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Conical Turbulent Boundary Layer Experiments and a Correlation With Flat Plate Data

Measurements in the turbulent boundary layer of unyawed cones for Mach numbers from 1 to 6 are presented. Specifically, the first measurements of total temperature profiles, directly determined skin friction, and local heat flux in the turbulent boundary layer of cone models are given. In addition, these experimental data are shown to justify the calculation of turbulent friction and heat transfer on unyawed cones by means of an incompressible plate friction law and simple auxiliary relations.

Experimental Methods

The experimental investigation was performed in the wind tunnel facilities of the University of Minnesota Rosemount Aeronautical Laboratory. In Fig. 1 is shown a photograph of the 15-deg cone model with boundary layer probes installed in the $M_\infty = 3$, 6 × 9-in. test section. An impact temperature probe [1][1] was mounted on the upper surface of the cone and an impact pressure probe on the lower surface at the meridional plane. The probes were adjustable from outside the tunnel during operation by means of push rods projecting through the model skin as shown. The probe tips were approximately 5 inches downstream from the tip of the model.

The impact pressure probe was made of $^1/_{32}$-in. OD stainless steel tubing. The tip opening was 1.5 mils[2] high and 17 mils wide with outside dimensions of 3.2 × 20 mils. The probe was designed for fast response [2], the response time being about 15 seconds.

The locations of the probes relative to the surface of the model were measured with a 20-power stereoscopic microscope.

The surface of the model was instrumented with static pressure taps and thermocouples [3]. Static pressure orifices were 14 mils

[1] Numbers in brackets designate References at end of paper.

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[2] 1 mil = 0.001 in.

Nomenclature

A = area of surface
C_f = local skin friction coefficient, τ/q_1
C_F = average or total skin friction coefficient $= \frac{1}{A}\int_A C_f(A')dA'$
c_p = specific heat at constant pressure
h = heat-transfer coefficient (quantity of heat per unit time per unit area per temperature difference)
k = thermal conductivity
M = Mach number without subscript indicates local value of Mach number in undisturbed boundary layer
mil = one thousandth of an inch . . . roughly 1/40 of a millimeter
n = exponent in turbulent velocity profile
Pr = Prandtl number $= \frac{c_p\mu}{k}$ based on thermal equilibrium at the wetted surface
$q = \frac{\rho u^2}{2}$
St = Stanton number $= \frac{h}{\rho_1 c_p u_1}$ based on conditions just outside boundary layer
T = temperature
u = velocity in the x-direction
x = distance downstream from boundary layer origin on cone or plate
y = distance perpendicular to the wetted surface
δ = boundary layer thickness determined by taking $y = \delta$ when $u = 0.995u_1$
δ^* = boundary layer displacement thickness, $\int_0^\delta \left[1 - \frac{\rho u}{\rho_1 u_1}\right] dy$
γ = ratio of specific heats c_p/c_v
θ = boundary layer momentum defect thickness, $\int_0^\delta \frac{\rho u}{\rho_1 u_1}\left[1 - \frac{u}{u_1}\right] dy$
ν = kinematic viscosity μ/ρ
μ = viscosity
ρ = density
σ = boundary layer recovery factor $\frac{T_e - T_1}{T_0 - T_1}$

Subscripts

∞ = upstream conditions
1 = conditions locally at outer edge of boundary layer
0 = stagnation conditions
e = properties based on recovery temperature (surface temperature at condition of zero heat transfer)
i = incompressible
t = either "transition" or "thermal boundary layer" as appropriate
w = conditions on surface of model
x = as in R_x, characteristic length used in calculating R_t

Superscripts

* = indicates values corresponding to equation (9)

