On the Heat Transfer of a Moving Composite Strip Compressed by Two Rotating Cylinders

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Two influential parameters in rolling mill analysis are the heat loss to the rolls and the strip temperature distribution. In this paper, the heat transfer process occurring in rolling is modeled by a moving three-layer composite strip, compressed between two rotating cylinders. It is shown that, for the range of parameters representative of strip rolling, the problem can be reduced, to leading order, to one of heat transfer beween slabs with plane parallel boundary contact. The heating effects due to deformation energy generated in the strip, friction energy generated at the strip/cylinder interfaces, and the strip/cylinder bulk temperature difference are considered. Application of the results to rolling analysis is demonstrated.

1 Introduction

The mechanical properties of sheet steel are influenced by the temperature of the steel on exit from the hot strip mill. For products of prime quality, close tolerances are required on this final strip temperature, which depends on the slab (steel plate at the rolling mill entry) temperature as well as the thermal processes occurring during rolling. These include air cooling between the rolling stands, water cooling in the descaling boxes (which remove the oxide layer formed on the strip surface) and heat loss in the rolling stands. Whereas simple heat transfer theory, involving radiation and convection, serves to describe the first two processes, the latter involves frictional and deformation heating, together with heat loss by conduction to the work rolls. Another important factor which should be considered for hot rolling is the buildup of a scale layer (oxide layer) on the strip surface. This scale layer, though normally very thin, is a poor thermal conductor and has been found to cause a significant reduction in heat loss from the strip [1, 2, 3]. Only recently have analytical studies been directed at quantifying this phenomenon.

The strip is reduced in thickness as it passes between the work rolls of a rolling stand. Figure 1 shows a sectional view of the process with a scale layer which remains intact in the contact region. Heat energy is generated in the strip as it is deformed in the roll gap region, and also along the roll/scale layer interfaces due to friction resulting from the differential speeds of the strip and rolls. Hence the heat transfer process near the contact region in a rolling stand is equivalent to that of a moving three-layer composite strip compressed by two rotating cylinders, with energy generated within the strip and at the cylinder/strip interface, and with thermal energy transfer due to the cylinder/strip bulk temperature difference.

Analytical solutions of this problem, disregarding the influence of the scale layer, have been obtained by Cerni [4] and Cerni et al. [5]. Finne et al. [6] derived a set of differential equations, which was solved numerically under the assumption of a constant scale layer thickness. Polukhin et al. [1, 2] obtained an analytical solution using similar model equations which treated the strip and the roll as two semi-infinite slabs. These solutions [1, 2, 6] take the heat capacity of the scale layer to be negligible (an assumption which requires validation). Recently, Pawelski et al. [3] have published results on the strip temperature distribution and heat transfer coefficient with the effects of a scale layer included. Unfortunately this paper [3] is rather obscure and does not give any analytical detail.

In this study, a detailed mathematical model is developed to describe the heat transfer near the contact region in a rolling stand, including the effect of the scale layer and its heat capacity. (The formulation for heating and cooling of the rolls may be found in [7, 8], and from the references quoted therein.) The set of differential equations describing the heat transfer is reduced to a simpler set by introducing the small nondimensional parameters related to the rolling conditions, thus allowing development of a perturbation solution.

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2 Problem Definition

Because of the large strip width in relation to its thickness, this study is confined to a two-dimensional analysis of the three-component system: the rolls, the scale layer, and the strip. From symmetry, only the upper half of the system needs to be considered. It is justifiable to assume that the heat transfer in the roll gap is a quasi-steady-state process because variations which occur along the strip, or during processing of a coil, take place on a much longer time scale.

For a coordinate system fixed in space, when the material properties are assumed to be constant in the temperature range under consideration, the heat conduction equation for the three-body system is

$$\mathbf{v}_i \cdot \nabla T_i = \alpha_i \nabla^2 T_i + Q_i / (\rho_i c_i) \qquad (1)$$

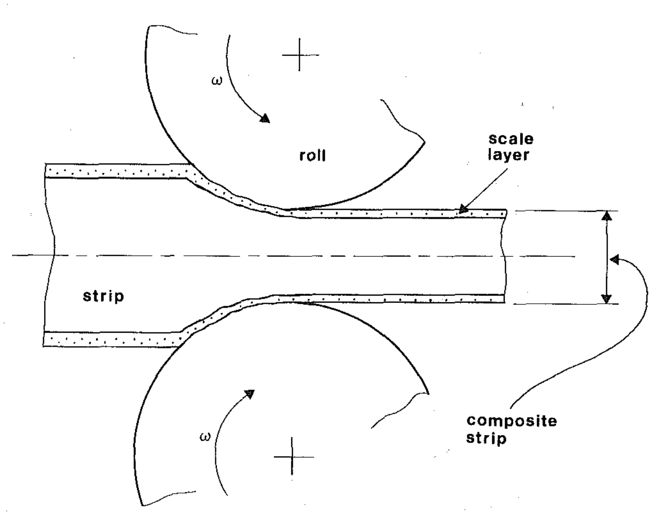

Fig. 1 Roll gap geometry (strip, scale layer, and roll not to scale)

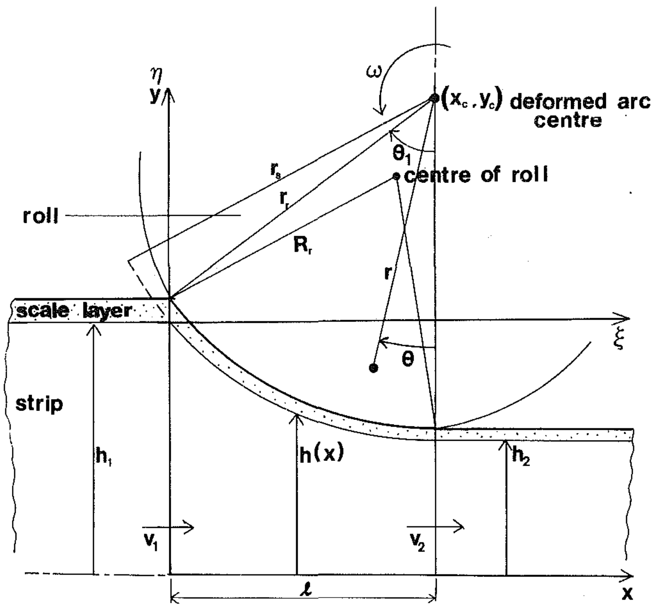

Fig. 2 Roll gap geometry assuming a circular arc of contact (strip, scale layer, and roll not to scale)

where the subscript i can be either r, s, or c, referring to the roll, strip and scale layer respectively. Here c_i is the specific heat; ρ_i the density; α_i the thermal diffusivity; T_i the temperature; $\mathbf{v}_i$ the material velocity; and Q_i the heat sources. It is not the purpose of this paper to develop theoretical expressions for Q_i. Suffice it to say that these may be obtained from any suitable roll gap model (see, for example, [6, 9]).

