the geometry of the extra dimensions and the observed Planck scale by

$$M_{Pl}^{2} = 2M_{Pl5}^{3}\int_{0}^{\pi r_c} e^{-2k|y|}dy = \frac{M_{Pl5}^{3}}{k}[1 - e^{-2kr_c\pi}] \quad (13)$$

Randall and Sundrum also found something else interesting. They discovered that Λ_5, the five-dimensional cosmological constant (vacuum energy) of the bulk, was negative. This showed that the space-time between the two branes was a slice of Anti-de Sitter (AdS_5) geometry. It was also found that the relations

$$V_{brane} = -V_{brane'} = 24M_{Pl5}^{3}k \qquad \text{and} \qquad \Lambda_5 = -24M_{Pl5}^{3}k^{2} \quad (14)$$

had to be satisfied to ensure four-dimensional Poincaré invariance. Although nothing initially sets the size of r_c, the hierarchy scale is strongly dependent on this value. The two branes have tension (and thus energy), and there is also an energy in the bulk. There needed to be some mechanism that stabilized the distance between the branes without fine-tuning. One proposed mechanism is that there exists an additional scalar field that is a particle whose energy is minimized for a particular size of the extra dimension [15].

The second model to be described by Randall and Sundrum (RS2) started in a way similar to RS1, with two branes, and achieved the same results as they did previously.

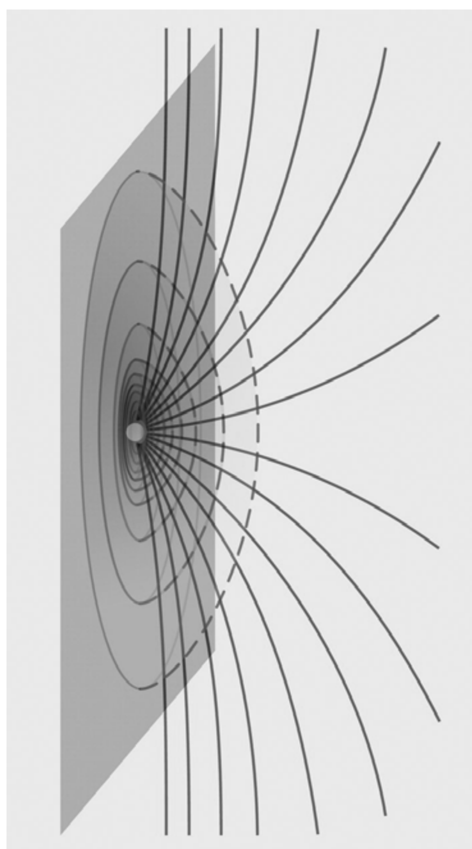

Figure 5. Basic setup for RS2. In this scenario, the extra dimension is infinite. Gravity is localized on our brane. The solid lines are the gravitational force lines, and the dashed lines represent equipotential surfaces. Note that the force lines are denser near the brane in this warped geometry [11].

The difference between their initial setup in this brane world and that of their previous one was that they interchanged the branes. The solution for the background metric was the same, involving the exponential decrease from one brane to the other. Previously, there was the "hidden" Planck brane to which the graviton was bound. Our "visible" brane saw the hierarchy because of the exponential decrease of the mass parameter on our TeV brane that was some distance away from the Planck brane. However, it is not necessary to consider the Planck scale as fundamental. Instead, one can consider the electroweak scale as fundamental and the Planck scale as derived. In this scenario, it is essential that the graviton be bound to our visible brane. This is possible, because one can show, using Einstein's equations, that the tension of the brane and the negative cosmological constant of the bulk (l4) allow a bound-state mode of the five-dimensional graviton that is highly concentrated on the brane [11]. See Figure 5. Therefore, it behaves as if it were a four-dimensional graviton. Now, we can take $r_c \rightarrow \infty$, removing the second brane from the setup. The size of the extra dimension is now infinite! Note that this is possible in the RS brane world using (13), but it is not possible in the context of ADD and (9). It was initially thought that larger dimensions lead to weaker gravitational forces (9); therefore, infinite dimensions should not be possible, because they would indicate that gravity is infinitely weak and thus decoupled from the brane world. Due to the concentration of the graviton on the brane, we would only be sensitive to the short distance scale over which gravity decreases and not the size of the extra dimension. Now, an infinite extra dimension could exist, and we would have never observed it. So, what would gravity look like on our brane? It was found that the static potential generated by the exchange of the zero-mode and continuum KK mode propagators between two masses is given by [3]

$$V(r) = -G_N \frac{m_1 m_2}{r} - \int_0^\infty \frac{G_N}{k} \frac{m_1 m_2 e^{-mr}}{r} \frac{m}{k} dm \,. \tag{15}$$

This integral can easily be calculated, resulting in a gravitational potential

$$V(r) = -G_N \frac{m_1 m_2}{r} \left(1 + \frac{1}{r^2 k^2} \right). \tag{16}$$

where k is expected to have a value that is of the same order as the fundamental Planck scale. As one can see, the addition of an infinite extra dimension still produces an effective four-dimensional theory of gravity. Equation (16) also implies that no deviations from Newtonian gravity are expected at the separation distance of 100 μm that is being tested [13]. Thus, none of the current constraints on large extra dimensions apply to the RS brane world as they do to the ADD brane world.

The traditional belief is that there are (3 + 1) dimensions, because all of the observations that we have performed are consistent with three spatial dimensions. In fact, it seems that three and only three spatial dimensions are required in order to agree with experiment. Compactification was one way to hide the extra dimensions, but they were believed to be so small (comparable to the Planck length) that we would never be able to probe them. Thus, it provided the possibility of extra dimensions, but it was uninteresting because it could not be tested. With the realization that everything in the Standard Model (photon, W/Z bosons, and fermions) could be confined to a 4-dimensional brane, large extra dimensions suddenly became possible. However, gravitons would still be able to probe the extra dimensions. So, any theory with large extra dimensions had to be consistent with experiments. Gravity had been measured at millimeter and now micron distances with no deviation from Newtonian gravity, confining the size of the dimensions in ADD brane worlds. RS brane worlds suffered from no such constraint. Separations of much less than micron distance would have to be used to test their theory. Another way to test both of these theories is through the use of particle accelerators. It is possible that the electroweak symmetry breaking scale (TeV) is one of the fundamental scales of nature, and this is the energy regime at which the LHC is set to run. In the ADD model, SM fields are only localized within m_{ew}^{-1} in the extra dimensions. Therefore, a particle involved in a sufficiently hard collision could acquire enough momentum in the extra dimension to escape from our brane. This would be signaled by a sharp upper limit in the transverse momentum that can be seen in 4 dimensions. In the RS models, the KK modes might be observed at the LHC as spin-2 resonances. According to RS2, due to the infinite size of the extra dimension, the gravitons created in collisions would decay immediately and appear as missing energy. Both ADD and RS models go further than probing the possibility of extra dimensions; they seek to answer the question of why extra dimensions could be important. Their answer is that they solve problems in physics in completely natural ways. For example, the hierarchy problem is naturally explained by large extra dimensions. Hopefully in the next few years, when LHC comes online and when the full details of these theories are flushed out, we will be able to see whether or not we live in a universe with large extra dimensions.

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