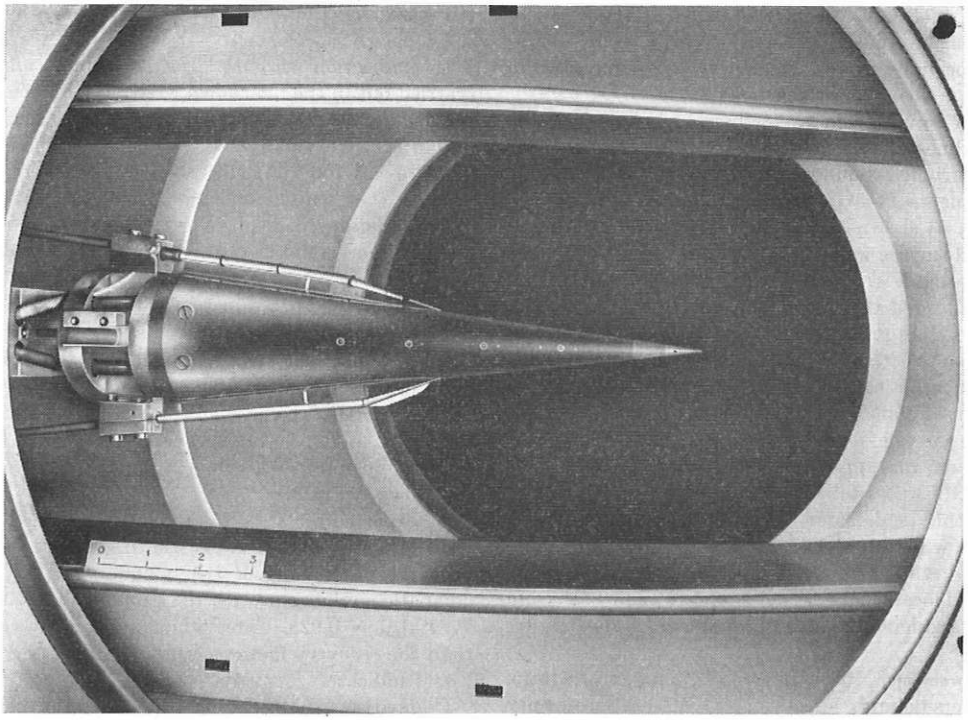

Fig. 1 Installation of impact pressure and impact temperature probes on 15-deg total angle cone