Since perfect contact is assumed at the interfaces, the boundary conditions to be satisfied at the strip/scale layer and roll/scale layer interfaces are based on the continuity of temperatures and heat fluxes across them, except that the total heat flow into the roll and the scale layer at the roll/scale layer interface equals the frictional energy generated therein due to slipping.

Equation (1) may be written in a two-dimensional Cartesian (x–y) form for the strip region and in a polar coordinate (r–θ) system for the roll and scale layer. For convenience, the origin of the x–y coordinates is located at the roll gap entry along the strip axis (centerline). The center of the (assumed) circular arc of the deformed roll is taken as the origin of the r–θ coordinates as shown in Fig. 2.

A nondimensionalization of the heat conduction equations is now carried out in order to identify their most significant terms.

(*a*) For the strip:

$$\hat{x} = x/(2r_s\Delta h)^{1/2}$$
$$\hat{y} = y/h_1$$
$$\hat{v}_{sx} = v_{sx}/v_1$$
$$\hat{v}_{sy} = \frac{v_{sy}}{v_1(2\Delta h/r_s)^{1/2}}$$
$$\hat{T}_s = T_s/T_{s1}$$

where r_s is the distance from the deformed roll center to the strip/scale layer interface at the roll gap entry; $\Delta h = h_1 - h_2$ the half of the reduction in strip thickness; h_1 and h_2 are half of the strip thicknesses at the roll gap entry and exit respectively; v_1 is the horizontal strip velocity prior to the roll gap entry; T_{s1} a certain reference temperature of the strip; and v_{sx} and v_{sy} are the strip velocity components in the x and y directions respectively.

In the above, $(2r_s\Delta h)^{1/2}$ is approximately the horizontal contact length, l (since $\Delta h/r_s << 1$); $v_1(2\Delta h/r_s)^{1/2}$ is approximately the vertical speed of the strip surface at the roll gap entry point.

(*b*) For the scale layer:

$$\hat{\theta} = \theta/(2\Delta h/r_s)^{1/2}$$
$$\hat{s} = (r - r_r)/s_1$$
$$\hat{v}_{cr} = \frac{v_{cr}}{v_1 \Delta s (2r_s\Delta h)^{-1/2}}$$
$$\hat{v}_{c\theta} = v_{c\theta}/v_1$$
$$\hat{T}_c = T_c/T_{c1}$$

Nomenclature

c = specific heat
f_1 = $(\gamma_r - 1)/(\gamma_r + 1)$
f_2 = $(\gamma_s - 1)/(\gamma_s + 1)$
F_c = Fourier number for the scale layer, equation (22)
$\hat{F}_c$ = Fourier number defined in equation (39)
F_r = Fourier number for the roll, equation (23)
F_s = Fourier number for the strip, equation (21)
$\hat{F}_s$ = Fourier number defined in equation (34)
h = half of the strip thickness
k = thermal conductivity
l = contact length
q = rate of heat transfer to a roll
q_f = rate of frictional heat energy generated per unit area at the roll/scale layer interface
Q = rate of heat energy generated per unit volume
Q_s = rate of deformation heat energy generated per unit volume in the strip
r,θ = coordinates defined in Fig. 2
r_r = deformed roll radius, Fig. 2
r_s = radius defined in Fig. 2
R_r = roll radius
s = scale layer thickness
t = time
t_2 = contact time
T = temperature
T_0 = $T_{s1} - T_{r1}$
T' = average temperature
v = horizontal strip velocity
$\mathbf{v}$ = velocity vector
x,y = coordinates defined in Fig. 2
α = thermal diffusivity
β_c = term defined in equation (6)
β_r = term defined in equation (7)
β_s = term defined in equation (5)
γ_r = $(\rho_r k_r c_r)^{1/2}/(\rho_c k_c c_c)^{1/2}$
γ_s = $(\rho_s k_s c_s)^{1/2}/(\rho_c k_c c_c)^{1/2}$
Δh = half of the strip thickness reduction
Δs = scale layer thickness reduction
ϵ_1 = h_1/r_s
ϵ_2 = $\Delta h/h_1$
ϵ_3 = s_1/h_1
ϵ_4 = $\Delta s/s_1$
$\hat{\lambda}$ = term defined in equation (19)
ξ,η = coordinates, Fig. 2
ρ = density
ω = angular roll velocity

Subscripts

1 = roll gap entry
2 = roll gap exit
c = scale layer
d = deformation energy
f = friction energy
r = roll (as a first subscript)
r = radial direction (as a second subscript)
s = strip
t = initial strip/roll temperature difference
x = x-direction
y = y-direction
θ = peripheral direction

Symbols

$\hat{}$ = nondimensionalized parameter (see text for detail of nondimensionalization) where r_r is the deformed roll radius; T_{c1} a certain reference temperature of the scale layer; $\Delta s = s_1 - s_2$ the reduction in scale layer thickness in the radial direction; s_1 and s_2 are the scale layer thicknesses at the roll gap entry and exit respectively (in the radial direction); and v_{cr} and $v_{c\theta}$ are the radial and circumferential velocity components respectively for the scale layer.

Here, $(2\Delta h/r_s)^{1/2}$ is approximately the total angle subtended, θ_1; $v_1 \Delta s(2r_s \Delta h)^{-1/2}$ is approximately the scale layer thickness reduction divided by the contact time. (A general case of nonzero Δs is assumed in the above but, if Δs is zero, $v_{cr} = \hat{v}_{cr} = 0$.)

(*c*) For the roll:

$$\hat{\theta} = \theta/(2\Delta h/r_s)^{1/2}$$
$$\hat{r} = (r_r - r)/h_1$$
$$\hat{v}_{rr} = \frac{v_{rr}}{\omega(r_r - R_r)(2\Delta h/r_s)^{1/2}}$$
$$\hat{v}_{r\theta} = v_{r\theta}/(\omega R_r)$$
$$\hat{T}_r = T_r/T_{r1}$$

where ω is the angular roll speed; R_r the original roll radius; T_{r1} a certain reference temperature of the roll; and v_{rr} and $v_{r\theta}$ are the radial and circumferential velocity components respectively for the roll.

Here, r is nondimensionalized by the entry half strip thickness, h_1; ωR_r is approximately the circumferential speed of the roll surface at the roll gap entry; $\omega(r_r - R_r)(2\Delta h/r_s)^{1/2}$ is approximately the radial speed of the roll surface just prior to the roll gap entry.