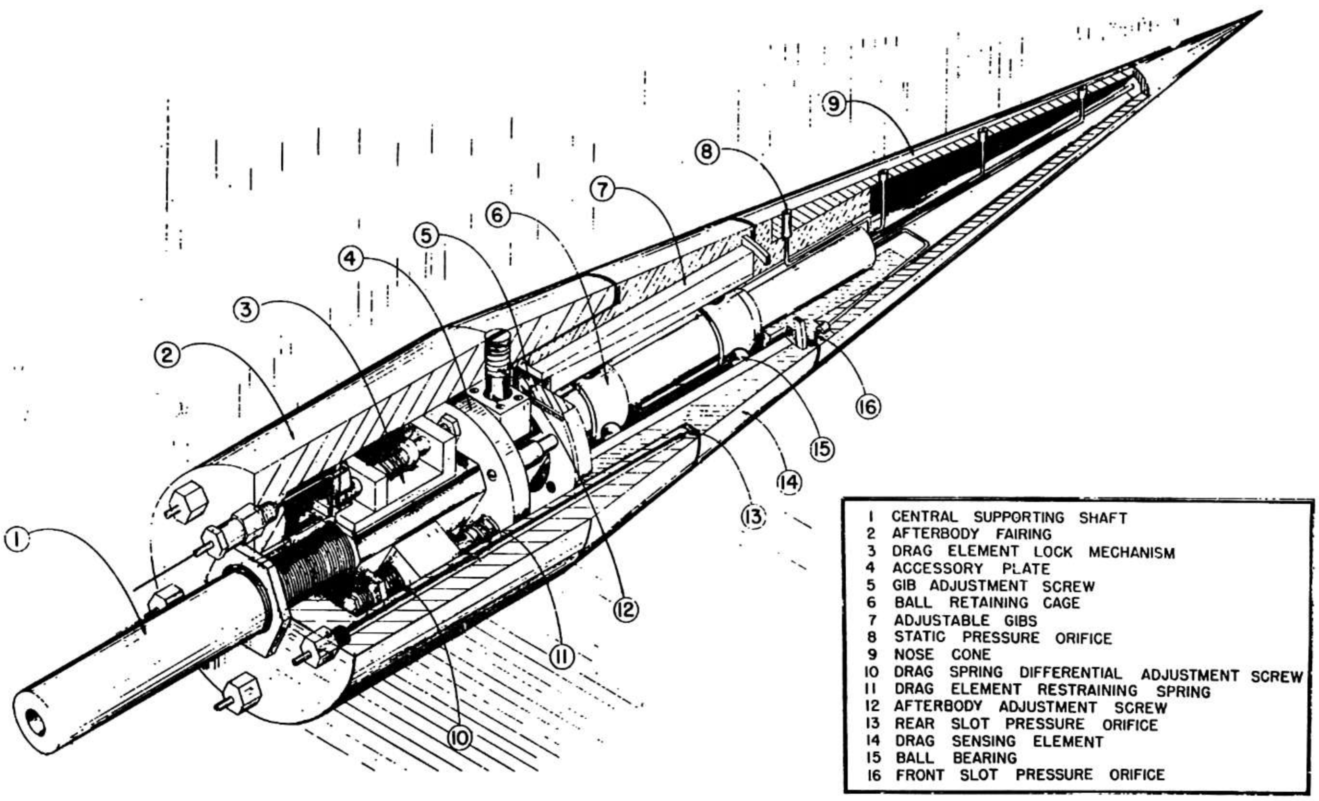

Fig. 2 15-deg cone drag balance

in diameter and opened into $^1/_{16}$-in. OD brass tubing pressure leads. Orifice fittings were soldered into the model skin. Iron-constantan thermocouples were installed by soft soldering in holes drilled through the $^1/_{32}$-in-thick model skin.

A combination of an upstream directed air jet issuing from the cone tip and a roughness trip (shown in Fig. 1) was used to insure the existence of a turbulent boundary layer.

Direct measurements of skin friction were made using the balance system [4, 5] shown as Fig. 2. A segment (truncated cone) of the conical surface was supported from the central shaft on 6 steel balls in such a way that it was free to "float" with a clearance of 1 mil afforded at the leading edge and trailing edge of the floating surface. Lateral adjustment of the floating element was achieved by the selective adjustment of an internal parallel inclined wedge system. A conservative estimate of the maximum floating element misalignment with the conical surface was $^3/_{10}$ mil. However, the average boundary layer thickness for the skin friction measurements was approximately 120 mils giving a ratio of boundary layer thickness to surface step of 400. During the testing no waves could be detected as originating from either the front or rear crack when observed with a double pass schlieren system at high sensitivity.

Determinations of local values of heat flux were made both from electrically heated segmented models [6] and from a "heat sink" cone [7].

Accuracy of Measurements. Random errors in temperature ratio, or in flow parameters determined by probing due to uncertainty in pressure and temperature measurement are:[3] $T_e/T_0 = \pm 1$ per cent; momentum thickness ± 5 per cent to ± 7 per cent; displacement thickness ± 3 per cent to ± 5 per cent.

For measurements on the model with the balance system, the experimental uncertainties are as follows: Reynolds number ± 3 per cent; friction coefficient ± 3.4 per cent; Mach number ± 1.5 per cent.

[3] See reference [7] for details.

In connection with the direct determinations of skin friction (carried out in the blowdown wind tunnel), the pressure force on the floating element was determined by measurements of pressure on both the exterior and interior surfaces of the floating element. The pressure force was always less than 20 per cent of the friction drag for these measurements.

The accuracy of heat-transfer measurements was determined from the uncertainties in the terms in the heat-balance equation. They are: $(T_w - T_e) \pm 2$ deg F; slope $\pm$ 4%; $\Delta W/\Delta A \pm$ 1%. Thus the uncertainty in h was $\pm 5\%$. The corresponding uncertainty in Stanton number was $\pm 8\%$.

Mach Number and Total Temperature Profiles. The first measurement of a cone flow turbulent thermal boundary layer total temperature profile was accomplished for this investigation. Simultaneously, an impact pressure profile was obtained. The measurements were obtained at $\mathbf{M}_1 = 2.8$ and $R_x = 4 \times 10^6$. The results are shown as Fig. 3.

As expected for a gas (Prandtl number less than unity), local total enthalpy values above the freestream value are observed near the outer edge of the boundary layer. The surface, in adiabatic equilibrium with the boundary layer flow, stabilized at $T_0/T_{01}]_{y=0} = 0.928$. This temperature ratio is one per cent lower than the recovery factor, according to the relation $\sigma = (\mathrm{Pr})^{1/3}$, would indicate.

Comparison [7] of the present data with existing two dimensional, nearly adiabatic data [8, 9], indicates that the cone boundary layer total temperature profile is similar to two-dimensional profiles in the y/δ_t versus $T_0(y)/T_{01}$ presentation. Further, indications are that similarity with respect to Reynolds number variations is also obtained over a moderate Reynolds number range at $\mathbf{M}_1 = 3$.

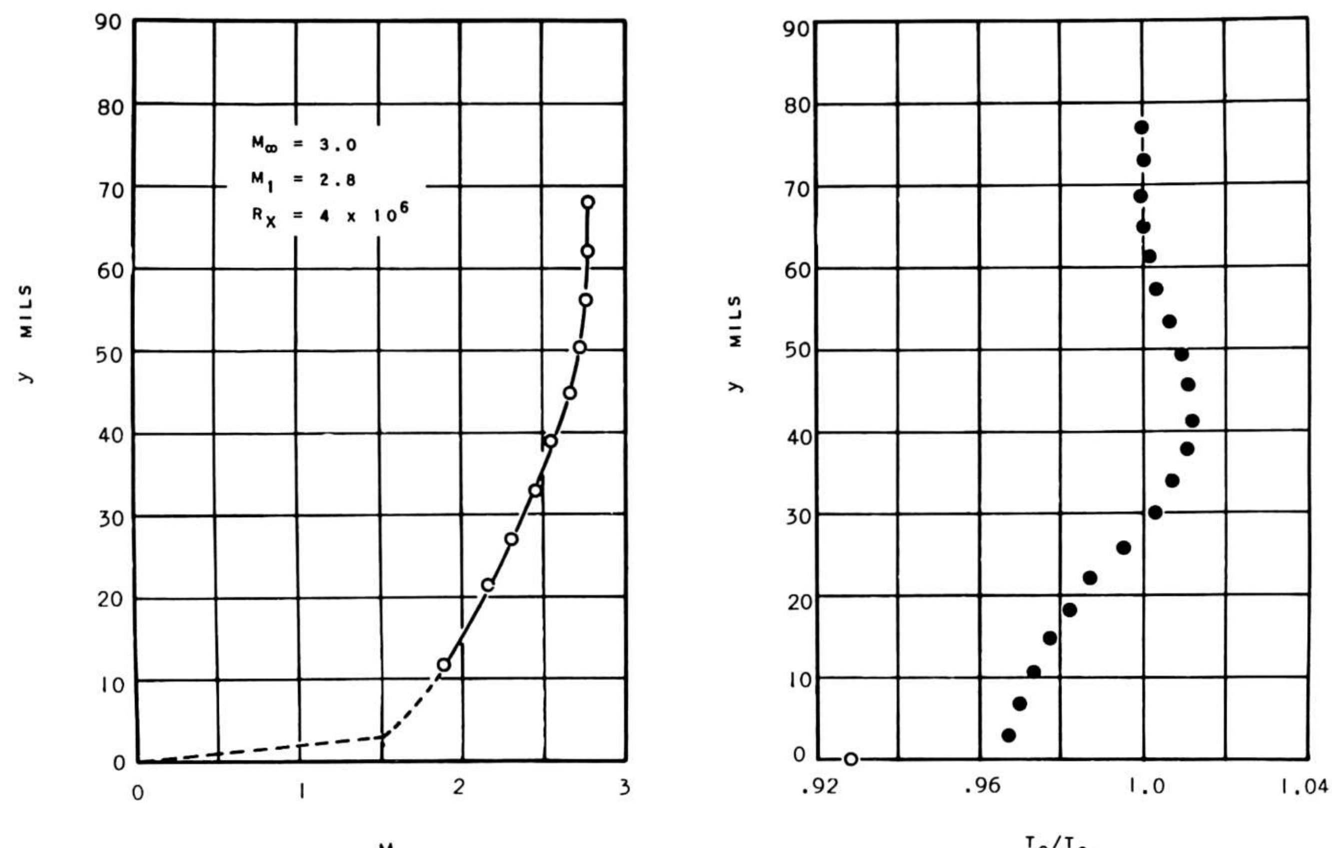

Fig. 3 Mach number and total temperature profiles in the turbulent boundary layer of 15-deg total angle cone

Turbulent Boundary Layer Velocity Profiles. Velocity profiles may be calculated from the Mach profile number and free stream total temperature. A set of turbulent velocity profiles from a cone at $M_1 = 3.7$ is presented as Fig. 4. The data were obtained from a fifteen-degree total angle cone at zero yaw and in thermal equilibrium with the flow. A combination roughness (sand) and air trip [4, 5] was used to produce fully turbulent boundary layer for three of the five sets of data. One set was obtained with sand trip only and one with air trip only.