Substitution of the nondimensional variables in the conduction equation (1) gives:

$$\hat{v}_{sx}\frac{\partial \hat{T}_s}{\partial \hat{x}} + 2(\Delta h/h_1)\hat{v}_{sy}\frac{\partial \hat{T}_s}{\partial \hat{y}} = \beta_s\left[\frac{1}{2}\left(\frac{h_1/r_s}{\Delta h/h_1}\right)\frac{\partial^2 \hat{T}_s}{\partial \hat{x}^2} + \frac{\partial^2 \hat{T}_s}{\partial \hat{y}^2}\right] + \hat{Q}_s \tag{2}$$

$$(1+\hat{s}s_1/r_r)(\Delta s/s_1)(r_r/r_s)\hat{v}_{cr}\frac{\partial \hat{T}_c}{\partial \hat{s}} + \hat{v}_{c\theta}\frac{\partial \hat{T}_c}{\partial \hat{\theta}} = \beta_c\left[\frac{\partial^2 \hat{T}_c}{\partial \hat{s}^2} + \left(\frac{s_1/r_r}{1+\hat{s}s_1/r_r}\right)\frac{\partial \hat{T}_c}{\partial \hat{s}} + \frac{1}{2}\left(\frac{s_1}{r_r}\right)^2 \frac{r_s/\Delta h}{(1+\hat{s}s_1/r_r)^2}\frac{\partial^2 \hat{T}_c}{\partial \hat{\theta}^2}\right] + \hat{Q}_c \tag{3}$$

$$-2(1-\hat{r}h_1/r_r)[(r_r/R_r)-1]\times (\Delta h/h_1)(r_r/r_s)\hat{v}_{rr}\frac{\partial \hat{T}_r}{\partial \hat{r}} + \hat{v}_{r\theta}\frac{\partial \hat{T}_r}{\partial \hat{\theta}} = \beta_r\left[\frac{\partial^2 \hat{T}_r}{\partial \hat{r}^2} - \left(\frac{h_1/r_r}{1-\hat{r}h_1/r_r}\right)\frac{\partial \hat{T}_r}{\partial \hat{r}} + \frac{1}{2}\left(\frac{h_1}{r_r}\right)^2 \frac{r_s/\Delta h}{(1-\hat{r}h_1/r_r)^2}\frac{\partial^2 \hat{T}_r}{\partial \hat{\theta}^2}\right] + \hat{Q}_r \tag{4}$$

where

$$\beta_s = [(2r_s\Delta h)^{1/2}/v_1]/(h_1^2/\alpha_s) \tag{5}$$
$$\beta_c = [r_r(2\Delta h/r_s)^{1/2}(1+\hat{s}s_1/r_r)/v_1]/(s_1^2/\alpha_c) \tag{6}$$
$$\beta_r = [r_r(2\Delta h/r_s)^{1/2}(1-\hat{r}h_1/r_r)/(\omega R_r)]/(h_1^2/\alpha_r) \tag{7}$$
$$\hat{Q}_s = [Q_s(2r_s\Delta h)^{1/2}/v_1]/(\rho_s c_s T_{s1}) \tag{8}$$
$$\hat{Q}_c = [Q_c r_r(2\Delta h/r_s)^{1/2}(1+\hat{s}s_1/r_r)/v_1]/(\rho_c c_c T_{c1}) \tag{9}$$

and

$$\hat{Q}_r = [Q_r r_r(2\Delta h/r_s)^{1/2}(1-\hat{r}h_1/r_r)/(\omega R_r)]/(\rho_r c_r T_{r_1}) \tag{10}$$

The relevance of the nondimensionalization is now apparent since, in physical terms, β_s, β_c, and β_r are approximately the ratio of contact time to diffusion time for the media, and $\hat{Q}_s$, $\hat{Q}_c$, and $\hat{Q}_r$ are approximately the ratio of heat energy generated in the roll gap to the "stored" energy at the roll gap entry for the media.

In rolling, the entry strip thickness, while normally much larger than the strip thickness reduction, is very much smaller than the deformed roll radius. Moreover, the scale layer, if present, is very thin compared to the strip thickness, and its reduction in thickness will also be very small.

Therefore,

$$\epsilon_1 = h_1/r_s << \epsilon_2 = \Delta h/h_1 << 1$$
$$\epsilon_3 = s_1/h_1 << 1$$

and

$$\epsilon_4 = \Delta s/s_1 << 1$$

From geometry, $r_r/r_s = 1 - s_1/r_s = 1 - \epsilon_1\epsilon_3$. It follows that

$$(s_1/r_r)^2(r_s/\Delta h) = (\epsilon_1/\epsilon_2)[\epsilon_3^2/(1-\epsilon_1\epsilon_3)] << 1$$
$$(h_1/r_r)^2(r_s/\Delta h) = (\epsilon_1/\epsilon_2)(1-\epsilon_1\epsilon_3)^{-2} << 1$$

Thus the dominant terms in equations (2-4) may be identified by writing them in terms of these small parameters. In addition, it is often desirable to express the roll gap variables in terms of the time variable t so that the contact time will then appear explicitly. For the nondimensional variables employed, the relationships are:

$$\hat{v}_{sx} = \frac{(2r_s\Delta h)^{1/2}/v_1}{h_1^2/\alpha_s}\frac{\partial \hat{x}}{\partial \hat{t}}$$

$$\hat{v}_{c\theta} = (r_r/r_s)(1+\hat{s}s_1/r_r)\frac{(2r_s\Delta h)^{1/2}/v_1}{h_1^2/\alpha_s}\frac{\partial \hat{\theta}}{\partial \hat{t}} = \frac{(2r_s\Delta h)^{1/2}/v_1}{h_1^2/\alpha_s}\frac{\partial \hat{\theta}}{\partial \hat{t}} + O(\epsilon_1\epsilon_3)$$

$$\hat{v}_{r\theta} = (r_r/r_s)(1-\hat{r}h_1/r_r)\frac{(2r_s\Delta h)^{1/2}/(\omega R_r)}{h_1^2/\alpha_s}\frac{\partial \hat{\theta}}{\partial \hat{t}} = \frac{(2r_s\Delta h)^{1/2}/(\omega R_r)}{h_1^2/\alpha_s}\frac{\partial \hat{\theta}}{\partial \hat{t}} + O(\epsilon_1, \epsilon_1\epsilon_3)$$

where $\hat{t} = t/(h_1{}^2/\alpha_s)$ is the time parameter, nondimensionalized by the diffusion time across half the strip thickness. Since the effects of temperature gradient and heat transfer are confined mainly to the interface regions, it is more appropriate to transform the $(\hat{x}, \hat{y})$ axes to the $(\hat{\xi}, \hat{\eta})$ axes with the origin set on the strip surface at the roll gap entry (Fig. 2), using the transformations $\hat{\xi} = \hat{x}$ and $\hat{\eta} = \hat{y} - 1$. The relations between $(\hat{r}, \hat{\theta})$ [or $(\hat{s}, \hat{\theta})$] and $(\hat{\xi}, \hat{\eta})$ are

$$\hat{\xi} = \begin{cases} \hat{x}_c - [r_r/(2r_s\Delta h)^{1/2}](1+\hat{s}s_1/r_r)\sin[(2\Delta h/r_s)^{1/2}\hat{\theta}] \\ = \hat{x}_c - \hat{\theta} + O[(\epsilon_1\epsilon_2)^{3/2}, \epsilon_1\epsilon_3] & \text{for the scale layer} \\ \hat{x}_c - [r_r/(2r_s\Delta h)^{1/2}](1-\hat{r}h_1/r_r)\sin[(2\Delta h/r_s)^{1/2}\hat{\theta}] \\ = \hat{x}_c - \hat{\theta} + O(\epsilon_1, \epsilon_1\epsilon_2, \epsilon_1\epsilon_3) & \text{for the roll} \end{cases}$$

$$\hat{\eta} = \begin{cases} \hat{y}_c - 1 - (r_r/h_1)(1+\hat{s}s_1/r_r)\cos[(2\Delta h/r_s)^{1/2}\hat{\theta}] \\ = \epsilon_2(\hat{\theta}^2 - 1) + \epsilon_3(1-\hat{s}) + O(\epsilon_1\epsilon_2, \epsilon_1\epsilon_3) & \text{for the scale layer} \\ \hat{y}_c - 1 - (r_r/h_1)(1-\hat{r}h_1/r_r)\cos[(2\Delta h/r_s)^{1/2}\hat{\theta}] \\ = \hat{r} + O(\epsilon_2, \epsilon_3, \epsilon_1\epsilon_2, \epsilon_1\epsilon_3) & \text{for the roll} \end{cases}$$

It follows that equations (2-4), after the transformation and simplification, become

$$\frac{\partial \hat{T}_s}{\partial \hat{t}} = \frac{\partial^2 \hat{T}_s}{\partial \hat{\eta}^2} + Q_s\,(h_1^2/\alpha_s)/(\rho_s c_s T_{s1}) + O(\epsilon_2, \epsilon_1/\epsilon_2) \tag{11}$$

$$\frac{\partial \hat{T}_c}{\partial \hat{t}} = (\alpha_c/\alpha_s)\frac{\partial^2 \hat{T}_c}{\partial \hat{\eta}^2} + Q_c\,(h_1^2/\alpha_s)/(\rho_c c_c T_{c1}) + O(\epsilon_4, \epsilon_1/\epsilon_2, \epsilon_1\epsilon_2, \epsilon_1\epsilon_3, \epsilon_3^2) \tag{12}$$

$$\frac{\partial \hat{T}_r}{\partial \hat{t}} = (\alpha_r/\alpha_s)\frac{\partial^2 \hat{T}_r}{\partial \hat{\eta}^2} + Q_r\,(h_1^2/\alpha_s)/(\rho_r c_r T_{r1}) + O(\epsilon_1, \epsilon_2, \epsilon_1/\epsilon_2, \epsilon_1\epsilon_2, \epsilon_1\epsilon_3) \tag{13}$$

The boundary conditions in the new coordinate system $(\hat{t}, \hat{\eta})$ are

(a) on the strip axis, $\hat{\eta} = -1$, from symmetry:

$$\frac{\partial \hat{T}_s(\hat{t}, -1)}{\partial \hat{\eta}} = 0 \tag{14}$$

(b) on the strip/scale layer interface, $\hat{\eta} = 0 + O(\epsilon_2, \epsilon_1\epsilon_2)$, for continuity of temperatures and heat fluxes:

$$\hat{T}_s(\hat{t}, 0) = (T_{c1}/T_{s1})\hat{T}_c(\hat{t}, 0) \tag{15}$$

$$\frac{\partial \hat{T}_s(\hat{t}, 0)}{\partial \hat{\eta}} = (k_c/k_s)\,(T_{c1}/T_{s1})\frac{\partial \hat{T}_c(\hat{t}, 0)}{\partial \hat{\eta}} \tag{16}$$

(c) on the roll/scale layer interface, $\hat{\eta} = \epsilon_3 + O(\epsilon_2, \epsilon_1\epsilon_2, \epsilon_1\epsilon_3)$:

$$\hat{T}_r(\hat{t}, \epsilon_3) = (T_{c1}/T_{r1})\hat{T}_c(\hat{t}, \epsilon_3) \tag{17}$$

$$-\frac{\partial \hat{T}_r(\hat{t}, \epsilon_3)}{\partial \hat{\eta}} = -(k_c/k_r)\,(T_{c1}/T_{r1})\frac{\partial \hat{T}_c(\hat{t}, \epsilon_3)}{\partial \hat{\eta}} + h_1 q_f/(k_r T_{r1}) \tag{18}$$

where q_f is the rate of friction energy (per unit area) generated at the roll/scale layer interface due to slipping;

(d) on a circular layer in the roll at a sufficient distance from the interfaces such that heat flow across the layer during contact time may be neglected (this assumption is valid since the heating and cooling of the roll confines to a very small region near the roll surface [7, 8]), say, at $\hat{r} = \hat{\lambda}$ where $1 >> \hat{\lambda} >> \epsilon_2\epsilon_3$, i.e., $\hat{\eta} = \hat{\lambda} + O(\epsilon_2, \epsilon_3, \epsilon_1\epsilon_2, \epsilon_1\epsilon_3)$:

$$\frac{\partial \hat{T}_r(\hat{t}, \hat{\lambda})}{\partial \hat{\eta}} = 0 \tag{19}$$

It can be seen that the leading order problem for the heat transfer in the roll gap reduces to equations (11-13) which, with the above set of boundary conditions, describes one-dimensional heat flow between two thick flat slabs separated by a slab of finite thickness. The perturbation formulation, developed here, clarifies the conditions under which this model may be employed and justifies the use of the simplified system introduced as the starting point of the previous analyses [1-6].

3 Solution

In order to obtain an analytic solution, a certain temperature distribution at the roll gap entry must be assumed. Since the strip and roll speeds (or, more precisely, the Peclet numbers, which are defined as $v_{sx}h_1/\alpha_s$ and $\omega R_r^2/\alpha_r$ for the strip and roll respectively) are high, the conduction component is small compared to the advective component, and it is justified to assume a uniform temperature distribution for all three bodies at the roll gap entry, i.e., the strip and scale layer have the same initial temperature, set to equal their reference temperatures, $T_{s1} = T_{c1}$, hence $\hat{T}_s(0, \hat{\eta}) = \hat{T}_c(0, \hat{\eta}) = 1$; and the initial roll temperature is equal to its reference temperature T_{r1} giving $\hat{T}_r(0, \hat{\eta}) = 1$. The initial time, $\hat{t} = 0$, is assumed to be at the roll gap entry.

Since the deformation heat energy generated in the roll and scale layer regions is usually negligible in comparison with that generated in the strip, it is reasonable to set $Q_r = Q_c = 0$.

A first-order solution of equations (11-13) subject to boundary conditions (14-19) can be obtained, using Laplace transforms, with further assumptions that the deformation heat energy Q_s and frictional energy q_f, which may be calculated using a roll gap model (see, for example, [6]), are distributed uniformly throughout the roll gap; and that the transport time is small compared to the diffusion time (i.e., $\hat{t} << 1$) such that the approximation of semi-infinite slabs is valid.