Just as in the case of existing flat-plate data over a relatively wide range of Reynolds number and Mach number, the cone data are well represented by $^1/_7$-power profiles for the outer region in combination with linear profiles assumed for the laminar sublayer.

An Engineering Prediction of Turbulent Skin Friction and Heat Transfer on a Cone at Zero Yaw at Supersonic Speeds. It was indicated by the results of the present investigation [7] that velocity profiles from the turbulent boundary layer of both plate and cone at the same M_1 are congruent when presented as u/u_1 versus y/θ. In other words

$$\frac{u}{u_1} = a(y/\theta)^{1/n} \tag{1}$$

for both plate and cone where a has the same dependence on Mach number for both cases over the range of Mach number investigated. It was found that $n = 7$ for the moderate range of Reynolds numbers investigated.

If it is now assumed that a friction law of form $C_f = B(R_\delta)^k$ holds, this law together with equation (1) results in

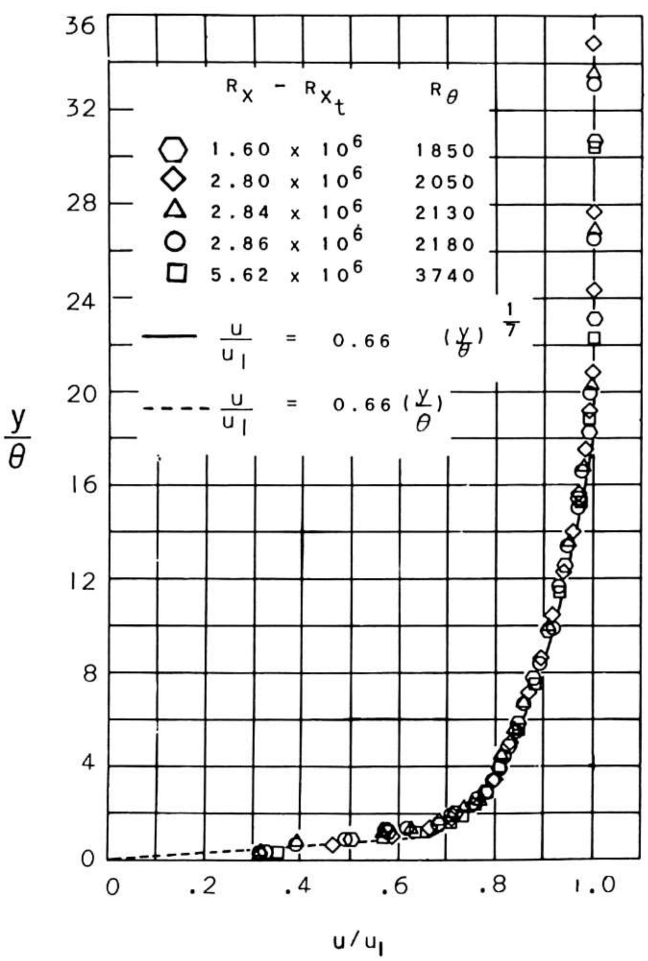

Fig. 4 Turbulent velocity profiles on a cone

$$C_f = \frac{B}{\left[\frac{u_1\delta}{\nu_1}\right]^{\frac{2}{n+1}}} \tag{2}$$

for both cases. It is further assumed that B, a parameter depending on Mach number, has the same dependence on Mach number for both cases. It follows that the friction coefficients will be related by

$$\frac{C_f}{\bar{C}_f} = \left[\frac{\bar{\delta}}{\delta}\right]^{\frac{2}{n+1}} \tag{3}$$

Here and in what follows, a bar over a quantity indicates a two-dimensional (plate or axial flow cylinder) boundary layer quantity.

For the plate,

$$\frac{\bar{C}_f}{2} = \frac{d\bar{\theta}}{dx} \tag{4}$$

while for the cone

$$\frac{C_f}{2} = \frac{d\theta}{dx} + \frac{\theta}{x} \tag{5}$$

Again, as a consequence of the congruence of velocity and temperature profiles for plate and cone, the momentum defect thickness for both cases has the same dependence on M_1. That is,

$$\frac{\theta}{\delta} = \frac{\bar{\theta}}{\bar{\delta}} = f(M_1, n) \tag{6}$$

Using equations (2) and (6), equations (4) and (5) can be integrated. These results together with equation (3) give

$$\frac{C_f}{\bar{C}_f} = \left[\frac{2(n+2)}{n+1}\right]^{\frac{2}{n+3}} \tag{7}$$

at a specified M_1, n, u_1/ν_1, and $x = \bar{x}$. This development, which explicitly considers variable Mach number, yields precisely the same result as that obtained by Gazley [10] for the case of constant properties.