For convenience, and since the problem is linear, the solution is written in component form:

$$\hat{T}_i(\hat{t}, \hat{\eta}) = 1 + \hat{T}_{it}(\hat{t}, \hat{\eta}) + \hat{T}_{id}(\hat{t}, \hat{\eta}) + \hat{T}_{if}(\hat{t}, \hat{\eta}) \tag{20}$$

where $\hat{T}_{it}$, $\hat{T}_{id}$, and $\hat{T}_{if}$ are the temperature changes due to the roll/strip bulk temperature difference, deformation energy, and frictional energy respectively. Although the temperature distribution in the scale layer may also be derived, it is not included here since it has no practical significance.

The solution may best be expressed in terms of Fourier numbers, F_s, F_c, and F_r, which are defined as follows:

$$F_s(\hat{t}, \hat{\eta}) = \alpha_s t/\eta^2 = \hat{t}/\hat{\eta}^2 \tag{21}$$

$$F_c(\hat{t}) = \alpha_c t/s_1^2 = (\alpha_c/\alpha_s)\,\hat{t}/\epsilon_3^2 \tag{22}$$

$$F_r(\hat{t}, \hat{\eta}) = \alpha_r t/(\eta - s_1)^2 = (\alpha_r/\alpha_s)\hat{t}/(\hat{\eta} - \epsilon_3)^2 \tag{23}$$

where

$$\eta = h_1\hat{\eta}.$$

It should be noted that in the above, the Fourier numbers for the strip and roll, F_s and F_r, are functions of both $\hat{t}$ and $\hat{\eta}$. They give an indication on the extent of heating or cooling of the strip/roll element of interest after the elapsed time. On the other hand, F_c, the Fourier number for the scale layer, is a function only of $\hat{t}$ and is a measure of the elapsed time in comparison with the diffusion time across the scale layer.

It can be shown, after some lengthy mathematical manipulations, that the solutions for the roll and strip temperature distributions are:

$$\hat{T}_{rt}(\hat{t}, \hat{\eta}) = \frac{T_0}{T_{r1}}\frac{1}{\gamma_r + 1}\left\{\sum_{n=0}^{\infty}\left[(f_1 f_2)^n \operatorname{erfc}\left(\frac{1}{2}F_r^{-1/2} + nF_c^{-1/2}\right)\right] + f_2\sum_{n=0}^{\infty}\left[(f_1 f_2)^n \operatorname{erfc}\left(\frac{1}{2}F_r^{-1/2} + [n+1]F_c^{-1/2}\right)\right]\right\} \tag{24}$$

$$\hat{T}_{rd}(\hat{t}, \hat{\eta}) = 8\hat{t}\frac{h_1^2 Q_s}{k_s T_{r1}}\frac{\gamma_s}{(\gamma_r + 1)(\gamma_s + 1)} \times \sum_{n=0}^{\infty}\left\{(f_1 f_2)^n i^2\operatorname{erfc}\left[\frac{1}{2}F_r^{-1/2} + \frac{1}{2}(2n+1)F_c^{-1/2}\right]\right\} \tag{25}$$

$$\hat{T}_{rf}(\hat{t}, \hat{\eta}) = 2\hat{t}^{1/2}\frac{h_1 q_f}{k_s T_{r1}}\frac{\gamma_s}{\gamma_r + 1}\sum_{n=0}^{\infty}\left\{(f_1 f_2)^n \times \left[i\operatorname{erfc}\left(\frac{1}{2}F_r^{-1/2} + nF_c^{-1/2}\right) - f_2\, i\operatorname{erfc}\left(\frac{1}{2}F_r^{-1/2} + [n+1]F_c^{-1/2}\right)\right]\right\} \tag{26}$$

$$\hat{T}_{st}(\hat{t}, \hat{\eta}) = -2\frac{T_0}{T_{s1}}\frac{\gamma_r}{(\gamma_r + 1)(\gamma_s + 1)} \times \sum_{n=0}^{\infty}\left\{(f_1 f_2)^n \operatorname{erfc}\left[\frac{1}{2}F_s^{-1/2} + \frac{1}{2}(2n+1)F_c^{-1/2}\right]\right\} \tag{27}$$

$$\hat{T}_{sd}(\hat{t}, \hat{\eta}) = \hat{t}\frac{h_1^2 Q_s}{k_s\, T_{s1}}\left\{1\right.$$

$$-\frac{4}{\gamma_s+1}\sum_{n=0}^{\infty}\left[(f_1f_2)^n\left(i^2erfc\left[\frac{1}{2}F_s^{-1/2}+nF_c^{-1/2}\right]\right.\right.$$

$$\left.\left.\left.+f_1 i^2 erfc\left[\frac{1}{2}F_s^{-1/2}+\{n+1\}F_c^{-1/2}\right]\right)\right]\right\} \quad (28)$$

$$\hat{T}_{sf}(\hat{t},\hat{\eta})=4\hat{t}^{1/2}\frac{h_1 q_f}{k_s T_{s1}}\frac{\gamma_s}{(\gamma_r+1)(\gamma_s+1)}\times$$

$$\sum_{n=0}^{\infty}\left\{(f_1f_2)^n ierfc\left[\frac{1}{2}F_s^{-1/2}+\frac{1}{2}(2n+1)F_c^{-1/2}\right]\right\} \quad (29)$$

where

$$\gamma_r=(\rho_r k_r c_r)^{1/2}/(\rho_c k_c c_c)^{1/2}$$

$$\gamma_s=(\rho_s k_s c_s)^{1/2}/(\rho_c k_c c_c)^{1/2}$$

$$f_1=(\gamma_r-1)/(\gamma_r+1)$$

$$f_2=(\gamma_s-1)/(\gamma_s+1)$$

$$T_0=T_{s1}-T_{r1}$$

and k_s is the strip thermal conductivity.

Here, $i^m erfc(w)$ is the complementary error function defined as

$$i^m erfc(w)=\int_w^{\infty} i^{m-1}erfc(u)du \quad m=0,1,2,\ldots$$

where

$$i^0 erfc(w)=erfc(w)=1-erf(w)$$

On integration of equations (27–29) with respect to $\hat{\eta}$, the average strip temperature across its section, $\hat{T}_s'(\hat{t})$, may be obtained:

$$\hat{T}_s'(\hat{t})=1+\hat{T}_{st}'(\hat{t})+\hat{T}_{sd}'(\hat{t})+\hat{T}_{sf}'(\hat{t}) \quad (30)$$

where

$$\hat{T}_{st}'(\hat{t})=4\hat{t}^{1/2}\frac{T_0}{T_{s1}}\frac{h_1}{h(t)}\frac{\gamma_r}{(\gamma_r+1)(\gamma_s+1)}\sum_{n=0}^{\infty}\left\{(f_1f_2)^n\times\right.$$

$$\left.\left[ierfc\left(\frac{1}{2}\hat{F}_s^{-1/2}+\frac{1}{2}[2n+1]F_c^{-1/2}\right)-ierfc\left(\frac{1}{2}[2n+1]F_c^{-1/2}\right)\right]\right\} \quad (31)$$

$$\hat{T}_{sd}'(\hat{t})=\hat{t}\frac{h_1^2 Q_s}{k_s T_{s1}}\left\{1-\frac{8}{\gamma_s+1}\hat{t}^{1/2}\frac{h_1}{h(t)}\sum_{n=0}^{\infty}\left[(f_1f_2)^n\times\right.\right.$$

$$\left(i^3erfc[nF_c^{-1/2}]-i^3erfc\left[\frac{1}{2}\hat{F}_s^{-1/2}+nF_c^{-1/2}\right]\right.$$

$$+f_1 i^3 erfc[\{n+1\}F_c^{-1/2}]-f_1 i^3 erfc\left[\frac{1}{2}\hat{F}_s^{-1/2}\right.$$

$$\left.\left.\left.\left.+\{n+1\}F_c^{-1/2}\right]\right)\right]\right\} \quad (32)$$

and

$$\hat{T}_{sf}'(\hat{t})=8\hat{t}\frac{h_1 q_f}{k_s T_{s1}}\frac{h_1}{h(t)}\frac{\gamma_s}{(\gamma_r+1)(\gamma_s+1)}\times$$

$$\sum_{n=0}^{\infty}\left\{(f_1f_2)^n\left[i^2erfc\left(\frac{1}{2}[2n+1]F_c^{-1/2}\right)\right.\right.$$

$$\left.\left.-i^2erfc\left(\frac{1}{2}\hat{F}_s^{-1/2}+\frac{1}{2}[2n+1]F_c^{-1/2}\right)\right]\right\} \quad (33)$$

In the above,

$$\hat{F}_s(\hat{t},h)=F_s[\hat{t},h(t)/h_1]=\alpha_s t/h^2(t) \quad (34)$$

Although the average roll temperature may be obtained by a similar approach, the details will not be included here.

The heat flow rate to a roll q_r may be obtained by integrating the heat conduction along the roll/scale layer interface:

$$\hat{q}_r=\frac{q_r}{k_r T_{r1}}\frac{\hat{t}_2}{\hat{l}}=-\int_0^{\hat{t}_2}\frac{\partial\hat{T}_r(\hat{t},\epsilon_3)}{\partial\hat{\eta}}d\hat{t}=\hat{q}_{rt}+\hat{q}_{rd}+\hat{q}_{rf} \quad (35)$$

where

$$\hat{q}_{rt}=\frac{1}{2}\frac{T_0}{T_{r1}}\left(\frac{\alpha_s}{\alpha_r}\right)^{1/2}\frac{1}{\gamma_r+1}\left\{\sum_{n=0}^{\infty}\left[(f_1f_2)^n I_1(n\hat{F}_c^{-1/2})\right]\right.$$

$$\left.+f_2\sum_{n=0}^{\infty}\left[(f_1f_2)^n I_1([n+1]\hat{F}_c^{-1/2})\right]\right\} \quad (36)$$

$$\hat{q}_{rd}=4\frac{h_1^2 Q_s}{k_r T_{r1}}\frac{\gamma_r}{(\gamma_r+1)(\gamma_s+1)}\times$$

$$\sum_{n=0}^{\infty}\left\{(f_1f_2)^n I_3\left[\frac{1}{2}(2n+1)\hat{F}_c^{-1/2}\right]\right\} \quad (37)$$

and

$$\hat{q}_{rf}=\frac{h_1 q_f}{k_r T_{r1}}\frac{\gamma_r}{\gamma_r+1}\times$$

$$\sum_{n=0}^{\infty}\left\{(f_1f_2)^n[I_2(n\hat{F}_c^{-1/2})-f_2 I_2([n+1]\hat{F}_c^{-1/2})]\right\} \quad (38)$$

In the above,

$$\hat{F}_c=F_c(1)=(\alpha_c/\alpha_s)/\epsilon_3^2 \quad (39)$$

Here, $l=\hat{l}h_1$ and $t_2=\hat{t}_2(h_1{}^2/\alpha_s)$ are the contact length and contact time respectively. If a detailed roll gap model is not available, these parameters may be approximated by $l=(2r_s\Delta h)^{1/2}$ and $t_2=v_1 l$. The integrals I_1, I_2, and I_3, which may be integrated by parts, are defined, with the results of integration given, as follows:

$$I_1(u)=\int_0^{\hat{t}_2}\hat{t}^{-1/2}i^{-1}erfc(u\hat{t}^{-1/2})d\hat{t}$$

$$=2[\hat{t}_2^{1/2}i^{-1}erfc(u\hat{t}_2^{-1/2})-2u\ erfc(u\hat{t}_2^{-1/2})]$$

$$I_2(u)=\int_0^{\hat{t}_2}erfc(u\hat{t}^{-1/2})d\hat{t}$$

$$=(\hat{t}_2+2u^2)erfc(u\hat{t}_2^{-1/2})-u\hat{t}_2^{1/2}i^{-1}erfc(u\hat{t}_2^{-1/2})$$

$$I_3(u)=\int_0^{\hat{t}_2}\hat{t}^{1/2}ierfc(u\hat{t}^{-1/2})d\hat{t}$$

$$=\frac{2}{3}\left\{\hat{t}_2^{3/2}ierfc(u\hat{t}^{-1/2})-\frac{1}{2}u[\hat{t}_2+2u^2erfc(u\hat{t}_2^{-1/2})\right.$$

$$\left.-u\hat{t}_2^{1/2}i^{-1}erfc(u\hat{t}_2^{-1/2})]\right\}$$

where u is a function independent of $\hat{t}$.

When the scale layer is absent, the above solutions can be further simplified, and are found to agree with previous results under the same conditions ([1,2] with the "heat transfer coefficient from the strip to the roll" set to infinity).

4 Numerical Results

Unfortunately, a direct comparison with previous numerical results, and, in particular, with those of [3], has been made difficult by uncertainty of the data values employed therein. (It should be pointed out that the formulation in [3] clearly differs from the present formulation in that the frictional energy is inexplicably taken to be generated at the strip/scale layer interface, and that the strip center-line temperature is assumed to remain unchanged even when deformation energy is generated in the roll gap.) Typical thermal values for hot rolling conditions (Table 1) have been used for all the calculations given in this paper. Terms in the

Table 1 Typical thermal data used in the numerical calculations

Strip		
Conductivity, k_s	(W/m°C)	28
Diffusivity, α_s	(m^2/s)	5.9×10^{-6}
Scale Layer		
Conductivity, k_c	(W/m°C)	2.5
Diffusivity, α_c	(m^2/s)	4.6×10^{-7}
Roll		
Conductivity, k_r	(W/m°C)	31
Diffusivity, α_r	(m^2/s)	5.4×10^{-6}