It is general practice to obtain heat-transfer coefficients from skin friction coefficients for the turbulent boundary layer by use of the Reynolds analogy (see, for example, reference [11], p. 107). For compressible flow plate experiments, a form of the Reynolds analogy modified to account for property value variation has proved useful [12]. This relation, proposed by A. P. Colburn [18], for incompressible flow, is

$$\bar{\text{St}} = \frac{\bar{C}_f}{2}(\text{Pr}^*)^{-2/3} \tag{8}$$

where the Prandtl number (Pr*) is evaluated at the reference temperature

$$T^* = T_1 + 0.5(T_w - T_1) + 0.22(T_e - T_1) \tag{9}$$

proposed by Eckert [12].

If it is assumed that equation (8) holds for cone flow as well as for plate flow, then

$$\frac{\text{St}}{\bar{\text{St}}} = \frac{C_f}{\bar{C}_f} \tag{10}$$

if $T^* = \overline{T^*}$.

For $n = 7$, equation (7) yields for local values of Stanton number and skin friction

$$\frac{\text{St}}{\overline{\text{St}}} = \frac{C_f}{\overline{C}_f} = 1.18 \qquad (11)$$

for $T^* = \overline{T^*}$ and $R_x = \overline{R_x}$ independent of Reynolds number and Mach number over the range covered by the experimental data discussed here. Note that for $5 < n < 10$, equation (7) shows $1.23 < C_f/\overline{C}_f < 1.13$. Thus the effect of variations of n on the transformation relation is small in this range.

For obtaining the relation between mean skin friction and heat transfer on plates and cones, the relations

$$\overline{C_F} = \frac{1}{\bar{x}} \int_0^{\bar{x}} \overline{C}_f(x')dx' \quad \text{and} \quad C_F = \frac{2}{x^2} \int_0^x C_f(x')x'dx'$$

are employed. It follows from the above that for average friction and heat-transfer coefficients

$$\frac{C_F}{\overline{C}_F} = \frac{\text{S}_T}{\overline{\text{S}}_T} = 2\left[\frac{n+1}{2(n+2)}\right]^{\frac{n+1}{n+3}} = 1.05 \qquad (12)$$

for $n = 7$. The effect of variations of n on the transformation relation equation (12) is negligible; for $5 < n < 10$, $1.065 < C_F/\overline{C_F} < 1.035$.

Correlations of Direct Measurements of Turbulent Skin Friction and Heat Transfer. The results of the measurements at $M_1 = 3.7$ are shown as Fig. 5 in comparison with the data of Coles [13] on a flat plate in the form C_f versus $(R_x - R_{x_t})$ where the dashed line and the line through the cone data are drawn parallel to the faired line representing Coles' data. R_{x_t} is the Reynolds number based on the distance between the measuring station and the beginning of transition as determined from schlieren observations. The dashed line is higher than the plate data by 18 per cent in accord with equation (11). The cone data are about 10 per cent higher than predicted by equation (11).

Fig. 6 is adapted from Fig. 2 of reference [14]. It shows the relation between the heat-transfer data (at constant surface temperature) from two cones and a secant ogive and from the flat plate data of Pappas [15]. The mean line of the cone flow data is higher than the corresponding mean line of the flat plate data and parallel to the flat plate data mean line. The lines were obtained by least squares procedure from the data points. As predicted by equation (11), the mean line of the cone data is about 18 per cent higher than the plate data mean line.

Fig. 7 shows a comparison between measured heat transfer and transformed skin friction measurements on cones at $M_1 = 3.7$. Stanton numbers are calculated from measured skin friction coefficients by the Colburn relation, equation (8), where the Prandtl number is evaluated at the reference temperature proposed by Eckert [12], equation (9). The agreement is satisfactory.

Fig. 8 shows a comparison between heat-transfer results and skin friction results for cones and plates over the present Mach number range. The data are evaluated at $R_x = \bar{R}_x = 3 \times 10^6$. In order to provide a comparison of results on a $T^* = \overline{T^*}$ basis, all Stanton number data were adjusted to $T_w = T_e$ using the theory of Van Driest [17] (the same procedure was used in comparing the data of Fig. 7). The maximum adjustment required amounted to 5 per cent of the measured value. The average adjustment required was 3 per cent of the measured values.