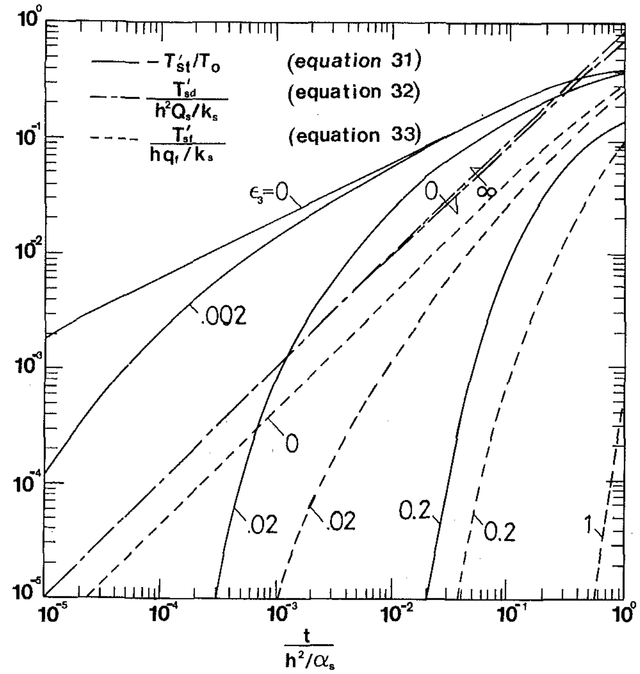

Fig. 3 Variations of the average strip temperature with the scale layer thickness and time

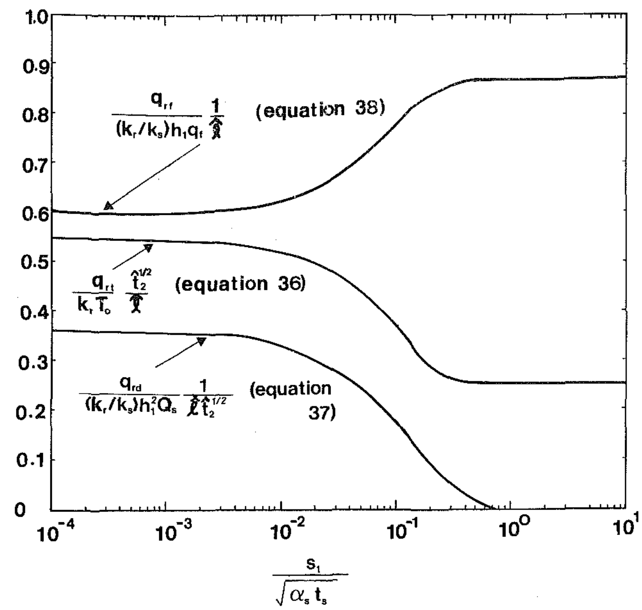

Fig. 4 Variations of heat transfer to the roll with the scale layer thickness and contact time

solution were rearranged to eliminate redundant variables so that the most compact form of information may be presented. The figures, which will be described below, are extremely useful for rapid calculation of strip temperature and heat transfer, without the necessity of having to calculate the roll gap parameters accurately. Rational approximation and recurrence relations of the error functions, given in [10], were used in the numerical calculations and it was found that reasonable accuracy could be achieved using only the first few terms of the infinite series solution.

4.1 Average Strip Temperature. The average strip temperature in the roll gap at any instance is shown in Fig. 3, which is a plot of equations (31–33), with the temperature change components, T'_{st}, T'_{sd}, T'_{sf} normalized by T_0, h^2Q_s/k_s, and hq_f/k_s respectively, and time t by the diffusion time, h^2/α_s. The second and third normalizing parameters for the temperature components represent, respectively, an insulated strip temperature change due to the deformation energy generated during the strip diffusion time, and the steady-state temperature difference between the strip center line and strip surface due to heat flux caused by all the frictional energy passing across the half strip. It can be seen from Fig. 3 that the scale layer has a pronounced influence on the bulk temperature difference (T_0) and frictional heating (q_f) effects but is insignificant for the deformation heating (Q_s) effect. In the latter case the normalized average strip temperature is found to approximate a linear relation with the normalized time, signifying that the heat loss to the rolls is insignificant compared to the energy generated. It should be further noted that the curves shown in Fig. 3 are independent of the actual strip thickness. However, for the theory to be valid, the normalized time should be sufficiently small (certainly less than unity) for the assumption of the "thick slabs" to be justified.

4.2 Heat Transfer to the Rolls. Equations (36–38) may be plotted by rearranging terms such that the heat transfer to the rolls is described in a single diagram. The heat transfer components q_{rt}, q_{rd}, q_{rf} have been normalized by $k_rT_0\hat{l}\hat{t}^{-1/2}$, $(k_r/k_s)h_1{}^2Q_s\hat{l}\hat{t}_2{}^{1/2}$, and $(k_r/k_s)h_1q_f\hat{l}$ respectively, and plotted against $s_1(\alpha_st_2)^{-1/2}$, a parameter dependent on the scale layer thickness and contact time. The three curves shown in Fig. 4 give the normalized heat transfer components to a roll; each curve approaches an upper and a lower limit, and all are dependent on the thermal properties of the strip, scale layer, and the roll. These limits, approached when $s_1(\alpha_st_2)^{-1/2}$ tends to zero and infinity, are equivalent physically to the case of thermal exchanges between two semi-infinite slabs, with the appropriate thermal data adopted. The heat transfer due to T_0 and Q_s is reduced as $s_1(\alpha_st_2)^{-1/2}$ increases due to the insulating effect of the scale layer. Where the scale layer is sufficiently thick (or, more precisely, the diffusion time across the scale layer is large compared to the elapsed time), no deformation energy will reach the roll, whereas the heat transfer due to T_0 will arise solely from the heat capacity stored in the scale layer. Consequently, q_{rt} tends to a finite but nonzero limit while q_{rd} tends to zero. The heat transfer due to q_f is found to increase as $s_1(\alpha_st_2)^{-1/2}$ increases, again due to the insulating effect of the scale layer which, in this case, reduces the frictional energy being transferred to the strip. (The reader is reminded that friction energy is assumed to be generated at the roll/scale layer interface.) Conversion of Pawelski and Bruns' results [3] (only available for q_{rt}), using the thermal data of Table 1, gives excellent agreement for the upper limit of the heat transfer component. However, the fact that their results for the lower limit are approximately 40% lower than those herein may be simply due to the difference in the thermal data values used for the scale layer. It is obvious from Fig. 4 that the heat transfer for all three components has reached its upper and lower limits in regions outside the range $0.003 < s_1(\alpha_st_2)^{-1/2} < 1$.

4.3 Effect of Heat Capacity of the Scale Layer. Most

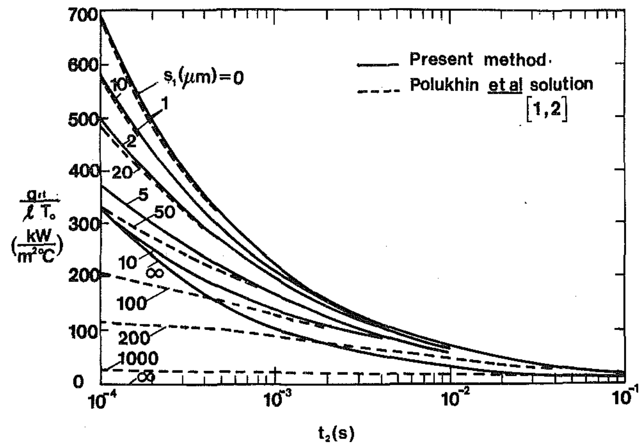

Fig. 5 Comparison of heat transfer to the roll due to an initial strip/roll temperature difference with the Polukhin et al. solution [1,2]

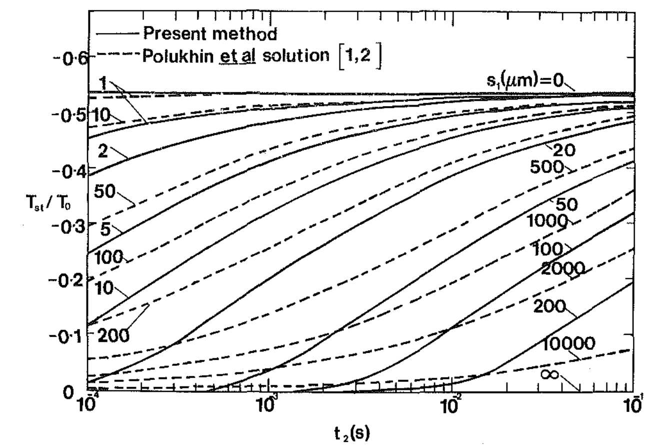

Fig. 7 Comparison of temperature change at the strip/scale layer interface due to an initial strip/roll temperature difference with the Polukhin et al. solution [1,2]

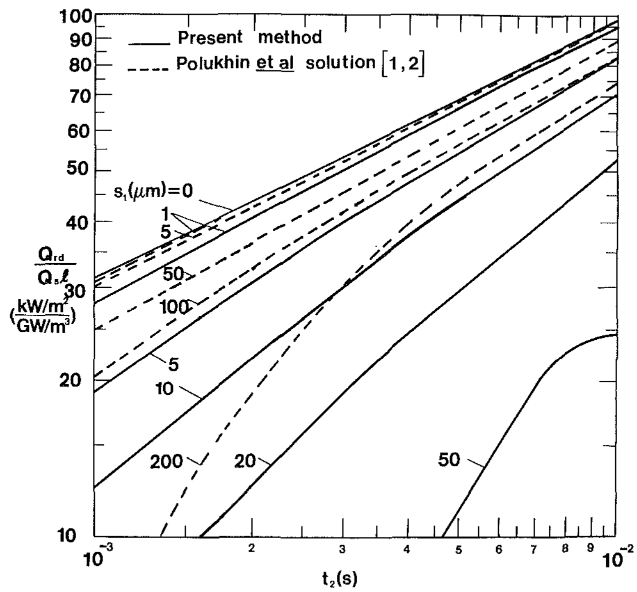

Fig. 6 Comparison of heat transfer to the roll due to deformation energy with the Polukhin et al. solution [1,2]

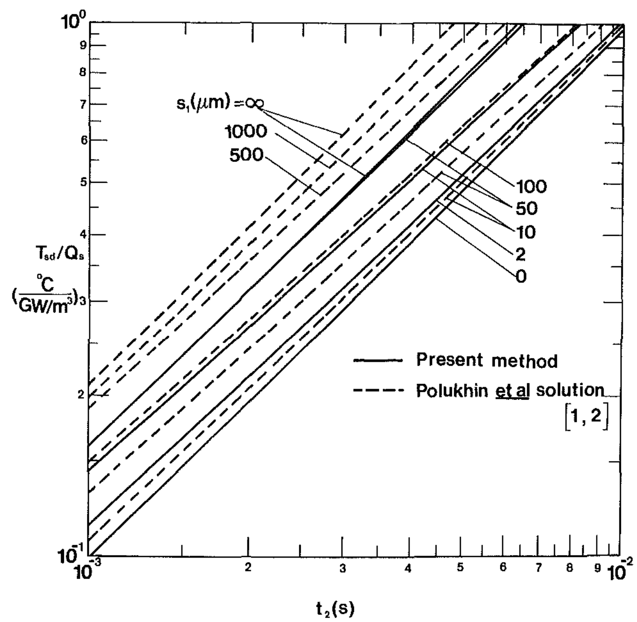

Fig. 8 Comparison of temperature change at the strip/scale layer interface due to deformation energy with the Polukhin et al. solution [1,2]

previous workers have neglected the heat capacity of the scale layer in their calculations because of its negligible thickness. The validity of this approximation can be studied with the current analysis. The present solution is compared, in particular, with that of Polukhin et al. [1, 2], who obtained a solution analytically, on the assumption that the heat transfer coefficient equals (k_c/s_1), i.e., the scale layer has only thermal resistance and no inertia. From their formulae (with the minor typographical errors corrected), the heat transfer to the roll due to T_0 and Q_s, and the strip temperature components at the strip/scale layer interface due to T_0 and Q_s are compared in Figs. 5–8 respectively (expressions for friction energy are not available in [1, 2]). When the scale layer is absent, the two solutions are, of course, identical. However, the results diverge as the scale layer thickness increases, q_{rt} tending to the zero limit in Polukhin's solution but a finite nonzero limit in the present solution due to the heat stored in the scale layer. Similar differences are observed in the other comparisons. These differences appear significant because of the short contact time involved in rolling; the heat capacity in even thin scale layers can thus be quite pronounced. Since under normal rolling conditions the contact time and scale layer are in the range 0.0003 to 0.1 s and 0.005 to 0.5 mm respectively, it is crucial to include the heat capacity of the scale layer in the formulation.

5 Conclusion

An analytical solution for the transfer of heat between the strip and roll, including the effect of an oxide layer on the strip surface, has been obtained. The results are consistent with those of the previous workers when the scale layer is absent. Simple graphs have been generated for the rapid evaluation of the heat transfer and strip average temperature when the heat energy terms, scale layer thickness, and contact time are given. The scale layer has a dominant effect on the heat transfer process. It acts as an insulating layer which can reduce the heat transfer significantly (by up to 50% for typical strip thicknesses and contact times, as evidenced in Fig. 4). It has also been shown that the heat capacity of the scale layer plays an important role in hot rolling thermal analysis and therefore should not be neglected.

The solution presented here, which is applicable to both hot and cold rolling conditions, is valid when the contact time is reasonably small and the roll diameter is large compared to the strip thickness and reduction ratio such that the curvature effect may be neglected. (It has been found that the solution is applicable, for typical rolling conditions, to strip thicknesses of greater than 1 mm for hot rolling and even less for cold rolling.) The assumed uniform distribution of the frictional and deformation energy is expected to be valid in hot rolling since the overall heat transfer is dominated by the strip/roll bulk temperature difference.

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