In nondimensionalizing the Stanton numbers and skin friction coefficients, incompressible flat plate theory was used in both cases. Values of Stanton number (incompressible) were calculated utilizing equations (8) and (9). The reference temperature value used to calculate St. was calculated using the wind tunnel adiabatic wall temperature at each Mach number.

The vertical bars through many of the data points in Fig. 8 indicate the maximum scatter of the data at that Mach number. The symbol location in each case represents the mean value of the data. Where no vertical bar appears, the scatter of the data is represented by the symbol size.

The solid curve is the experimentally determined mean curve through the plate data points. Obviously, this relation should not be expected to hold for flow around cones at subsonic speeds or for cones with detached shocks.

The data correlations indicated by Figs. 5 through 8 are regarded as justification for the calculation of turbulent friction

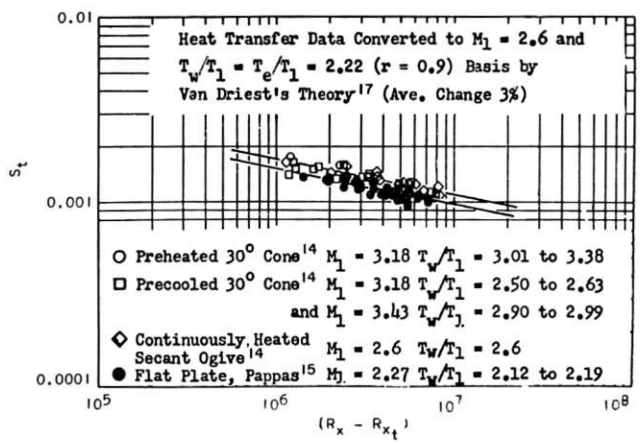

Fig. 6 Comparison of direct measurements of turbulent heat transfer on cones and a plate at $M_1 = 2.6$

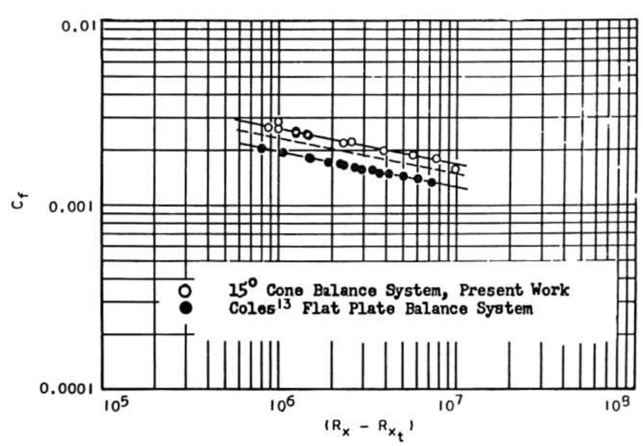

Fig. 5 Comparison of direct measurements of turbulent skin friction on a cone and a plate at $M_1 = 3.7$

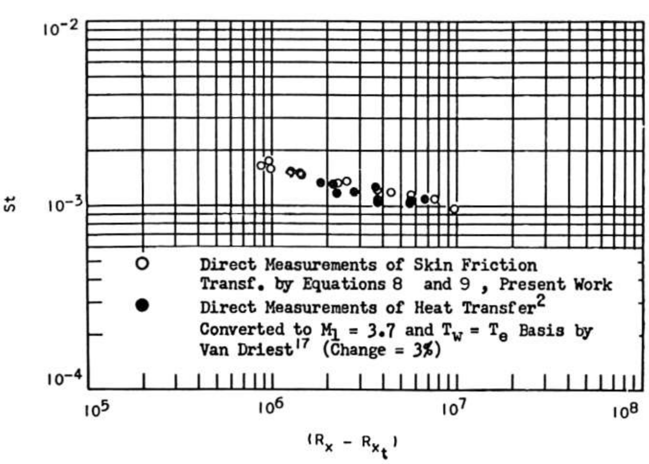

Fig. 7 Comparison of heat transfer on a cone with skin friction on a cone at $M_1 = 3.7$

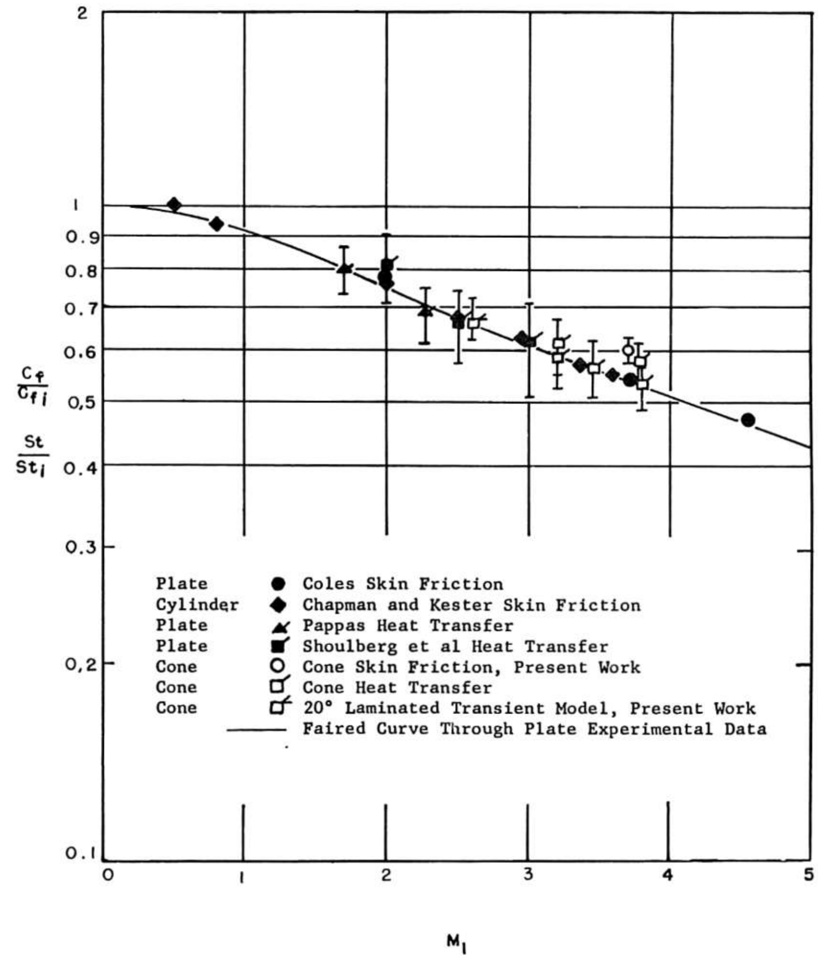

Fig. 8 A comparison between heat-transfer and skin-friction measurements on plates, axial flow cylinders, and cones

and heat transfer on unyawed cones at supersonic speeds by means of an incompressible plate friction law and the simple auxiliary relations given as equations (8), (9), (11), and (12).

As a summary of this procedure, suppose it is desired to calculate the local heat flux at the surface of a cone at Mach number M_1 and R_x at the outside edge of the boundary layer. Starting with the Schultz-Grunow local skin friction law

$$\overline{C_{fi}} = \frac{0.370}{(\log_{10} \overline{R_x})^{2.584}} \tag{13}$$

where $\overline{R_x} = R_x$, the corresponding incompressible plate Stanton number $\overline{St}_i$ is calculated by equation (8). The compressible flow flat plate $\overline{St}$ is obtained by multiplying St_i by $\overline{St}/\overline{St}_i$ from Fig. 8. The cone Stanton number St is obtained by increasing $\overline{St}$ by 18 per cent in accordance with equation (11). The same procedure is followed for average values except the relation between cone and plate values is given by equation (12).

Summary and Conclusions. The first measurements of the total temperature distribution through a turbulent conical boundary layer are presented.

A comparison of directly measured local skin friction values on the cone with those on the plate at constant Mach number indicates that cone skin friction is higher but that it has the same dependence on Reynolds number as plate skin friction (Fig. 5). A comparison of local heat-transfer values on cone and plate similarly shows higher heat transfer on a cone at the same M_1 and Reynolds number based on property values just outside the boundary layer (Fig. 6).

It is confirmed that the modified Reynolds analogy of Colburn, equation (5), together with the reference temperature of Eckert, equation (6), permits the calculation of cone heat-transfer values from known cone skin-friction values (Figs. 7 and 8).

Finally, an analysis is presented which predicts that skin-friction and heat-transfer values on unyawed cone in supersonic flow should be approximately a constant times the corresponding flat-plate values and this prediction is confirmed by experiment.

Acknowledgment

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